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R. F. Fowler

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STUDIES IN OFFICIAL STATISTICS
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Further problems of index number construction

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The effect of sampling variability of the weights on the accuracy of the Index of Retail Prices in the United Kingdom

I Introduction*

Since January 1962 the official Index of Retail Prices has been calculated as a chain Laspeyres index with the weighting basis revised each year. The weights are derived from the expenditure of a changing representative sample of households taking part in the continuous Family Expenditure Survey. The sample of households whose expenditure is used in compiling the Index of Retail Prices covers all classes of households in the United Kingdom except

(i) those the head of which had a recorded gross income exceeding a certain limit, and

(ii) those in which at least three quarters of the total income was derived from National Insurance retirement or similar pensions and/or supplementary pensions or allowances paid in supplementation or instead of such pensions.

This sample is representative of about nine-tenths of all households in the United Kingdom. From 1962 to 1966 the sample of 'index' households whose expenditures were used to form the weighting basis of the index averaged about 3,000 a year. In 1967 this sample was doubled in size. A new sample has been selected each year in such a way that the average expenditure in each quarter of the year can be regarded as representative of the average expenditure of all 'index type' households in the United Kingdom.

2 In their Report(1) of March 1962 recommending the introduction of the present Index of Retail Prices, the Cost of Living Advisory Committee said that 'the number of households co-operating in any one year would not be large enough to provide an acceptable basis for re-weighting the index in view of the sampling error on certain items, in particular those of large unit value purchased infrequently, such as motor vehicles, furniture, carpets and other major household appliances' (para. 11). The Committee thought that these difficulties could be met by basing the weights of the index on an average of the expenditure of index households over a period of years and recommended that an average of three years covering about 9,000 households would provide an acceptable basis. Moreover such an average would have the advantage, not obtainable from a single year sample *however large*, of reducing the impact on the index of non-sampling fluctuations from year to year particularly of durable goods. Accordingly the weights of the index during any calendar year are derived from the expenditure of index households in the three years ended in the previous June valued at the prices obtaining at the beginning of the calendar year for which the index is being computed. In other words the index measures the percentage change in cost during a calendar year of buying the *quantities* of goods and services actually bought by index households during the three years ended in June of the previous year. In 1967 the Family Expenditure Survey was doubled in size and the number of index households whose expenditure is used in compiling the weights of the index has been, since 1969, about 6,000 a year or 18,000 for the three years.

3 The Cost of Living Advisory Committee's recommendation to use a moving average of three years expenditure as weights was designed, first to reduce sampling errors due to the comparative smallness of the sample of index households in any one year, and secondly to reduce the effect of abnormal variations in expenditure which might be due to certain non-sampling fluctuations from year to year particularly of durable consumers' goods. At that time it was not possible to make any real estimate of the standard error of the index of retail prices due to sampling errors in the weights nor of the effect on the index of retail prices of using a moving average of three years' expenditure as weights instead of two years' or one year's and any decision had necessarily to be based largely on informed judgment. The completion of the matrix of revaluations for the years 1958 to 1968 given in Appendices A, B and C of *Some problems of index number construction* now makes it possible to calculate exactly the effect during the ten years 1958 to 1968 of using a moving average of three years' expenditure as weights and also to make some estimates of the sampling error in the Index of Retail Prices during this period due to sampling variability in the weights.

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*This paper was completed in September 1972.

Part II of this paper is a slightly expanded version (including the effect of averaging three years' weights and including the figures for 1968) of Section III of *Some problems of index number construction*, which was published as No. 3 in the *Research Series* of *Studies in Official Statistics* (HMSO, March 1970). Part III of the paper represents the results of a study carried out during a six-month special assignment in Ottawa with Statistics Canada. The method of analysis is an expansion and development of the method used in a simple form in Appendix 1 of the Retail Prices Index Advisory Committee's *Report on proposals for retail price indices for regions* (Cmnd. 4749, HMSO, August 1971). The analysis in this section would have been impossible without the aid of a modern electronic computer. I am greatly indebted to Mr Vishnu Lagoo of Prices Division, Statistics Canada, for his help in preparing the computer input and in discussing a number of technical problems which arose in the interpretation of the results. My thanks are due also to Professor Sir Claus Moser (Head of the United Kingdom Government Statistical Service) and to Mr Harry Champion (Prices Division, Statistics Canada) for reading and commenting on the paper.

(1) *Report on Revision of the Index of Retail Prices* (Cmnd. 1657, HMSO, March 1962).

II The effect on chain indices of prices of using moving averages of weights

4 In *Research Series no. 3* an attempt was made to examine the effect on a chain Laspeyres index of retail prices of using a moving average of weights. But the analysis was confined to a comparison of chain Laspeyres indices of prices using, first, weights for one calendar year and, secondly, an average of weights for two successive calendar years and a comparison of these indices with the Index of Retail Prices using weights derived from the expenditure of three years ending in June of each year. The completion of the matrix of revaluations (carried out separately for each of the 92 sections of expenditure distinguished in the compilation of the Index of Retail Prices) now makes it possible to extend the analysis to include chain Laspeyres indices using weights based on moving averages of three years' as well as of two years' and of one year's expenditure. That is, we compile the three chain Laspeyres indices of prices:

(1) $\frac{(Q_{n-1}P_n)}{(Q_{n-1}P_{n-1})}$ ..... Single year's quantities

(2) $\frac{(Q_{n-2}+Q_{n-1})P_n}{(Q_{n-2}+Q_{n-1})P_{n-1}}$ ..... Two years' quantities

(3) $\frac{(Q_{n-3}+Q_{n-2}+Q_{n-1})P_n}{(Q_{n-3}+Q_{n-2}+Q_{n-1})P_{n-1}}$ .. Three years' quantities

The three sets of indices—all strictly comparable with each other—have been compiled for each of the years 1961 to 1968 for (a) All Items, (b) Food and (c) Transport and Vehicles. The results are given in Table 1.

5 For All Items and for the Food Group, the effect on the indices of prices of using a moving average of three years' expenditures as weights is very small. For All Items the index records a rise of 31·42 per cent between 1960 and 1968 using three years' expenditure as weights, compared with a rise of 31·28 per cent using two years and 31·61 per cent using one year. For Food the rises are 27·23, 27·38, and 27·31 respectively. For Transport and Vehicles, however, the differences are bigger, as one would expect in view of the very large year-to-year variations in expenditure and quantum(²). Averaging of weights raises the index for Transport and Vehicles from 117·32 in 1968 (with 1960 = 100) using one year's expenditures to 118·42 using two years and to 120·51 using three years. Even so, the differences are surprisingly small. The table also shows the percentage changes from year to year. For All Items, the biggest difference made by using three years' expenditures instead of one year's expenditures as weights was 0·31 in 1965. For Food the biggest difference was 0·13 in 1964. For Transport and Vehicles, the biggest difference was 1·69 in 1961 but in 1967 and 1968 the difference was less than 0·08.

6 When prices are changing at uneven rates a moving average of weights used in calculating a chain Laspeyres price index will have the effect of producing a lag in the adjustments to the relative quantities associated with changes in relative prices. A Laspeyres index of the change in prices between two consecutive years, using an average of two or three years' expenditures as weights will therefore tend to show a greater rate of change in the average level of prices than that shown by an index using only one year's expenditures as weights. On the other hand the weights are subject to sampling and other forms of error which will sometimes be such that the index using a moving average of weights will show a smaller, instead of a larger, rate of change of prices than that shown by an index using only one year's expenditures as weights. Moreover in the calculation of a dynamic index measuring changes in prices over time, changes in income may outweigh the usual substitution effects so that, even in the absence of sampling errors in the weights, an index using a moving average of weights may show a smaller change than an index using one year's weights. These different influences will, in general, be confounded. For Transport and Vehicles the substitution effect clearly predominated particularly in the period 1961 to 1964. The Laspeyres price index with fixed 1958 weights rose by 38·8 per cent between 1958 and 1968 while the Paasche index with 1968 weights rose by only 5·7 per cent. The sub-group index for public transport fares rose more than the sub-group index for motoring and cycles while there was an increase in the relative weight for motoring. This explains why averaging of weights raises the index for Transport and Vehicles in each of the years 1961, 1962 and 1964. For Food and All Items the effect of averaging is smaller but the figures in Table 1 suggest that substitution effects were more important in the period 1961 to 1964 than in 1965 to 1968.

7 The results of this further analysis confirm the conclusions reached in *Research Series no. 3* that the use of a three-year moving average of expenditures in compiling a chain Laspeyres index of retail prices during the period 1958 to 1968 makes very little difference compared with the use of single year or an average of two years, provided that the index covers a group of expenditures with a fairly large number of items. Where the group covers a small number of items exhibiting considerable variability in movements of prices and of quantities bought, such as Transport and Vehicles, the effect of averaging weights is significant but is nevertheless smaller than might have been expected.

