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First published February 1995

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# INPUT-OUTPUT TABLES FOR THE UNITED KINGDOM 1990

Central Statistical Office - February 1995

Editor: Duncan Millard

London: HMSO

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# INPUT-OUTPUT TABLES FOR THE UNITED KINGDOM 1990

by Duncan Millard, Central Statistical Office

## Introduction

This article describes the 1990 derived input-output tables, based on the input-output framework underlying the 1990 national accounts. The first part provides background information on input-output including, links with the national accounts, how input-output is used, its history and some basic theory. The second part concentrates on the basic tables, the make and use matrices. The third part gives the theory behind the derived tables and presents the tables in an aggregated form. Tables in the full 123 input-output group form are available from the input-output section of the Central Statistical Office - a contact point is given at the end of the article. The tables presented here are consistent with the 1990 input-output balances published in *Economic Trends* No 480, October 1993<sup>1</sup> and with the 1993 *Blue Book*<sup>2</sup>.

## What are input-output tables?

Input-output tables display the flow of goods and services in the economy in matrix form. They illustrate the relationship between producers and consumers and the interdependence among the different industries.

## Relationship with the national accounts

Input-output tables add an extra dimension to the way the national accounts are compiled and presented. The national accounts are concerned with the composition and value of goods and services entering into final demand, and the factor incomes generated in the economic process. The national accounts do not display the inter-industry transactions which are the intermediate processes that supply the economy with final goods from primary inputs. Input-output tables show these intermediate transactions for over a hundred and twenty different industries and corresponding commodity groups. The use matrix also provides the only detailed breakdown of the production accounts for industries available in the UK national accounts. (Throughout this article the term commodity is used to mean the characteristic product of an industry group. The word commodity has become the established in input-output terminology, however readers should be aware that commodities are more correctly described as products).

In the 1984<sup>3</sup> and earlier input-output tables, the figures used to compile the tables were constrained to those already published in the national accounts. For 1990 and subsequent years the situation has changed: the input-output framework is now integrated with the national accounts. Annual input-output balances are the mechanism used to ensure consistency between the output, income and expenditure components of GDP in the compilation of the accounts (see below).

A fuller description of the link between national accounts and input-output tables, including how all 3 measures of GDP can be calculated from the tables, is in the 1990 balance article<sup>1</sup> in *Economic Trends*.

## Uses of input-output tables

There are two main uses for input-output, both of which make use of the framework that allows the whole economy to be analysed in a tabular form.

- On an annual basis, input-output balances are used to achieve consistency in the national accounts aggregates by linking the components of value added, output and final demand. It is possible to reconcile the 3 measures of GDP and produce the definitive level, without statistical discrepancies, by resolving imbalances between the supply and demand for goods and services and reconciling them with the corresponding value added estimates.
- Analytical input-output tables are used to model the economy through a disaggregated view of industrial behaviour. This allows economic questions to be answered such as: what will be the direct and indirect effect on the output of specific commodities given an increase in final demand? In addition, it is possible to quantify the import content of exports, to break down each industry's output into its ultimate primary input components, and to show how final demand is generated by the value added of the different industries.

## History of input-output tables

Input-output tables were first developed in 1936 by Wassily Leontief. In 1941, Leontief produced the first input-output tables for the US economy for 1919 and 1929. The compilation of input-output tables was initially carried out by universities and planning and research institutes solely for input-output analysis. However, in many countries, input-output tables have evolved to be the key mechanism used in checking the internal consistency of the national accounts. Some countries such as the Netherlands use the input-output framework as the basis of their national accounting system.

The first official input-output tables for the United Kingdom were for the year 1954, published in 1961. Since then, tables based upon comprehensive inquiries into purchases of materials and fuels by the manufacturing (and in later years service) industries have been published at regular intervals for the years 1963<sup>4</sup>, 1968<sup>5</sup>, 1974<sup>6</sup>, 1979<sup>7</sup>, 1984<sup>3</sup> and now 1990. Input-output balances (a purchaser price use matrix and domestic output and total supply tables) have also been published for 1989<sup>8</sup> and 1990<sup>1</sup> and we are currently working on the balances for 1991 and 1992.

## Input-output theory

The UK input-output tables follow the rules recommended by the United Nations in their *System of National Accounts*<sup>9</sup> and the subsequent volume *Input-Output Tables and Analysis*<sup>10</sup>.

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Input-output concentrates on the industry-production accounts, and a highly simplified accounting framework for input-output is shown in the diagram below:

Production sectors	Final demand	Totals
Production sectors	W	f
Primary inputs	y	q
Totals	q	

To describe input-output theory at its simplest, consider an economy with no foreign trade, no taxes and where no distinction is made between industries and commodities. Industries, referred to here as production sectors, do not engage in any secondary production, and so produce only their own characteristic products.