(²) See Table C3 of *Research Series no. 3.*

8 To the extent that averaging of weights from successive years reduces the effect on the index of prices of sampling error in the weights, the results of this analysis appear to suggest that the effect of sampling error in the indices for All Items and for large and relatively homogeneous groups such as Food is small. But these results do not provide any estimate of the standard error of the price index due to sampling error in the weights since, as shown in paragraph 6, the process of averaging successive years confounds two separate and distinct effects. It is necessary therefore to find a more direct method of estimating this standard error.

III Sampling error of price indices due to sampling error in weights

9 The sampling errors in indices of retail prices arise from two sources, *viz.*, the weights and the price quotations used. Since the Family Expenditure Survey is a sample survey sampling variations in the weights derived from expenditures recorded in the Survey will affect the index even if the prices used are without error. Similarly the price quotations used in compiling the index are only one possible sample and sampling variations in the prices used will affect the index even if the weights used are without error. In addition, of course, there are non-sampling errors in both weights and prices which may often be more important than errors due to sampling variability. In this section attention has been confined to the effect on the Index of Retail Prices of sampling variability in the weights and the analysis proceeds on the assumption that there is no error in the price relatives used(3). For any given distribution of price relatives between two unit time periods t_0 and t_1 there will be a distribution of estimates of the average change in prices varying with the sampling distribution of the weights used. This will be true even if the given price relatives are without error. It should be noted that the effect on the index of prices of sampling error in the weights is not independent of the distribution of price relatives. If all prices rose in the same proportion, sampling error in the weights would have no effect on the index. On the other hand sampling error in prices will affect the index even if there is no error in the weights(4). The magnitude of the standard error of the price index due to sampling variability of the weights depends therefore on the dispersion of price relatives and there is no reason to suppose that the effect of any given sampling error in the weights will be related in any simple way to the magnitude of the average percentage change in prices.

10 One way (the simplest) of estimating the sampling error of the Index of Retail Prices due to sampling variability of the expenditure weights derived from the Family Expenditure Survey would be to divide randomly, within each stratum, the sample of household budgets used in compiling the weights, into two or (preferably) more sub-samples quarter by quarter. Each year there would then be two or more random samples of weights which could be used, in conjunction with the given set of price relatives to compile two or more independent estimates of average price change (either Laspeyres or Paasche). This would require the production of two or more tables similar to Appendix A and two or more tables similar to Table 2a. The cost of producing such tables for the years 1958 to 1968 would be unacceptably high but their production could be incorporated easily in a revised computer programme in the future. The disadvantage of this method however is that the estimated variances will be subject to a large margin of error owing to the small number of degrees of freedom available.

11 The second method would be to estimate the standard error of the index due to the sampling variability in the expenditure weights using a similar, but more extensive, method to that employed in estimating standard errors of

(3) The Retail Price Index is, in fact, calculated as a weighted average of relative price changes:

$$\frac{\sum w_o r_n}{\sum w_o}$$

where w_o represents average expenditure $q_o p_o$ in a base period and r_n is a price relative $\frac{p_n}{p_o}$. The expenditure weight $q_o p_o$ is obtained from the Family Expenditure Survey and is subject to sampling error. The price relatives r_n are obtained independently from a (non-random) representative selection of retail outlets and the price relatives are obtained directly as the percentage changes in prices for the same items each month. That is, the price relative is not the ratio of prices p_n and p_o obtained from independent random samples. Strictly speaking, the sampling error of the weights is the net effect of variations in quantities bought and prices paid by the sample households included in the Family Expenditure Survey. The FES however does not obtain separate information about quantities and prices paid. In this study the price relatives r_n have been assumed to be without error and attention has been concentrated on the effects on the index of prices of sampling variability in the expenditure weights. It is convenient to express the results of revaluing expenditure weights in a given year t_o at the prices of another year t_n in the form

$$\frac{\sum q_o p_n}{\sum q_o p_o}$$

and to say that we are examining the effects on the index of sampling variability in the q_o. But it must be remembered that this means that the effect of sampling variability of the (implicit) prices in the expenditure weight $q_o p_o$ is included in the estimated variance of the index due to sampling variability of the weights.

(4) The converse is true in compiling indices of quantum. If all quantities increase in the same proportion, sampling errors in the price relatives will have no effect on the index of quantum while sampling error in the quantities will affect the index even if there is no error in the prices.

each item of expenditure as published in the annual reports of the Family Expenditure Survey. This method involves some computational difficulties and requires the calculation of a very large number of covariances, in addition to the variances normally required.

12 The method adopted in this paper uses variability between total expenditures of different samples of households, thus avoiding the necessity of computing covariances, and can be thought of as a variation of method 1. Although the method involves a large amount of computation it is thought that it is computationally easier than that involved in method 2. Apart from its interest as an alternative method it appears to have some advantages and there is no reason to suppose that the final estimates of standard error will differ significantly from those obtained by either method 1 or method 2.

13 The revaluations at prices in each of the years 1958 to 1968 given in Appendix A provide a means of compiling estimates of standard errors which, though subject to qualifications, can nevertheless be regarded as reasonably reliable. From Appendix A we can compile 110 indices of the form

$$\frac{\sum Q_m P_n}{\sum Q_m P_{n-1}}$$

where m varies from 1958 to 1968 and n from 1959 to 1968. The 110 indices computed in this way are given in Table 2a for All Items. For any index, say 1959 compared with 1958, there are 11 estimates obtained by using the 11 different weights appropriate to each year. The first will be $\frac{\sum Q_{58} P_{59}}{\sum Q_{58} P_{58}}$ and the last will be $\frac{\sum Q_{68} P_{59}}{\sum Q_{68} P_{58}}$. The differences between the indices in any column cannot however be ascribed solely to sampling fluctuations in the weights from year to year since there will have been systematic shifts between 1958 and 1968 due to long term movements in relative prices and changes in demand due to other factors. Table 2a shows that for most years (though not for 1967 and 1968) there is a clear downward movement, greater for some years than for others. The successive yearly weights cannot therefore be regarded as random samples from an unchanging universe and the successive estimates of the change in prices between two consecutive years cannot be used as such to provide an unbiased estimate of the sampling error of the price index for these two years due to sampling variability of the weights. If there were no systematic shift in the ten successive estimates of the change in prices between any two successive years, these estimates could be regarded as reflecting random fluctuations in the weights due to sampling variability together with any irregular (non-systematic) fluctuations. It is therefore necessary to examine the possibility of removing the influence of the systematic shifts from the estimates given in Table 2a.

14 It is clear from Table 2a that the extent of the downward shift varies from column to column and does not appear to be related to the magnitude of the price change between two successive years. The downward shift is to be expected since it reflects the tendency for an index of price change between two periods to be lower if computed with weights of a later rather than of an earlier period. It will be seen that the chain Paasche indices (shown in italics) are lower than the chain Laspeyres indices (shown in bold type) in all columns except the last two. Over the period there will be a general tendency towards substitution of items whose prices have risen less than average. But income changes and changes in taste may compensate or more than compensate. The eleven estimates of the price change between 1967 and 1968 show no shift either upwards or downwards.

15 In most columns, but not all, the downward shift appeared to be linearly related to time. Regression equations were fitted, by least squares, of the form

$$y = a + bx \text{ and also } \log y = a + bx$$

The sample of households whose budgets were used as weights varied slightly from year to year and was doubled in 1967 and 1968. The regressions were therefore computed by minimizing the sum of *weighted* residuals. Furthermore the coefficient of serial correlation was computed by least squares and the regression equation recalculated successively after adjusting for autoregressive schemes of order 1 to 8. The regression equation selected was that for which the absolute value of the coefficient of serial correlation was smallest. It was found that generally the linear equation (computed by least squares) gave a better fit than the logarithmic equation and that where the curvilinear regression appeared to be better the difference was small and probably not significant. There appears to be no *a priori* reason why the relationship should be curvilinear and the further analysis (in paragraphs 18 *et seq.* below) for changes in prices over longer periods than a year appears to support the assumption of a linear relationship. It was therefore decided to use only the results given by the linear regressions. These are set out in Table 2b for All Items. Similar calculations using both linear and logarithmic regressions were made for Food and Transport and Vehicles and the results given by the linear regressions are set out in Tables 3b and 4b respectively.

All Items Indices

16 It will be seen, from Table 2b, that the standard error of the percentage change in the All Items index of prices between two successive years due to sampling variability of the weights derived from samples of 18,000 households varied from 0·0234 to 0·1022. It is noticeable that there appears to be no clear relationship between the magnitude of the standard error for changes in prices between two successive years and the magnitude of the change in the average level of prices between those years.