Final demand (f) consists of consumers' expenditure, government final consumption and capital formation. Primary inputs (y) are the factor incomes generated in the production process i.e. income from employment, self-employment and gross profits.

In the diagram, matrix W records the value of transactions between the production sectors in the economy and is known as a use matrix. A typical entry is  $w_{ij}$ , the amount bought by sector j of sector i's output. Commodity output is represented by q which, along with f and y, is a vector.

It is now possible to define the output of each production sector in terms of the amounts purchased by other production sectors (intermediate demand) and the amounts sold to final consumers (final demand).

For the whole economy we can write:

$$q_1 = w_{11} + w_{12} + w_{13} + \dots + w_{1n} + f_1$$

$$q_2 = w_{21} + w_{22} + w_{23} + \dots + w_{2n} + f_2$$

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$$q_n = w_{n1} + w_{n2} + w_{n3} + \dots + w_{nn} + f_n$$

The above set of structural equations express the input-output relations in terms of the entries in the use matrix, but the matrix in coefficient form is more useful. A coefficient matrix records not the value of each transaction, but the amount of each commodity purchased per unit of output of the purchasing sector. To form such a matrix, each column of the use matrix W must be divided by the total gross output of the purchasing sector. This coefficient matrix is denoted by A where a typical cell  $a_{ij}$  is defined as the amount of commodity i used in the production of a unit of commodity j. In algebraic notation

$$w_{ij} = a_{ij} q_j, \text{ or } A = W q^{-1},$$

where q is the diagonal matrix form of the vector q.

A new set of structural equations can now be written as follows:

$$q_1 = a_{11} q_1 + a_{12} q_2 + a_{13} q_3 + \dots + a_{1n} q_n + f_1$$

$$q_2 = a_{21} q_1 + a_{22} q_2 + a_{23} q_3 + \dots + a_{2n} q_n + f_2$$

$$q_n = a_{n1} q_1 + a_{n2} q_2 + a_{n3} q_3 + \dots + a_{nn} q_n + f_n$$

Here, each of the input-output relations is expressed in terms of a coefficient  $a_{ij}$ , expressing the input as a proportion of the output of the purchasing sector, and  $q_j$  the output of that sector. These equations can be written in matrix form as:

$$q = Aq + f.$$

Equations in this form are suitable for model-building and analysis. If the values of the coefficients are known and the level of final demand known or assumed, it is possible to solve this set of equations to find the level of output of various commodities q. This leads to the well-known Leontief equation (of which more will be said later) where  $(I-A)^{-1}$  is the Leontief inverse:

$$q = (I-A)^{-1} f \dots \dots \dots (1).$$

### The basic tables

#### Valuation of transactions

All the matrices have been valued at basic prices, as distinct from purchasers' or producer prices. This means that the purchases in the purchaser price use matrix have had distribution margins deducted and reallocated to the distribution commodity groups. Commodity and production taxes less subsidies are also deducted from purchases and redistributed to the tax row within primary inputs. The production taxes have similarly been removed from the value of goods supplied in the producer price make matrix. Imports are valued free on board plus any carriage, insurance and fr

A uniform valuation of goods and services is necessary so that the supply and demand for each input-output group balances. The basic price of a commodity is just that: the price excluding distribution margins and taxes. Because a basic price is the "true" price free from any impositions of taxation policy or non-production costs it is the preferred valuation to use for further analyses.

#### Industries and commodities

To simplify the above basic description, no distinction was made between industries and commodities (products). However, it is important that the actual difference is understood. Industries are defined using the 1980 version of the Standard Industrial Classification<sup>11</sup> and commodities are defined as the principle output of each industry. Producing units are classified to a particular industry according to which commodity they produce. If a unit produces more than one commodity, they are classified according to the commodity which accounts for the greatest part of their output. Because producing units also produce commodities that are the characteristic product of other industries, it is not possible to define the elements in the two classifications in such a way that there is a one-to-one correspondence between them. In tables 1 - 3 commodities are shown in the rows and industries are shown in the columns.

**Table 1 - The 1990 make matrix**

IO groups at 8 level (equivalent 123 groups)	£ million								
	Agric (1-3)	Energy (4-9)	Manuf (10-90)	Constrn (91)	Distribn (92-95)	Transport (96-102)	Business (103-114)	Other Ser (115-123)	Total
Agriculture	19,110	-	-	-	-	-	-	-	19,110
Energy	2	62,521	352	-	-	-	-	-	62,875
Manufacturing	23	207	270,902	-	-	-	-	-	271,133
Construction	155	1,140	417	88,389	-	-	-	-	90,101
Distribution	120	727	9,456	952	123,386	433	406	217	135,697
Transport	-	211	981	-	-	68,118	-	-	69,310
Business Services	227	478	6,217	378	1,768	1,403	150,779	2,043	163,294
Other services	73	26	324	147	355	129	1,770	154,996	157,821
<b>Total</b>	<b>19,711</b>	<b>65,310</b>	<b>288,649</b>	<b>89,866</b>	<b>125,509</b>	<b>70,083</b>	<b>152,956</b>	<b>157,257</b>	<b>969,340</b>

**Table 1: the make matrix**

This table provides a breakdown of domestic supply for each of the 123 commodities in terms of the producing industry (ie. "who makes what"). It shows for each commodity how much is produced by the industry for which it is the principal product (the diagonal entries) and how much is produced by other industries as secondary products (the off diagonal entries).

The nature (subsidiary or by-product) of the non-characteristic production is used in calculating the derived input-output tables. A subsidiary product is one with its own input structure and production process independent of the characteristic product of the industry in question. A by-product is produced as a part of the main production process (eg. sawdust from sawing logs). Using this structure industries can be transformed onto a commodity basis, so that commodity by commodity use matrices can be produced. The make matrix can also be used to transform commodities into industries to derive industry by industry use matrices.

**Tables 2 and 3: the domestic and imports use matrices**

Use matrices show the input structure of industries in terms of domestic and imported goods and services (ie. "who uses what"). They provide an analysis of primary inputs by industry, and show a commodity analysis of the categories of final demand. Sales are shown in the rows and purchases in the columns.

In the routine input-output balances, a combined use matrix is produced. This matrix makes no distinction between an input which is domestically produced or an imported one. However, for analytical work it is normal to isolate the use of domestic production. The use matrix is therefore split into a domestic use matrix (Table 2) and an imports use matrix (Table 3).

Each column in a use matrix breaks down the inputs to an industry between intermediate and primary inputs. In the domestic use matrix, apart from purchasing inputs of the products of other industries in the economy, industries also buy imports which are shown as a row in primary inputs. An industry pays wages and salaries to workers and indirect taxes to government. The excess of its output over the payment for intermediate inputs, wages and salaries and indirect taxes yields the gross operating surplus.

On the other hand, each row shows how a particular commodity is distributed to other industries, as an intermediate purchase, or as final demand to consumers' expenditure, general government final consumption (GGFC), gross domestic fixed capital formation (GDFCF), change in stocks and exports.

**Table 2- The 1990 domestic use matrix**

£ million

(equivalent 123 grps)	Agric (1-3)	Energy (4-9)	Manuf (10-90)	Constrn (91)	Distribn (92-95)	Transport (96-102)	Business (103-114)	Other Ser (115-123)	Adjustment	Tot int
Agriculture	2,874	-	9,130	3	444	31	-	88	-	12,571
Energy	554	22,494	5,608	596	2,326	2,209	1,658	711	-	36,157
Manufacturing	3,828	3,208	62,312	14,710	12,262	4,312	7,897	3,549	-	112,080
Construction	242	28	1,006	22,991	631	146	1,701	951	-	27,695
Distribution	887	1,267	12,633	2,869	4,131	2,681	2,266	773	-	27,507
Transport	273	1,345	8,896	888	11,498	8,145	10,129	1,392	-	42,564
Business Services	688	1,543	21,137	8,083	14,692	7,246	35,794	5,199	24,972	119,354
Other services	383	349	3,804	401	1,014	966	3,010	6,139	-	16,065
Total intermediate	9,729	30,234	124,526	50,541	46,999	25,736	62,454	18,801	24,972	393,993
Imports	1,470	8,069	48,013	4,318	2,705	3,028	2,472	1,421	-	71,495
Sales by f demand	22	61	1,949	159	143	211	1,240	95	-	3,881
Taxes less subs	-433	3,148	3,596	319	7,587	1,036	4,997	836	-	21,086
Income from empl	3,085	8,807	75,106	16,358	47,385	23,042	49,055	89,520	-	312,358
Gross profits etc	5,838	14,992	35,459	18,171	20,691	17,029	32,737	46,583	-24,972	166,527
Total inputs	19,711	65,310	288,649	89,866	125,509	70,083	152,956	157,257	-	969,340