17 Since the households were selected by a random process within strata each year it may be reasonable to assume that for any given set of price relatives between two successive years say 1958 and 1959, the eleven successive estimates (after eliminating systematic trend and serial correlation in the residuals) can be regarded as representing the results of using eleven independent random samples of weights. We cannot, however, assume that the differences between the ten estimates of standard error for each of the ten sets of

price relatives would have arisen from chance only. For the ten sets of price relatives between successive pairs of years cannot be assumed to represent a random selection from a universe of all possible sets of relative prices between pairs of years([5]). In fact the application of the Bartlett test of the significance of differences between several sample variances([6]) shows that for All Items the probability that the differences are due to sampling fluctuations is less than 0·01. The interpretation of the results is therefore that for each of ten sets of price relatives between pairs of successive years from 1958 to 1968, varying from + 0·86 per cent on average to + 5·27 per cent on average there is an estimate of the standard error due to sampling variability of the weights and this varies from 0·0234 to 0·1022 when the weights are based on samples of 18,000 households. The differences between the ten standard errors are too great to be ascribed only to chance and the mean standard error of 0·0770 (weighted by degrees of freedom) cannot be taken to be an unbiased estimate of the standard error in the universe of all possible sets of price relatives.

18 The analysis in Table 2b gives estimates of standard errors due to sampling variability of the weights for changes in prices from one year to the next. There is no reason to suppose that the standard error is unaffected by the length of the period over which the price relatives are measured. The dispersion of price relatives and the correlation between the price relatives and the residual 'random' fluctuations of the weights from 1958 to 1968 will almost certainly vary according to the period of time over which the price relatives are measured. Since the absolute difference between Laspeyres and Paasche indices increases with the length of time over which the prices are being compared and the magnitude of the increase in prices it is to be expected that both the regression coefficient measuring the 'shift' and the absolute value of the standard error will increase. From the figures for the eleven years 1958 to 1968 it is possible to compute ten separate sets of price relatives over periods of one year (the denominator being the earlier of the two years), nine sets over periods of two years and so on making 55 separate sets of price relatives([7]). A similar analysis to that carried out in Table 2b (including adjustment for serial correlation) can therefore be applied to these 55 sets of price relatives yielding estimates of standard error corresponding to each of these 55 price comparisons. Since it was clear that over longer periods of time than one year the regression was linear, only straight line regressions were computed.

19 The standard errors for each of these 55 separate price comparisons for All Items are given in Table 2c. Horizontally each line shows the movement over time of the standard error for Laspeyres price indices with reference base 1958, 1959 . . . to 1967 respectively. Vertically each column shows the movement of the standard error for indices measuring changes from varying base years of prices over periods of one year, (given in Table 2b) two years, three years . . . to ten years.

20 Particular interest attaches to the standard errors and regression coefficients for comparisons for each year from 1959 to 1968 with 1958 as base or denominator, that is the Laspeyres index with 1958 weights. The results of the analysis are given in Table 2d in the same detail as those given for the year-to-year changes of a chain Laspeyres index given in Table 2b. It will be seen that the straight line regression accounts for a very large proportion of the variance. The negative value of the regression coefficient, as expected, increases steadily throughout the period. The standard error however, though higher in the second half of the decade, fluctuates violently. By 1963 when the 1958 base-weighted Laspeyres index had recorded a rise of 11·72 per cent, in prices the standard error was 0·1722. The standard error then fell to 0·1518 by 1967 when prices had risen to 29·45 per cent above 1958 and rose again to 0·1789 in 1968 when prices were 35·64 per cent above the 1958 level.

21 One of the most interesting results shown in Table 2c is that although the standard error is generally greater the longer the period of time over which the price changes are measured, it is still less than 0·20 by 1968 when prices had risen by 35·6 per cent above the 1958 level.

Food Group

22 It will be seen from Table 3b that sampling variations in the weights used have only a small effect on the estimates of the change in the general level of food prices between successive years. The standard error of the food price index for samples of 18,000 households varies from 0·0174 for the slight fall in food prices between 1959 and 1960 to 0·0780 for the increase of 3·81 per cent, in prices between 1961 and 1962. But again there appears to be no relationship between the size of the standard error and the size of the change in the general level of food prices. Although the regression coefficients are all negative they are small and with a few exceptions, statistically not significant when tested individually. The Bartlett test gives $\frac{M}{C} = 21{\cdot}10$ for 9 degrees of freedom and shows that as for All Items the probability that the differences between the ten estimates of standard error are due to sampling fluctuations is less than 0·01. Although the weighted mean of the ten standard errors (0·0464) is less than the weighted mean of the ten standard errors given in Table 2b for All Items (0·0770) the standard errors for Food are greater than those for All Items for changes between 1958, 1961 and 1967 and the respective succeeding years.

([5]) This would be so even if the price relatives between any two successive years had been selected by a random process. The price quotations used in compiling the Index of Retail Prices were not selected by a random process but were obtained from the same retailers and other sources throughout the year and were selected to provide price relatives which taken together would be representative of all price relatives during the year.

([6]) For All Items in this example

$$k = 10$$
$$M = 28{\cdot}43766$$
$$C = 1{\cdot}064614$$
$$\frac{M}{C} = 26{\cdot}71$$

and for $g = 9$, $P =$ less than 0·01.

([7]) There will also be 55 sets of price relatives where the denominator is the later of the two years. But these will be simply a mirror of the first set.

23 Table 3d shows that for the 1958 base weighted Laspeyres index of food prices the standard error moves in a similar way to that shown in Table 2d, rising to 0·076 by 1965, then declining before finally rising to 0·072 by 1968 when prices had risen by 28·34 per cent. It will be seen that the regression coefficients are, with two exceptions, highly significant and remove a large proportion of the total variance. Table 3c shows that the standard error rises only from an average of 0·0407 over periods of one year to 0·0718 over periods of ten years in 1968 when the Food Group index had risen to 128·34.

Transport and Vehicles Group

24 The analysis of the movements of the price index for the Transport and Vehicles Group is of particular interest in view of the relatively large sampling errors in expenditure on the purchase of motor cars and the abnormal year-to-year variations in expenditure due to non-sampling causes. Table 4a shows that the general downward trend in the estimates is accentuated, being particularly large for the price change between 1960 and 1961. But for the change between 1967 and 1968 there is a small upward shift. The regression coefficients, after adjusting for serial correlation, in Table 4b confirm the conclusion that there was a fairly high degree of negative correlation throughout this period between changes in relative price and changes in relative quantities. This explains why there was such a large difference between the 1958 weighted Laspeyres price index for 1968 (138·82) and the Paasche index with 1968 weights (105·70). This high negative correlation occurred because, during this period, there was a significant shift from public to private transport associated with an average price rise for private transport which was less than the price rise for public transport.

25 Not unexpectedly the standard errors due to sampling variation in the weights are, in general, higher than those found for the Food group and for All Items. The standard errors (for samples of 18,000 households) varied from 0·457 for the small change in prices between 1960 and 1961 to 0·050 for the rise of 6·31 per cent between 1967 and 1968 with a weighted mean standard error of 0·2451. Again there is no apparent relationship between the magnitude of the standard errors and the magnitude of the price changes when taken over periods of one year. The Bartlett test gives $\frac{M}{C} = 47{\cdot}07$ and the probability that the differences between the ten estimates of variance were due to chance was less than 0·0001.

26 As for All Items and Food, the standard errors for Transport and Vehicles increase, as the interval of time over which the price relatives are measured increases, from an average of 0·2034 for intervals of one year to 2·6428 for intervals of ten years (Tables 4c and 4d).

Standard error of a chain index

27 Since the Index of Retail Prices in the United Kingdom is a chain Laspeyres index with weights changed every year, it is important to know how the standard error of a chain index with weights changed every year compares with that of a base weighted Laspeyres index for periods of time extending over several years. If the errors in each link of the chain index were uncorrelated an estimate could be obtained without difficulty from the figures in Table 2b by using the formula

$$\left(\frac{\sigma_{u_n}}{u_n}\right)^2 = \sum_{t=0}^{t=n}\left(\frac{\sigma_{x_n}}{x_n}\right)^2$$

where σ_{x_n} represents the standard errors for each pair of years, x_n represents the year-to-year percentage changes (+ 100) in the chain index and u_n the chain index for the nth year. The standard error of the chain Laspeyres index for 1968 of 133·86, calculated in this way, is 0·2618. However an examination of the data shows that there is some correlation and that the covariance terms cannot be neglected. Accordingly the standard error of the chain index was estimated by calculating sums of squares and cross-products using the weighted residuals from the fitted regression equations for each pair of years given in Table 2b (i.e. after adjusting for autoregression) and using $\sum v^1 - k^1$ degrees of freedom where v^1 is the number of degrees of freedom of the residual variance in Tables 2b, 3b and 4b and k^1 (the number of additional unknown parameters i.e. cross-products estimated in successive years) takes the value 1, 2, 3 . . . 9 successively from 1960 to 1968. The results are set out in Table 5 for each year together with the standard errors for the 1958 weighted Laspeyres index of prices. The table includes also similar figures for the Food and the Transport and Vehicles Group([8]).