	Cons exp	GGFC	GDFCF	Stocks	Exports	Total FD	Total
Agriculture	4,702	146	-	94	1,597	6,539	19,110
Energy	15,464	2,465	-	-194	8,983	26,718	62,875
Manufacturing	41,651	11,697	22,732	-1,313	84,287	159,053	271,133
Construction	5,395	4,802	51,269	881	60	62,407	90,101
Distribution	88,287	1,869	2,376	-	15,657	108,189	135,697
Transport	14,344	2,503	612	-	9,287	26,745	69,310
Business Services	21,288	6,179	8,663	-	7,811	43,940	163,294
Other services	64,718	76,132	-	-	905	141,755	157,821
Total intermed	255,849	105,793	85,652	-533	128,586	575,348	969,340
Imports	43,968	10,091	21,294	-585	2,022	76,790	148,285
Sales by f demand	5,387	-7,584	-4,360	-	2,676	-3,881	-
Taxes less subs	42,323	4,634	4,190	-	-	51,146	72,232
Income from empl	-	-	-	-	-	-	312,358
Gross profits etc	-	-	-	-	-	-	166,527
Total inputs	347,527	112,934	106,776	-1,118	133,284	699,403	1,668,743

There is very little information on the domestic/imports split of the demand for goods and services. Trade data is used for total imports, which can in some cases be supplemented to allow an allocation of imported goods to a specific category of final demand such as capital formation. There is a limited amount of industry specific

information on purchases (eg DTI data on the coal industry) which allows a more definite allocation to be made for those industries. In general, the sum of the two matrices (the combined use matrix) is significantly more reliable than the two separate analyses of the demand for domestic and imported goods and services.

**Table 3 - The 1990 imports use matrix**

£ million

(Equivalent 123 grps)	Agric (1-3)	Energy (4-9)	Manuf (10-90)	Constrn (91)	Distribn (92-95)	Transport (96-102)	Business (103-114)	Other Ser (115-123)	Tot int
Agriculture	644	-	1,930	-	387	6	-	-	2,967
Energy	63	6,284	1,342	-	59	502	10	22	8,282
Manufacturing	749	978	43,671	4,182	1,675	695	83	215	52,247
Construction	-	-	-	-	-	-	-	-	-
Distribution	-	-	-	-	5	7	-	-	13
Transport	4	757	322	29	494	1,725	259	29	3,619
Business Services	10	46	674	107	82	55	2,096	44	3,115
Other services	-	4	73	-	3	37	24	1,111	1,253
Total intermediate	1,470	8,069	48,013	4,318	2,705	3,028	2,472	1,421	71,495
	Cons exp	GGFC	GDFCF	Stocks	Exports	Total FD	Total		
Agriculture	1,083	-	-	18	40	1,142	4,108		
Energy	554	31	-	-29	-	556	8,838		
Manufacturing	31,041	7,969	21,200	-574	1,982	61,619	113,865		
Construction	-	-	-	-	-	-	-		
Distribution	5,943	-	-	-	-	5,943	5,956		
Transport	4,059	483	93	-	-	4,635	8,254		
Business Services	191	899	-	-	-	1,090	4,205		
Other services	1,096	710	-	-	-	1,805	3,058		
Total intermediate	43,968	10,091	21,294	-585	2,022	76,790	148,285		

**The derived input-output tables**

**Symmetric tables**

The text above describes the difference between commodities and industries and explains that because industries produce non-characteristic production there is not a one-one link between them. However, to transform the relationship between supply and demand for commodities from the one represented by the equation

$$q = Bg + f \dots \dots \dots (2)$$

into an equation that can be solved for q, ie

$$q = Aq + f \dots \dots \dots (3)$$

the vectors of commodity and industry output need to be identical.

In equations (2) and (3) the following notation is used:

- q is the commodity output vector;
- B is the coefficient form of the commodity by industry use matrix;
- g is the industry output vector; and
- f is the final demand vector.

To achieve the identity between commodity and industry output the use matrix needs to be transformed into a symmetric commodity by commodity (or industry by industry) version. In equation (3), A represents the coefficient form of the symmetric commodity by commodity matrix. Once equation (3) is established it can be solved to give equation (1), ie:

$$q = (I-A)^{-1}f.$$

It is possible to generate a symmetric matrix using the structure of the make matrix (to identify the non-characteristic, off-diagonal, production) together with some simple assumptions on the nature of the technology used in the production of non-characteristic products.

Two alternative basic assumptions are used - the commodity technology assumption and the industry technology assumption.

- The commodity technology assumption is that a commodity has the same input structure no matter which industry produces it (this sort of production can be thought of as subsidiary production).
- The industry technology assumption is that all commodities produced by an industry have the same input structure (this sort of non-characteristic production can be thought of as by-products). Under the industry technology assumption, commodities have a different input structure depending on which industry produces them.