28 It will be seen that for All Items and for Food the estimated standard errors for the chain index and the 1958 weighted Laspeyres index are very close together. Both rise absolutely as the interval of time over which comparisons are made increases. For Transport and Vehicles the standard error of the chain index appears to be less than that of the 1958 weighted index during the last half of the decade. The differences can be tested for significance by means of the F ratio and it then turns out that not one of the differences shown for either All Items or for Food is significant at the 5 per cent point. For Transport and Vehicles however the differences for 1962, 1964, 1965 and 1966 are significant at the 5 per cent level.

Reliability of the estimates of standard error

29 All the estimates of standard error obtained by the method described above are themselves subject to both sampling and non-sampling errors. If the estimates are to be of use it is essential to have some measure of their reliability however approximate. Although the differences between the

([8]) Irma Adelman's paper *A New Approach to the Construction of Index Numbers* in the *Review of Economics and Statistics*, August 1958, contains an estimate of the variance of an index number of retail food prices compiled from a randomly selected sample of commodity prices and advocates the use of a chain index with samples of prices selected at frequent intervals. The article includes a formula, provided by R. Dorfman, of the ratio of the variance of an annual fixed base index to the variance of a chain index over the same period compiled from probabilistic samples of prices selected each month. This should not be confused with the estimates of standard error given above for chain and fixed base indices which reflect only the effect of sampling variations in the expenditure weights, price relatives being assumed to be without error.

Reliability of estimates of standard error

Indices for 1968 (1958 = 100)	Estimated standard error s_1	Degrees of freedom v_1	Upper fiducial limit of 95% confidence range $\bar{s}$†	Difference $\bar{s} - s_1$
	(1)	(2)	(3)	(4)
1958 weighted indices				
All Items	0·1789	4	0·4245	0·2456
Food	0·0719	4	0·1697	0·0979
Transport and Vehicles	2·6428	9	4·3506	1·7078
Chain indices				
All Items	0·2178	17	0·3049	0·0871
Food	0·0864	21	0·1162	0·0298
Transport and Vehicles	1·7625	21	2·3712	0·6087

†Obtained by putting $\frac{\bar{s}^2}{s_1^2}$ = value of F at the 0·05 level with $v_2 = \infty$.

This is equivalent to finding $\bar{s}$ from $\frac{s_1^2 v_1}{\bar{s}^2}$ = value of χ^2 for $P = 0{\cdot}95$ and v_1 degrees of freedom.

estimated standard errors for chain indices and for base weighted Laspeyres indices have been shown to be not significant at the 5 per cent point (except for Transport and Vehicles) the estimates of the standard errors are not equally reliable since the estimates for the 1958 weighted indices are based on a much smaller number of degrees of freedom. We are interested primarily in the possibility that the estimates of standard error are too low and that the true population values are greater than estimated. An appropriate method is to use the variance ratio to find the upper fiducial limit of the 95 per cent confidence range. These estimates have been calculated for the estimated standard errors of (a) the 1958 weighted and (b) the chain indices for 1968, for All Items, for Food and for Transport and Vehicles and are given in the Table above.

30 If the standard errors of the 1958 weighted price index for any given year and the chain index for that year can be assumed to be independent an estimate of the standard error of their difference can be made by multiplying by 1·4142 the higher of the standard errors in column (1) of the Table above. As an example the estimated standard errors of the difference are given for the indices for 1968 for All Items, Food and Transport and Vehicles in the Table below which compares them with the differences between the two types of index for 1968. The results suggest that the differences between the 1958 weighted indices and the chain indices for 1968 are real and not due to sampling error of the weights.

Indices of prices for 1968 (1958 = 100)

	1958 weighted	Chain index	Difference	Estimated standard error of the difference
All Items	135·640	133·863	1·778	0·308
Food	128·340	128·810	0·470	0·122
Transport and Vehicles	138·817	119·322	19·495	3·737

IV Conclusion

31 This paper has been limited to the consideration of two questions concerned with the effect of variability of the weights on the accuracy of the official General Index of Retail Prices in the United Kingdom. The first question, dealt with in Section II concerns the effect on a chain Laspeyres index of retail prices, with weights changed every year, of using a three-year moving average of weights derived from the results of the continuous Family Expenditure Survey in the United Kingdom. Section III represents an attempt to obtain an estimate of the standard error of the Index of Retail Prices due to sampling variability of these weights. In both sections of the paper the methods of analysis used are dependent on the availability of the matrix of revaluations (Appendices A, B and C) which is an important by-product of the Family Expenditure Survey.

32 The results of the analysis in Section II suggested that the use of a three-year moving average of weights in compiling a chain Laspeyres index of retail prices in the United Kingdom made less difference than might have been expected. So long as the index covers a large group of items, such as All Items or Food, not subject to violent year-to-year fluctuations in expenditure, averaging of weights has only a very small effect both on the level of the index after several years and on the magnitude of the year-to-year percentage changes. For Transport and Vehicles the effect of averaging is significant although it is smaller than might have been expected in view of the violent year-to-year fluctuations in expenditure during the period covered. A three-year moving average confounds the effect of averaging real changes over time with the effect of increasing the size of the sample of households from whose expenditures the weights are derived. It is necessary therefore to find a more direct method of estimating the standard error of the index due to sampling variability of the weights.

33 In order to obtain the estimates of the standard error due to sampling variability of the weights, given in Section III of the paper, a method was used which treated the weights from 1958 to 1968 obtained from the Family Expenditure Survey as a random sample by fitting regression lines to eliminate the systematic changes, functionally related to time, and to eliminate as far as possible any serial correlation in the residuals. It has been stated above that, in the absence of the information required to produce accurate estimates, this method of analysis cannot be regarded as providing more than an approximation of these standard errors. However, in spite of the limitations associated with this indirect method the results seem to be sufficiently reliable to permit some important conclusions to be drawn about the magnitude of the effect of sampling variability of the weights on the Index of Retail Prices. It is, therefore, all the more important to be aware of the assumptions, both explicit and implicit, in this analysis and of the limitations on any conclusions that can be drawn from the results.

34 For any one set of price relatives, say 1958 to 1959, eleven estimates of the average price change between 1958 and 1959 were computed by applying to these price relatives the eleven sets of weights derived from the expenditures recorded in each of the years 1958 to 1959 by samples of households co-operating in the Family Expenditure Survey. The samples of households were selected by a random process each year independently of each other. The application of the eleven sets of weights derived from these eleven randomly selected samples to the given set of price relatives produced eleven estimates of the average change in prices between the two years. Although the eleven samples of households were selected by a random process each year, the eleven sets of weights obtained from their recorded expenditures cannot be assumed to represent a set of randomly selected weights. For they will exhibit variations which reflect both random and non-random factors among which one of the most important is the constant adjustment *over time* of the relative quantities of different items bought to changes in relative prices. The eleven estimates of price change between 1958 and 1959 will therefore also exhibit variations which reflect both random and non-random factors. It is immediately obvious from an inspection of the resulting estimates of price changes obtained in this way that one of these non-random factors is responsible for a general downward shift in the estimates when these are listed in order from 1958 to 1968 according to the year to which the weights relate. This downward shift occurs because over the period 1958 to 1968 expenditure will have been adjusted to the changes in relative prices that occurred during the period. The stronger the degree of negative correlation between changes in relative quantities and changes in relative prices the greater this shift. It follows that the magnitude of the shift will vary according to the two years to which the price relatives relate and may not always be downward.

35 The fitting of the regression equations was designed to remove the effects of this non-random variation over time in the relative quantities of the items included in the weights for each of the years 1958 to 1968. The residuals from the fitted regression equations contain the effect not only of random variations in the weights but also of any non-

random factors other than that removed by the regression equation. To the extent that non-random factors that operate in a time dimension are reflected in serial correlation the choice of the regression equation after adjustment for autoregression removes or reduces their influence. But even though regression equations have been fitted for which serial correlation was generally very small it must be remembered that the residual variance contains the effect of any other non-random variations.

36 For any *one* set of price relatives, say between 1958 and 1959, the eleven estimates of the average price change after the removal of the systematic shift by the use of a regression equation selected after adjustment for serial correlation, have been *assumed* to be randomly distributed and the variance has been assumed to be a measure of the effect of sampling variability of the weights on the accuracy of the index when computed over this particular period of time.

37 Since the standard error of the price index due to sampling variability of the weights is not independent of the dispersion of the price relatives, it follows that the standard error will vary from one set of price relatives to another. Although it might be reasonable to suppose that sampling variability of the weights estimated in the way described does not vary significantly from one year to another, at least over short periods of a few years, the variability from one year to another of the dispersion of prices is almost certainly greater and more complex than the sampling variability of weights. The dispersion of prices will vary from year to year depending on the many factors responsible for price changes, and this variation will be non-random at least over relatively short periods of time such as 10 years.

38 The results of the analyses in Part III of this paper demonstrate quite clearly the over-riding importance of dispersion. Table 6 shows that the standard errors for the indices of Food prices are generally much smaller than those for Transport and Vehicles even for similar percentage increases in prices, reflecting the fact that during the period 1958 to 1968 the dispersion of Food price relatives was much smaller than the dispersion of the price components of the Transport & Vehicle group index. Furthermore it is clear that for both these groups and for All Items the standard error varies in a non-random manner according to the period of time over which the price comparisons are made. Though it is possible to compute estimates of standard error of the average change in a *given* set of price relatives over a *given* time period it is not possible to say what the standard error will be for any period in the future unless some assumption is made about the dispersion and the magnitude of the price relatives in this future period. Nevertheless it would seem safe to say that if conditions of price change in the future are similar to those which obtained in the period 1958 to 1968 the standard errors are likely to fall within the ranges given in the tables.

39 Although it might have been expected that the standard error of a chain index, due to sampling variability of the weights, would be greater than the standard error of a base-weighted Laspeyres index the results of the calculations given in Table 5 suggest that this is not so. Both for Food and for All Items the standard error of the chain-index was generally slightly greater but the differences were not statistically significant at the 5 per cent point. For Transport and Vehicles however the chain index had smaller standard errors than the 1958 weighted Laspeyres index, and some of these differences were significant. It would appear to be a safe conclusion that the standard error of a chain index due to sampling variability of the weights does not differ significantly from that of a base weighted Laspeyres index for periods of comparison up to ten years.

Chain Laspeyres Indices of Prices: Effect of averaging quantity weights

Table 1

	All Items			Food			Transport and Vehicles		
	Single year weights	2 years' weights	3 years' weights	Single year weights	2 years' weights	3 years' weights	Single year weights	2 years' weights	3 years' weights
	Average 1960 = 100								
1961	102·8623	103·0059	103·0808	101·2502	101·3161	101·3117	100·3744	101·4784	102·0612
1962	106·6941	106·9092	107·0714	105·1491	105·3087	105·2950	101·1178	102·3057	103·5005
1963	108·7999	108·9773	109·1453	107·6989	107·9281	107·8574	100·9841	101·4921	102·8603
1964	112·2980	112·4793	112·6315	110·8438	110·9991	110·8691	101·5317	102·5309	104·0583
1965	118·2185	118·1132	118·2181	114·8732	115·0350	114·8936	105·9797	106·6339	108·4892
1966	122·7433	122·4198	122·5374	119·0313	119·1175	118·9608	108·5471	109·5672	111·4451
1967	125·6716	125·3465	125·4956	121·9867	122·0690	121·9510	110·3517	111·3945	113·3828
1968	131·6090	131·2788	131·4214	127·3092	127·3785	127·2286	117·3160	118·4159	120·5072
	Year to Year changes per cent.								
1961	+2·8623	+3·0059	+3·0808	+1·2502	+1·3161	+1·3117	+0·3744	+1·4784	+2·0612
1962	+3·7252	+3·7894	+3·8713	+3·8508	+3·9407	+3·9317	+0·7406	+0·8152	+1·4102
1963	+1·9737	+1·9344	+1·9369	+2·4249	+2·4874	+2·4335	−0·1322	−0·7953	−0·6185
1964	+3·2152	+3·2135	+3·1941	+2·9201	+2·8454	+2·7923	+0·5423	+1·0235	+1·1647
1965	+5·2721	+5·0088	+4·9601	+3·6352	+3·6360	+3·6300	+4·3809	+4·0017	+4·2581
1966	+3·8275	+3·6462	+3·6537	+3·6197	+3·5489	+3·5400	+2·4225	+2·7508	+2·7246
1967	+2·3857	+2·3907	+2·4141	+2·4829	+2·4778	+2·5136	+1·6625	+1·6677	+1·7387
1968	+4·7245	+4·7327	+4·7219	+4·3632	+4·3496	+4·3276	+6·3110	+6·3032	+6·2835

Indices of Retail Prices: All Items

$$\frac{\sum Q_m P_n}{\sum Q_m P_{n-1}}$$

Table 2a

$\frac{n}{n-1}$ M	$\frac{P_{59}}{P_{58}}$	$\frac{P_{60}}{P_{59}}$	$\frac{P_{61}}{P_{60}}$	$\frac{P_{62}}{P_{61}}$	$\frac{P_{63}}{P_{62}}$	$\frac{P_{64}}{P_{63}}$	$\frac{P_{65}}{P_{64}}$	$\frac{P_{66}}{P_{65}}$	$\frac{P_{67}}{P_{66}}$	$\frac{P_{68}}{P_{67}}$
1958	**100·8582**	100·8767	103·2389	104·0868	102·1808	103·4641	104·8783	104·0117	102·6646	104·7853
1959	*100·5448*	**100·8610**	103·1549	104·0430	102·1702	103·2280	104·9661	104·0418	102·4924	104·6718
1960	100·5347	*100·5793*	**102·8623**	103·8553	101·9422	103·1141	104·9066	103·9649	102·4772	104·6988
1961	100·5687	100·4702	*102·6963*	**103·7252**	101·8957	103·1578	104·8115	103·9404	102·3950	104·7171
1962	100·5701	100·5766	102·8968	*103·6411*	**101·9737**	103·2117	104·8554	103·9859	102·3597	104·7414
1963	100·5868	100·3122	102·3939	103·2612	*101·6390*	**103·2152**	104·7514	103·6691	102·5173	104·7612
1964	100·5050	100·4393	102·6265	103·5087	101·8298	*102·9477*	**105·2721**	103·4527	102·4641	104·6770
1965	100·5094	100·2000	102·3348	103·2989	101·3798	102·8382	*104·6314*	**103·8275**	102·3957	104·6863
1966	100·3770	100·2972	102·4126	103·3864	101·4964	102·7820	104·6846	*103·7896*	**102·3857**	104·7267
1967	100·4942	100·1695	102·4031	103·2420	101·4440	102·9103	104·5648	103·5818	*102·5491*	**104·7101**
1968	100·4723	99·9646	102·2164	103·1465	101·7474	102·3368	104·4247	103·6985	102·2485	*104·7856*

Year-to-year chain Laspeyres indices in bold type.
Year-to-year chain Paasche indices in italics.

Estimated standard errors of Indices of Retail Prices due to sampling variability of weights: All Items

Table 2b

	After adjusting for autoregressive scheme of order	Serial correlation coefficient (1)	Regression coefficient		Mean sums of squares (Sample of 3,528 households)			R^2	Standard error for samples of		Per cent change in chain Laspeyres index of prices
			b	s.e. (b)	Regression	Residual (d.f.)	*F* ratio		6,000 households	18,000 households	
Change in prices from:											
1958 to 1959	2	−0·015	−0·0139	(0·0050)	0·024016	0·003069(7)	7·82*	0·528	0·0425	0·0245	+0·858
1959 to 1960	1	−0·347	−0·0765	(0·0070)	1·417955	0·011984(8)	118·32**	0·937	0·0839	0·0485	+0·861
1960 to 1961	4	+0·021	−0·0721	(0·0103)	0·736285	0·014960(5)	49·32**	0·908	0·0938	0·0542	+2·862
1961 to 1962	6	+0·199	−0·0535	(0·0090)	0·170453	0·004811(3)	35·43**	0·922	0·0532	0·0307	+3·725
1962 to 1963	3	+0·051	−0·0536	(0·0214)	0·333562	0·053306(6)	6·26*	0·511	0·1770	0·1022	+1·974
1963 to 1964	1	+0·093	−0·0797	(0·0174)	1·002220	0·047834(8)	20·95**	0·724	0·1677	0·0968	+3·215
1964 to 1965	1	+0·010	−0·0531	(0·0163)	0·468859	0·044013(8)	10·65*	0·571	0·1609	0·0929	+5·272
1965 to 1966	2	+0·013	−0·0354	(0·0181)	0·136895	0·030383(7)	4·51	0·392	0·1337	0·0772	+3·828
1966 to 1967	4	+0·050	−0·0002	(0·0040)	0·000012	0·007470(5)	0·00	0·000	0·0663	0·0383	+2·386
1967 to 1968	4	+0·117	+0·0067	(0·0095)	0·001392	0·002800(5)	0·50	0·090	0·0406	0·0234	+4·710

(1) Circular method of calculation. In fitting the regression equations however the coefficient of serial correlation has been estimated by computing the least squares regression for the residuals (e.g. $\hat{u}_t = r\hat{u}_{t-1} + e_t$ for a first order autoregressive scheme).

* Significant at 0·05 level.