If the make matrix is represented by M, the commodity output vector by q, the industry output vector by g, and the corresponding diagonal matrices by q and g, then:

$$C = M g^{-1} \dots \dots \dots (4)$$

and is known as the product mix matrix (each  $c_{ij}$  represents the amount of commodity i produced by industry j per unit level of

industry j output). It is simply the make matrix with each cell divided by the industry output relevant to that column.

$$D = M'q^{-1} \dots\dots\dots (5)$$

is known as the market shares matrix (each  $d_{ij}$  is the proportion of commodity j output produced by industry i per unit level of commodity j output). It is the make matrix transposed with each cell divided by the commodity output relevant to the new columns.

In order to treat the various elements of production according to the different technology assumptions, the make matrix has to be split into two matrices  $M_1$  and  $M_2$ :

- $M_1$  consists of all the diagonal elements plus those off-diagonal elements for which a commodity technology assumption seems most appropriate.
- $M_2$  contains those off-diagonal elements for which an industry technology assumption seems most appropriate.

The coefficient form of the commodity by commodity version of the use matrix is then given by:

$$A = B [C_1^{-1} (I - D_2'i) + D_2] = BR \dots\dots (6)$$

where i is the unit vector and

$$C_1 = M_1 g_1^{-1} \quad \& \quad D_2 = M_2' q^{-1}$$

R is known as the hybrid technology transformation matrix, because it encompasses both technology assumptions. A full description of the underlying theory of the transformations discussed in this section can be found in Technology assumptions in the construction of UK input-output tables<sup>12</sup>.

By multiplying matrix A by the commodity output vector q, the commodity by commodity matrix can be shown in value form. Table 4, created using this method, is an aggregated version of the commodity by commodity domestic use matrix for 1990. A commodity by commodity imports use matrix can be calculated in a similar manner, where the starting matrix B is the coefficient form of the imports use matrix.

**Table 4 - The 1990 commodity by commodity domestic use matrix**

	£ million								
(Equivalent 123 grps)	Agric (1-3)	Energy (4-9)	Manuf (10-90)	Constrn (91)	Distribn (92-95)	Transport (96-102)	Business (103-114)	Other Ser (115-123)	Tot int
Agriculture	2,853	0	9,059	3	536	32	0	88	12,571
Energy	550	22,203	5,377	598	2,670	2,241	1,824	694	36,157
Manufacturing	3,774	2,896	60,613	14,961	13,372	4,290	8,763	3,410	112,080
Construction	186	-	791	23,022	760	128	1,808	999	27,695
Distribution	873	1,220	12,171	2,891	4,438	2,700	2,472	742	27,507
Transport	266	1,253	7,251	846	12,956	8,091	10,599	1,303	42,564
Business Services	1,010	1,648	21,717	9,008	18,348	7,336	49,008	11,278	119,354
Other services	375	331	3,518	403	1,118	949	3,314	6,059	16,065
Total intermediate	9,888	29,550	120,498	51,732	54,197	25,767	77,789	24,572	393,993
Imports	1,453	8,005	47,084	4,334	3,190	3,019	2,999	1,412	71,495
Sales by final demand	10	52	1,899	136	109	184	1,411	80	3,881
Taxes on exp less subs	-434	3,096	2,690	273	8,427	1,022	5,271	741	21,086
Income from employment	2,966	7,918	69,787	16,703	50,599	22,953	52,776	88,656	312,358
Gross profits etc	5,228	14,255	29,176	16,923	19,175	16,365	23,047	42,359	166,527
Total inputs	19,110	62,875	271,133	90,101	135,697	69,310	163,294	157,821	969,340

	Cons exp	GGFC	GDFCF	Stocks	Exports	Total FD	Total
Agriculture	4,702	146	-	94	1,597	6,539	19,110
Energy	15,464	2,465	-	-194	8,983	26,718	62,875
Manufacturing	41,651	11,697	22,732	-1,313	84,287	159,053	271,133
Construction	5,395	4,802	51,269	881	60	62,407	90,101
Distribution	88,287	1,869	2,376	-	15,657	108,189	135,697
Transport	14,344	2,503	612	-	9,287	26,745	69,310
Business Services	21,288	6,179	8,663	-	7,811	43,940	163,294
Other services	64,718	76,132	-	-	905	141,755	157,821
Total intermediate	255,849	105,793	85,652	-533	128,586	575,348	969,340
Imports	43,968	10,091	21,294	-585	2,022	76,790	148,285
Sales by final demand	5,387	-7,584	-4,360	-	2,676	-3,881	-
Taxes on exp less subs	42,323	4,634	4,190	-	-	51,146	72,232
Income from employment	-	-	-	-	-	-	312,358
Gross profits etc	-	-	-	-	-	-	166,527
Total inputs	347,527	112,934	106,776	-1,118	133,284	699,403	1,668,743

The main focus in this article here has been on commodity by commodity tables rather than industry by industry tables. This is in line with the 1984<sup>3</sup> tables, but unlike previous tables produced for the UK. The main reason for showing industry by industry tables was that it was industry which was of interest to analysts of the United Kingdom economy. However, it can be argued that the commodity by commodity table is more in line with the assumption of homogeneous production than the industry by industry table. Since in the latter a single row may contain many products. The significance of homogeneity to input-output work makes the commodity by commodity tables a better starting point.

Industrial analyses can still be carried out using the full tables available associated to this article. A note describing a method for doing so is at Appendix 1 to this article. Further advice on how to use the commodity by commodity tables to do industrial analysis is available from input-output section of the CSO.

**The Leontief inverse**

The link between commodity output and final demand is given by the Leontief inverse repeated in the equation below:

$$q = (I-A)^{-1}f$$

Let  $l_{ij}$  represent any cell in the inverse matrix. The  $l_{ij}$  can be interpreted as the amount of gross output of commodity i needed both directly and indirectly to produce one unit of commodity j for final output. The aggregate form of this matrix (with all entries multiplied by 1,000) is given in Table 5.

**Table 5 - The Leontief inverse**

(Equivalent 123 grps)	Agric (1-3)	Energy (4-9)	Manuf (10-90)	Constrn (91)	Distribn (92-95)	Transport (96-102)	Business (103-114)	Other Ser (115-123)	Total
Agriculture	1,189.2	4.5	52.8	13.2	11.7	5.7	5.2	2.5	1,284.7
Energy	71.4	1,554.8	52.1	33.5	49.4	67.5	37.0	12.1	1,877.8
Manufacturing	337.2	109.3	1,334.2	324.6	171.3	121.6	125.4	44.0	2,567.7
Construction	20.3	2.3	9.6	1,349.2	13.0	7.1	23.4	11.0	1,435.9
Distribution	78.7	40.0	71.4	67.3	1,053.4	57.5	35.6	10.6	1,414.3
Transport	51.4	49.4	64.1	51.3	139.1	1,159.2	117.7	21.4	1,653.5
Business Services	160.4	87.9	186.0	254.7	250.7	206.7	1,476.6	119.1	2,742.1
Other services	34.1	13.0	24.9	17.8	19.2	23.5	35.3	1,043.6	1,211.2
Total	1,942.6	1,861.2	1,795.1	2,111.5	1,707.7	1,648.8	1,856.1	1,264.2	14,187.2

The data in the Leontief inverse can be interpreted in many ways. The column sum measures the direct and indirect result on the economy of a unit change in the final demand for the commodity at the head of the column. For example, using the above table, if the final demand for agriculture increased by 1,000 units the total effect on the economy would be to increase output by 1,942.6 units. The column sums, when shown in terms of a unit of domestic output, are known as the output multipliers.

Similarly, a row sum shows the total change in an industries' output of a uniform unit increase in the final demand for all commodities. If the final demand for all 8 commodities in the above table increased by 1,000 the output of the transport industry would increase by 1,653.5 units.

In equation (7) A is the direct demand for each commodity. The succeeding terms are the indirect demands resulting from each loop

of the economic process. For example, if the demand for commodity i increased there would be a direct increase in the output of commodity i. However, commodities j and k may be needed in the manufacture of commodity i and these may in turn require a certain amount of commodity i to produce them. Hence there will be a further indirect increase in the demand for commodity i, which in turn will generate further indirect demand for j and k and thus i. It can also be shown<sup>10</sup> that after 6 or 7 economic loops the indirect terms become insignificant.

The Leontief inverse is especially useful as it shows the amount of one commodity needed directly and indirectly to produce another. This can be seen by examining the power series expansion of the inverse, shown in the equation below (where A is the coefficient form of the commodity by commodity use matrix).

$$(I-A)^{-1} = I + A + A^2 + A^3 + A^4 + A^5 + \dots\dots\dots(7)$$

**Table 6 - Primary input content of final expenditure in 1990**

	£ million					
	Cons exp	GGFC	GDFCF	Stocks	Exports	Total
Imports of goods and services	70,409	14,883	33,943	-827	29,876	148,285
Sales by final demand	6,566	-7,323	-3,506	-29	4,292	-
Taxes on expend less subs	54,194	6,115	6,791	-15	5,147	72,232
Income from employment	126,422	81,918	43,811	-335	60,543	312,358
Gross profits etc	89,935	17,342	25,737	88	33,426	166,527
<b>Total inputs</b>	<b>347,527</b>	<b>112,934</b>	<b>106,776</b>	<b>-1,118</b>	<b>133,284</b>	<b>699,403</b>

**The primary input content of final demand**

By multiplying each row of the inverse by the appropriate ratio of primary input to gross output for that commodity, it is possible to generate a picture of final demand in terms of the primary inputs needed to generate it, both directly and indirectly. Repeating this analysis for all the primary inputs and summing over all the commodities produces the absolute content of final demand categories in terms of original primary inputs. This is shown in Table 6 below.