**Significant at 0·01 level.

Standard errors for price comparisons over varying periods of time: All Items

(For samples of 18,000 households)

Table 2c

Denominator	Standard errors for comparisons of prices over periods of									
	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
1958	0·0245	0·0608	0·1254	0·1109	0·1722	0·1515	0·1434	0·1192	0·1518	0·1789
1959	0·0485	0·0683	0·1252	0·1669	0·1784	0·1096	0·1569	0·1540	0·1779	
1960	0·0542	0·1058	0·1681	0·1548	0·1221	0·1403	0·1453	0·1493		
1961	0·0307	0·1311	0·1028	0·1278	0·0613	0·0835	0·1090			
1962	0·1022	0·0814	0·1489	0·1010	0·0988	0·1233				
1963	0·0968	0·1550	0·1124	0·1553	0·1524					
1964	0·0929	0·0570	0·0791	0·0639						
1965	0·0772	0·0595	0·0673							
1966	0·0383	0·0413								
1967	0·0234									
Mean	0·0589	0·0844	0·1161	0·1258	0·1309	0·1216	0·1386	0·1408	0·1648	0·1789

Estimated standard errors of Indices of Retail Prices due to sampling variability of weights: All Items

Table 2d

	After adjusting for autoregressive scheme of order	Serial correlation coefficient (1)	Regression coefficient		Mean sums of squares (Sample of 3,528 households)			R^2	Standard error for samples of		Per cent change since 1958 in 1958 weighted Laspeyres index
			b	s.e. (b)	Regression	Residual (d.f.)	*F* ratio		6,000 households	18,000 households	
Change in prices from 1958 to:											
1959	2	+0·022	−0·0139	(0·0050)	0·024016	0·003069(7)	7·82*	0·528	0·0425	0·0245	+ 0·858
1960	0	−0·007	−0·1024	(0·0107)	1·723988	0·018840(9)	91·51**	0·910	0·1053	0·0608	+ 1·742
1961	0	−0·196	−0·1958	(0·0221)	6·30111	0·080286(9)	78·48**	0·897	0·2173	0·1254	+ 5·038
1962	3	+0·066	−0·2441	(0·0149)	16·871643	0·062805(6)	268·63**	0·978	0·1922	0·1110	+ 9·330
1963	3	+0·069	−0·3159	(0·0250)	24·210327	0·151296(6)	160·02**	0·964	0·2983	0·1722	+11·715
1964	5	−0·050	−0·3745	(0·0394)	10·572186	0·117125(4)	90·26**	0·958	0·2624	0·1515	+15·585
1965	3	+0·031	−0·5011	(0·0123)	174·117126	0·104922(6)	1659·49**	0·996	0·2484	0·1434	+21·223
1966	5	−0·101	−0·5251	(0·0296)	22·823318	0·072452(4)	315·01**	0·987	0·2064	0·1192	+26·086
1967	5	−0·055	−0·5635	(0·0279)	47·779663	0·117535(4)	406·54**	0·990	0·2629	0·1518	+29·446
1968	5	−0·032	−0·5812	(0·0386)	37·014999	0·163208(4)	226·80**	0·983	0·3098	0·1789	+35·640

(1) Circular method of calculation: see note to Table 2b.
* Significant at 0·05 level.
**Significant at 0·01 level.

Indices of Retail Prices: Food Group

$$\frac{\sum Q_m P_n}{\sum Q_m P_{n-1}}$$

Table 3a

M $\frac{n}{n-1}$	$\frac{P_{59}}{P_{58}}$	$\frac{P_{60}}{P_{59}}$	$\frac{P_{61}}{P_{60}}$	$\frac{P_{62}}{P_{61}}$	$\frac{P_{63}}{P_{62}}$	$\frac{P_{64}}{P_{63}}$	$\frac{P_{65}}{P_{64}}$	$\frac{P_{66}}{P_{65}}$	$\frac{P_{67}}{P_{66}}$	$\frac{P_{68}}{P_{67}}$
1958	**101·8172**	99·1604	101·3030	103·7579	102·5656	102·7108	103·5900	103·4932	102·4610	104·5090
1959	*101·0668*	**99·4215**	101·3828	103·9135	102·4571	102·4814	103·5801	103·5540	102·4812	104·3857
1960	101·2209	*99·1839*	**101·2502**	104·0315	102·3251	102·6443	103·5484	103·5933	102·5041	**104·3663**
1961	101·3108	99·1903	*101·1786*	**103·8508**	102·5497	102·6858	103·5900	103·5210	102·4754	104·4362
1962	101·4235	99·2554	101·2493	*103·4916*	**102·4249**	102·7695	103·6180	103·5378	102·3710	104·4492
1963	101·4189	99·2021	101·2685	103·4564	*102·3136*	**102·9201**	103·6368	103·5224	102·4054	104·4178
1964	101·2968	99·2166	101·3862	103·7064	102·2448	*102·4200*	**103·6352**	103·4768	102·5869	104·2992
1965	101·2559	99·1638	101·3090	103·5826	102·2302	101·5303	*103·3872*	**103·6197**	102·4728	104·2338
1966	101·2673	99·1730	101·2617	103·5813	102·1698	102·5730	103·4198	*103·4986*	**102·4829**	104·2868
1967	101·2412	99·2034	101·1717	103·5128	102·1973	102·5992	103·4564	103·4449	*102·3629*	**104·3127**
1968	101·2266	99·0709	101·2424	103·6368	102·0760	102·5208	103·3523	103·5348	102·4115	*104·1375*

Year-to-year chain Laspeyres indices in bold type.
Year-to-year chain Paasche indices in italic type.

Estimated standard errors of Indices of Retail Prices due to sampling variability of weights: Food Group

Table 3b

	After adjusting for autoregressive scheme of order	Serial correlation coefficient (1)	Regression coefficient		Mean sums of squares (Sample of 3,528 households)			R^2	Standard error for samples of		Per cent change in chain Laspeyres index of prices
			b	s.e. (b)	Regression	Residual (d.f.)	*F* ratio		6,000 households	18,000 households	
Change in prices from:											
1958 to 1959	4	+0·183	−0·0440	(0·0268)	0·009066	0·003366(5)	2·69	0·350	0·0445	0·0257	+1·817
1959 to 1960	3	+0·088	−0·0105	(0·0018)	0·052845	0·001548(6)	34·13**	0·850	0·0302	0·0174	−0·578
1960 to 1961	1	+0·140	−0·0080	(0·0094)	0·005002	0·006850(8)	0·73	0·084	0·0635	0·0366	+1·250
1961 to 1962	2	+0·084	−0·0195	(0·0311)	0·012173	0·031031(7)	0·39	0·053	0·1351	0·0780	+3·851
1962 to 1963	1	−0·046	−0·0416	(0·0062)	0·308518	0·006942(8)	44·44**	0·847	0·0639	0·0369	+2·425
1963 to 1964	0	−0·011	−0·0133	(0·0121)	0·029198	0·024019(9)	1·22	0·120	0·1188	0·0686	+2·920
1964 to 1965	1	+0·061	−0·0301	(0·0107)	0·057608	0·007274(8)	7·92*	0·497	0·0654	0·0378	+3·635
1965 to 1966	5	−0·025	−0·0080	(0·0056)	0·008463	0·004149(4)	2·04	0·338	0·0494	0·0285	+3·620
1966 to 1967	5	−0·003	−0·0078	(0·0115)	0·003640	0·007977(6)	0·46	0·071	0·0685	0·0395	+2·483
1967 to 1968	4	−0·085	−0·0405	(0·0125)	0·075391	0·007209(5)	10·46*	0·677	0·0651	0·0376	+4·313

(1) Circular method of calculation: see note to Table 2b.
* Significant at 0·05 level.
**Significant at 0·01 level.