Examining the above table in percentage terms shows that the composition of expenditure groups varies substantially. For example, nearly 32% of investment (GDFCF) is made up of imports compared to just 13% of general government expenditure (GGFC). A similar comparison for income from employment, in the graph below, shows income from employment comprising three quarters of government final expenditure compared to less than two fifths for consumers' expenditure.

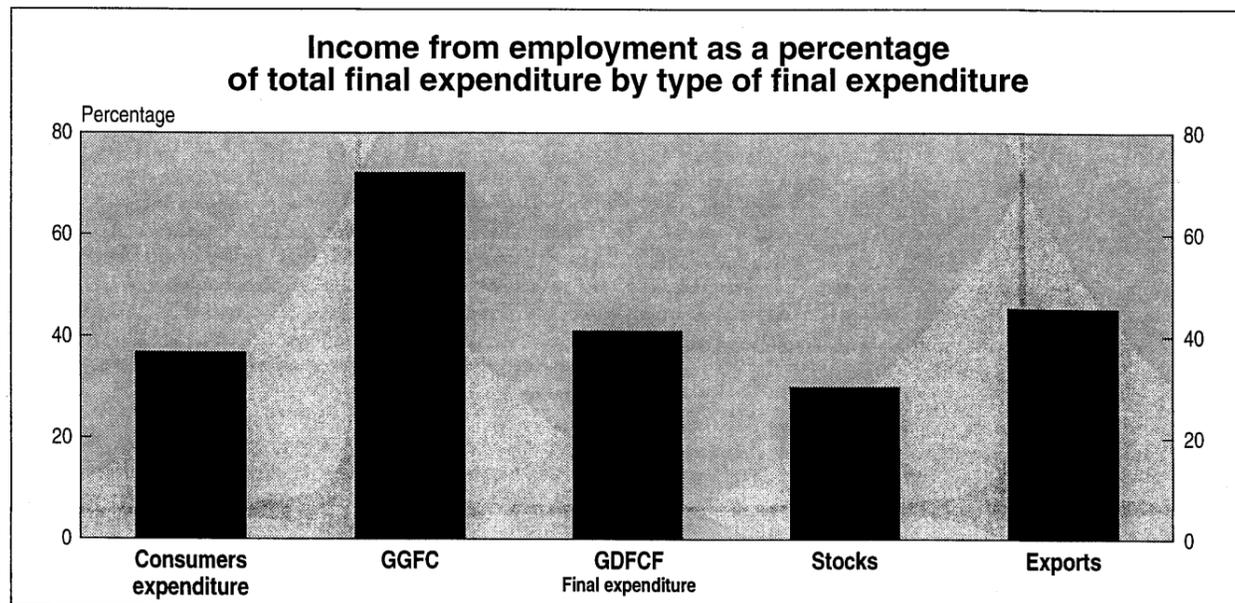
**Derived tables - general notes**

The simple Leontief model based upon input-output tables contains two major assumptions:

- Industry output can be represented as a linear combination of its inputs.
- The input-output industry and commodity groups are homogeneous.

Any significant departure from these two assumptions may affect the quality of analysis carried out.

The number of input-output groups was expanded for the 1990 balances and the table presented here to 123. The expansion was concentrated in the service sector where business services and other services have been divided up. There has also been some reorganisation of energy groups and a split in forestry and fishing. The definition of the groups in terms of the SIC(80)<sup>11</sup> for 1990 and 1984 can be found in references 1 and 3 respectively.



In the 1990 balance tables<sup>1</sup> a new treatment of general government final consumption was introduced. This entailed splitting general government purchases between the service industries of government eg education and health and including these transactions in the relevant industry columns in the use matrix. Government was then shown as purchasing all of its own output in the final demand column. For consistency with previous input-output tables the old treatment has been adopted here. This means that government is shown purchasing individual commodities in the final expenditure column.

The adjustment for financial services in the income measure of GDP as shown in the national accounts represents the net receipt of interest by financial institutions. This is, by convention, not included in the measure of output of the banking industry, which results in a negative profit for this industry. To overcome this presentational problem in the input-output tables, the adjustment for financial services has been added to the measure of profits for the banking industry and subtracted from total value added by an element in the profits row under the adjustments column. To prevent an imbalance between supply and demand, a figure for demand for this adjustment has also been placed in the adjustments column, and the measure of gross output of the banking industry in the make matrix reflects the addition of the financial services adjustment. Similar measures are taken for the financial services adjustment in insurance.