Standard errors for price comparisons over varying periods of time: Food Group

(For samples of 18,000 households)

Table 3c

Denominator	Standard errors for comparisons of prices over periods of									
	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
1958	0·0257	0·0471	0·0177	0·0579	0·0424	0·0418	0·0762	0·0535	0·0533	0·0718
1959	0·0174	0·0472	0·0734	0·0671	0·0257	0·0553	0·0653	0·0518	0·0421	
1960	0·0366	0·0817	0·0750	0·0314	0·0527	0·0482	0·0632	0·0493		
1961	0·0780	0·0759	0·0451	0·0777	0·0682	0·0376	0·0579			
1962	0·0369	0·0800	0·1081	0·0973	0·0622	0·0984				
1963	0·0686	0·0930	0·0931	0·0591	0·0437					
1964	0·0378	0·0199	0·0499	0·0437						
1965	0·0285	0·0612	0·0278							
1966	0·0395	0·0351								
1967	0·0376									
Mean	0·0407	0·0601	0·0613	0·0620	0·0492	0·0563	0·0657	0·0515	0·0477	0·0718

Estimated standard errors of Indices of Retail Prices due to sampling variability of weights: Food Group

Table 3d

	After adjusting for autoregressive scheme of order	Serial correlation coefficient (1)	Regression coefficient		Mean sums of squares (Sample of 3,528 households)			R^2	Standard error for samples of		Per cent change since 1958 in 1958 weighted Laspeyres index
			b	s.e. (b)	Regression	Residual (d.f.)	*F* ratio		6,000 households	18,000 households	
Change in prices from 1958 to:											
1959	4	+0·116	−0·0440	(0·0268)	0·009066	0·003366(5)	2·69	0·350	0·0445	0·0257	+ 1·817
1960	2	+0·036	−0·0388	(0·0207)	0·040927	0·011598(7)	3·53	0·335	0·0826	0·0477	+ 0·962
1961	5	+0·029	−0·0920	(0·0117)	0·098771	0·001500(4)	61·74**	0·939	0·0307	0·0177	+ 2·278
1962	4	−0·001	−0·0585	(0·0143)	0·291557	0·017084(5)	17·07**	0·773	0·1002	0·0579	+ 6·121
1963	4	−0·033	−0·1136	(0·0065)	2·804358	0·009182(5)	305·42**	0·984	0·0735	0·0424	+ 8·844
1964	4	+0·042	−0·1546	(0·0200)	0·531886	0·008924(5)	59·60**	0·923	0·0724	0·0418	+11·795
1965	4	−0·002	−0·2157	(0·0379)	0·958224	0·029607(5)	32·37**	0·866	0·1319	0·0762	+15·808
1966	4	+0·014	−0·2241	(0·0363)	0·554809	0·014576(5)	38·06**	0·884	0·0926	0·0535	+19·853
1967	5	−0·081	−0·2622	(0·0224)	1·988075	0·014469(4)	137·40**	0·972	0·0922	0·0533	+22·803
1968	5	+0·018	−0·3166	(0·0297)	2·992158	0·026337(4)	113·61**	0·966	0·1244	0·0718	+28·340

(1) Circular method of calculation: See note to Table 2b.
**Significant at 0·01 level.

Indices of Retail Prices: Transport and Vehicles Group

$$\frac{\sum Q_m P_n}{\sum Q_m P_{n-1}}$$

Table 4a

$\frac{n}{n-1}$ M	$\frac{P_{59}}{P_{58}}$	$\frac{P_{60}}{P_{59}}$	$\frac{P_{61}}{P_{60}}$	$\frac{P_{62}}{P_{61}}$	$\frac{P_{63}}{P_{62}}$	$\frac{P_{64}}{P_{63}}$	$\frac{P_{65}}{P_{64}}$	$\frac{P_{66}}{P_{65}}$	$\frac{P_{67}}{P_{66}}$	$\frac{P_{68}}{P_{67}}$
1958	**100·5435**	103·0059	103·4850	103·6285	100·2596	104·0242	105·7589	104·9419	102·1402	105·7175
1959	*101·7839*	**101·1597**	102·8938	102·9331	101·1017	102·4538	105·9517	104·4783	102·0232	105·7329
1960	101·5041	*99·1807*	**100·3744**	100·8932	99·7459	101·4645	104·7168	103·5800	101·9944	105·8677
1961	101·7788	98·4464	*99·6319*	**100·7406**	98·5598	101·4918	104·2001	103·2932	101·9302	105·9300
1962	101·6036	99·2271	100·9017	*101·1811*	**99·8678**	101·6728	104·9059	103·6041	102·0853	106·1703
1963	101·6171	96·9873	98·8407	99·0643	*98·1239*	**100·5423**	103·6632	102·6724	102·0544	106·0990
1964	101·3114	98·6902	99·2669	100·7015	98·4632	*100·9880*	**104·3809**	103·1504	101·9091	106·1770
1965	101·2132	97·5172	97·6734	99·1814	97·3836	100·2052	*103·4476*	**102·4225**	101·6729	106·2432
1966	100·6860	98·3558	98·2950	99·8441	97·7401	100·6135	103·8841	*102·9066*	**101·6625**	106·2947
1967	101·2966	97·5351	98·4642	99·4280	97·9151	100·3818	104·0575	102·4295	*101·9206*	**106·3111**
1968	101·4090	96·7182	97·6544	98·9078	97·3789	100·0854	103·5355	102·1173	101·9765	*106·1756*

Year-to-year chain Laspeyres indices in bold type.
Year-to-year chain Paasche indices in italic type.

Estimated standard errors of Indices of Retail Prices due to sampling variability of weights: Transport and Vehicles Group

Table 4b

	After adjusting for autoregressive scheme of order	Serial correlation coefficient (1)	Regression coefficient		Mean sums of squares (Sample of 3,528 households)			R^2	Standard error for samples of		Per cent change in chain Laspeyres index of prices
			b	s.e. (b)	Regression	Residual (d.f.)	*F* ratio		6,000 households	18,000 households	
Change in price from:											
1958 to 1959	0	−0·070	−0·0126	(0·0344)	0·026270	0·194468(9)	0·14	0·015	0·3382	0·1952	+0·544
1959 to 1960	6	+0·198	−0·1939	(0·0903)	2·422333	0·525143(3)	4·61	0·606	0·5557	0·3208	+1·160
1960 to 1961	0	+0·102	−0·4842	(0·0806)	38·535110	1·067442(9)	36·10**	0·800	0·7406	0·4574	+0·374
1961 to 1962	3	+0·040	−0·2490	(0·0275)	19·128204	0·233419(6)	81·95**	0·932	0·3705	0·2139	+0·741
1962 to 1963	1	+0·071	−0·3216	(0·0477)	22·452805	0·494637(8)	45·39**	0·850	0·5393	0·3114	−0·132
1963 to 1964	3	+0·012	−0·1888	(0·0250)	6·029918	0·105676(6)	57·06**	0·905	0·2493	0·1439	+0·542
1964 to 1965	4	−0·017	−0·1083	(0·0358)	1·375726	0·150624(5)	9·13*	0·646	0·2976	0·1718	+4·381
1965 to 1966	3	+0·044	−0·1673	(0·0115)	9·854872	0·046210(6)	213·26**	0·973	0·1648	0·0952	+2·423
1966 to 1967	3	+0·092	+0·0214	(0·0692)	0·002697	0·028244(6)	0·10	0·016	0·1289	0·0744	+1·663
1967 to 1968	1	+0·175	+0·0357	(0·0169)	0·057684	0·012965(8)	4·45	0·357	0·0873	0·0504	+6·311

(1) Circular method of calculation: See note to Table 2b.
* Significant at 0·05 level.
**Significant at 0·01 level.

Standard errors for price comparisons over varying periods: Transport and Vehicles Group

(For samples of 18,000 households)

Table 4c

Denominator	Standard errors for comparisons of prices over periods of									
	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
1958	0·1952	0·4223	0·8740	1·2868	0·9059	1·8198	2·1613	2·4382	2·5116	2·6428
1959	0·3208	0·4474	0·6380	0·8336	0·9915	1·1907	1·3094	1·3745	2·7055	
1960	0·4574	0·8458	1·1014	1·3677	1·6846	1·9374	2·0027	2·1057		
1961	0·2139	0·4210	0·5813	0·7713	0·8746	0·9412	0·9825			
1962	0·3114	0·5019	0·7731	0·9793	1·0266	1·0759				
1963	0·1439	0·3320	0·4295	0·7560	0·7885					
1964	0·1718	0·2616	0·3173	0·3198						
1965	0·0952	0·2032	0·1328							
1966	0·0744	0·0797								
1967	0·0504									
Mean	0·2034	0·3905	0·6059	0·9021	1·0453	1·3930	1·6140	1·9728	2·6086	2·6428

Estimated standard errors of Indices of Retail Prices due to sampling variability of weights: Transport and Vehicles Group

Table 4d

	After adjusting for autoregressive scheme of order	Serial correlation coefficient (1)	Regression coefficient		Mean sums of squares (Sample of 3,528 households)			R^2	Standard error for samples of		Per cent change since 1958 in 1958 weighted Laspeyres index
			b	s.e. (b)	Regression	Residual (d.f.)	*F* ratio		6,000 households	18,000 households	
Change in prices from 1958 to:											
1959	0	−0·011	−0·0126	(0·0344)	0·026270	0·194468(9)	0·14	0·015	0·3382	0·1952	+ 0·543
1960	0	−0·003	−0·4439	(0·0744)	32·380707	0·909792(9)	35·59**	0·798	0·7314	0·4223	+ 3·566
1961	0	+0·041	−0·9313	(0·1540)	142·533203	3·897697(9)	36·57**	0·802	1·5139	0·8740	+ 7·175
1962	0	+0·014	−1·3151	(0·2267)	284·197266	8·448188(9)	33·64**	0·785	2·2288	1·2868	+11·064
1963	4	+0·024	−1·1008	(0·1969)	130·931366	4·187344(5)	31·27**	0·862	1·5691	0·9059	+11·352
1964	0	+0·024	−1·9242	(0·3207)	608·479736	16·897156(9)	36·01**	0·800	3·1521	1·8198	+15·833
1965	0	+0·021	−2·2061	(0·3808)	799·822998	23·833786(9)	33·56**	0·789	3·7436	2·1613	+22·504
1966	0	+0·026	−2·5268	(0·4296)	1,049·251953	30·329956(9)	34·59**	0·794	4·2230	2·4382	+28·559
1967	0	+0·029	−2·5977	(0·4426)	1,108·903809	32·184922(9)	34·45**	0·793	4·3503	2·5116	+31·309
1968	0	+0·008	−2·6943	(0·4657)	1,192·910645	35·634644(9)	33·48**	0·788	4·5775	2·6428	+38·817

(1) Circular method of calculation: See note to Table 2b.
**Significant at 0·01 level.