The adjustment column is economically meaningless for the input-output analyses. Therefore, before the use matrix could be transformed to a commodity by commodity version the adjustment for financial services was distributed through the banking (insurance) product row increasing intermediate consumption. A counterbalancing amount was then subtracted from the profits of each industry leaving gross output unchanged. This reallocation is not strictly accurate as the adjustment for financial services includes payments by consumers, but to adjust final demand components would require a change to the measure of GDP and this has been avoided. The method used for 1990 is equivalent to the one used for 1984 except that for 1990 the allocation of the financial services adjustment to industries was done on the basis of bank deposits and not gross output.

**General sources and methods**

A full explanation of the data sources used to construct these tables including final demand is given in the 1990 balance article<sup>1</sup>.

For a fuller description of the methods used to compile the 1990 input-output tables see Armstrong<sup>10</sup> or Bulmer-Thomas<sup>13</sup>. Previous versions of input-output tables for the UK, referenced below, may also prove useful.

**Computer readable data**

Data are available for the full 123 by 123 matrix in computer readable form and as hard-copy. The computer readable tables are available as either ASCII files or Lotus 1-2-3<sup>TM</sup> spreadsheets on 3 1/2 or 5 1/4 inch floppy disks. They can be accessed from an IBM-PC<sup>TM</sup> compatible computer running DOS 3.0 or higher. Input-output balances for 1990, 1991 and 1992 are also available in the same formats.

**Contact points**

Enquiries on data availability should be addressed to:

Sales Desk  
Marketing and Sales Branch  
Room 131/4  
Central Statistical Office  
Great George Street  
London SW1P 3AQ

Tel : 0171-270 6081  
Fax: 0171-270 4986

All other enquiries should be made to Mr Duncan Millard of the input-output section in room 132A/2 at the above address, or by phone on 0171-270 6062.

**Acknowledgement**

I would like to thank everyone within the CSO and in other Government Departments who provided the underlying data for these tables. I would particularly like to thank my colleagues in the input-output section for their help and technical advice in producing the tables.

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Appendix 1

**Note on using commodity by commodity tables to carry out industrial analysis**

The Leontief equation,  $q = (I - A)^{-1}f$ , gives the change in commodity output associated with a change in final demand. It is possible to use the largely diagonal structure of the make matrix and the Market Share Mix (MSM) hypothesis to translate the change in commodity output into a change in industry output.

The diagonal nature of the make matrix demonstrates a strong (although not one - one) relationship between commodities and industries. So a change in one will be closely mirrored by a change in the other.

The MSM hypothesis is used in the derivation of the analytical input-output tables. It is represented by the equation:

$$D = M\hat{q}^{-1}$$

where  $\hat{q}$  is the diagonal matrix form of the commodity output vector. Each  $d_{ij}$  is defined as the proportion of commodity  $j$  output produced by industry  $i$  per unit level of commodity  $j$  output, ie all the industries that produce commodity  $j$  do so in fixed ratios. This means that if industry  $i$  produces a tenth of the total output of commodity  $j$  it will also produce a tenth of any extra commodity  $j$  required to meet an increase in final demand. Matrix  $D$  is the coefficient form of the make matrix under the MSM hypothesis.

Let  $\Delta$  (the delta symbol) represent a change in. Then we have:

$$\Delta q = (I - A)^{-1} \Delta f.$$

So  $\Delta q$  is the change in the commodity output resulting from the change in final demand. Then multiplying the change in commodity output by the matrix  $D$  generates the change in industry output, given by:

$$\Delta g = D \Delta q.$$

Once the change in industry output is calculated it can be used to calculate the change in any of the primary inputs (eg. income from employment, value added etc) for each industry. For example the change in income from employment,  $I_e$ , would be:

$$\Delta I_e = I_e (\Delta g/g),$$

or in matrix notation:

$$\Delta I_e = I_e [\Delta g g^{-1}].$$

Notation:

- $q$  = the commodity output vector
- $A$  = the coefficient form of the commodity by commodity use matrix
- $f$  = the final demand vector
- $g$  = the industry output vector.

Using the method described above will produce results of comparable (if not greater) accuracy to those produced using the Leontief inverse based on the industry by industry use matrix. The calculation of the industry by industry table requires the final demand components to be transformed from a commodity to an industry basis. Apart from introducing a further transformation, this can introduce errors of classification and interpretation because final demand is clearly defined in terms of commodities not industries. It is questionable whether the question 'what is the change in output of industry A resulting from an increase of  $x$  in the final demand for industry's B output?' actually makes economic sense. It is better to formulate the question in terms of identifiable commodities and then use the above methodology to work out the associated change in industrial output.



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