Standard errors of 1958 weighted and chain Laspeyres Indices of Retail Prices

(For samples of 18,000 households)

Table 5

	All Items		Food Group		Transport and Vehicles Group	
	1958 weighted (d.f.)	Chain index (d.f.)	1958 weighted (d.f.)	Chain index (d.f.)	1958 weighted (d.f.)	Chain index (d.f.)
Change in prices from 1958 to:						
1959	0·0245 (7)	0·0245 (7)	0·0257 (5)	0·0257 (5)	0·1952 (9)	0·1952 (9)
1960	0·0608 (9)	0·0454 (14)	0·0477 (7)	0·0202 (10)	0·4223 (9)	0·2846 (11)
1961	0·1254 (9)	0·0903 (17)	0·0177 (4)	0·0282 (16)	0·8740 (9)	0·5784 (18)
1962	0·1110 (6)	0·1135 (17)	0·0579 (5)	0·0744 (20)	1·2868 (9)	0·7775* (21)
1963	0·1722 (6)	0·1530 (19)	0·0424 (5)	0·0500 (24)	0·9059 (5)	1·0428 (25)
1964	0·1515 (4)	0·1865 (22)	0·0418 (5)	0·0487 (28)	1·8198 (9)	1·1866* (26)
1965	0·1434 (6)	0·2305 (24)	0·0762 (5)	0·0816 (30)	2·1613 (9)	1·4186* (25)
1966	0·1192 (4)	0·2064 (24)	0·0535 (5)	0·0618 (27)	2·4382 (9)	1·5461* (24)
1967	0·1518 (4)	0·2166 (21)	0·0533 (4)	0·0746 (25)	2·5116 (9)	1·6485 (22)
1968	0·1789 (4)	0·2178 (17)	0·0718 (4)	0·0864 (21)	2·6428 (9)	1·7625 (21)

*Difference from standard errors of 1958 weighted index significant at the 5% level.

Relationship between Laspeyres price indices and standard errors over varying periods of time between 1958 and 1968

Table 6

	No.	All Items			Food Group			Transport and Vehicles Group		
		Mean of price indices (1)	Mean of standard errors (2)	$\frac{\text{Col (2)}}{\text{Col (1)}} \times 100$ (3)	Mean of price indices (4)	Mean of standard errors (5)	$\frac{\text{Col (5)}}{\text{Col (4)}} \times 100$ (6)	Mean of price indices (7)	Mean of standard errors (8)	$\frac{\text{Col (8)}}{\text{Col (7)}} \times 100$ (9)
Measurements made over periods of:										
1 year	10	102·9691	0·0589	0·0572	102·5735	0·0407	0·0397	101·8005	0·2034	0·1998
2 years	9	105·9428	0·0844	0·0797	105·0904	0·0597	0·0568	103·7605	0·3905	0·3763
3 years	8	109·5690	0·1181	0·1078	108·0424	0·0613	0·0567	106·2490	0·6059	0·5703
4 years	7	113·2819	0·1258	0·1111	111·1570	0·0603	0·0542	108·9930	0·9021	0·8277
5 years	6	116·7183	0·1309	0·1122	114·0550	0·0492	0·0431	111·1082	1·0453	0·9408
6 years	5	120·5111	0·1216	0·1009	117·0619	0·0563	0·0481	114·9505	1·3930	1·2118
7 years	4	124·6934	0·1386	0·1112	120·2311	0·0657	0·0546	118·9626	1·6140	1·3567
8 years	3	127·1926	0·1408	0·1107	122·5667	0·0515	0·0420	124·6441	1·9728	1·5827
9 years	2	131·6030	0·1648	0·1252	124·4976	0·0477	0·0383	131·9146	2·6086	1·9775
10 years	1	131·6030	0·1789	0·1319	128·3402	0·0718	0·0559	138·8169	2·6428	1·9038

Average weekly expenditure by 'Index' households: All Items

Aggregate of quarterly weekly average expenditure—revalued

Appendix A

	Expenditure at prices obtaining in										
	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1958	14,455·30	14,579·36	14,707·17	15,183·52	15,804·04	16,148·70	16,708·10	17,523·17	18,226·15	18,711·80	19,607·22
1959	14,998·04	15·079·75	15,209·58	15,689·42	16,323·75	16,678·01	17,216·38	18,071·36	18,801·76	19,270·38	20,170·66
1960	15,602·31	15,685·74	15,776·61	16,228·18	16,853·82	17,181·15	17,716·18	18,585·44	19,322·33	19,800·98	20,731·39
1961	16,030·33	16,121·49	16,197·29	16,634·02	17,253·67	17,580·75	18,135·92	19,008·53	19,757·54	20,230·73	21,185·04
1962	15,743·26	15,833·01	15,924·31	16,385·60	16,982·21	17,317·39	17,873·57	18,741·40	19,488·42	19,948·29	20,894·12
1963	16,881·40	16,980·46	17,033·47	17,441·24	18,010·04	18,305·23	18,893·78	19,791·50	20,517·67	21,034·16	22,035·63
1964	16,433·17	16,516·15	16,588·70	17,024·40	17,621·74	17,944·19	18,473·13	19,447·06	20,118·51	20,614·26	21,578·38
1965	17,873·09	17,964·13	18,000·06	18,420·32	19,027·98	19,290·53	19,838·04	20,756·81	21,551·27	22,067·58	23,101·74
1966	17,597·95	17,664·29	17·716·78	18,144·22	18,758·65	19,039·35	19,569·03	20,485·76	21,262·08	21,769·32	22,798·29
1967	18,030·12	18,119·22	18,149·94	18,586·11	19,188·67	19,465·75	20,032·26	20,946·69	21,696·96	22,250·04	23,298·05
1968	18,633·08	18,721·09	18,714·47	19,129·29	19,731·19	20,075·92	20,545·05	21,454·11	22,247·59	22,747·83	23,836·44

d. = old pence (240 d. = £1)

Average weekly expenditure by 'Index' households: Food Group

Aggregate of quarterly weekly average expenditure—revalued

Appendix B

	Expenditure at prices obtaining in										
	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1958	4,941·10	5,030·89	4,988·65	5,053·65	5,243·56	5,378·09	5,523·88	5,722·19	5,922·08	6,067·82	6,341·42
1959	4,999·81	5,053·15	5,023·92	5,093·39	5,292·72	5,422·77	5,557·33	5,756·29	5,960·87	6,108·77	6,376·68
1960	5,055·34	5,117·06	5,075·30	5,138·75	5,345·92	5,470·22	5,614·87	5,814·11	6,023·03	6,173·85	6,443·42
1961	5,100·57	5,167·43	5,125·59	5,186·00	5,385·70	5,523·02	5,671·36	5,874·96	6,081·82	6,232·37	6,508·85
1962	5,095·22	5,167·75	5,129·27	5,193·35	5,374·68	5,505·01	5,657·47	5,862·16	6,069·55	6,213·46	6,489·91
1963	5,181·52	5,255·04	5,213·11	5,279·24	5,461·71	5,588·07	5,751·25	5,960·41	6,170·36	6,318·78	6,597·93
1964	5,096·25	5,162·34	5,121·90	5,192·90	5,385·37	5,506·26	5,639·51	5,844·52	6,047·72	6,204·17	6,470·90
1965	5,216·83	5,282·35	5,238·18	5,306·75	5,496·87	5,619·46	5,761·65	5,956·81	6,172·43	6,325·06	6,592·85
1966	5,240·92	5,307·34	5,263·45	5,329·86	5,520·74	5,640·53	5,785·66	5,983·52	6,192·86	6,346·62	6,618·69
1967	5,247·16	5,312·29	5,269·97	5,331·72	5,519·01	5,640·28	5,786·88	5,986·90	6,193·14	6,339·48	6,612·88
1968	5,218·34	5,282·35	5,233·27	5,298·29	5,490·98	5,604·97	5,746·26	5,938·89	6,148·82	6,297·10	6,557·64

d. = old pence (240d. = £1)

Average weekly expenditure by 'Index' households: Transport and Vehicles Group

Aggregate of quarterly weekly average expenditure—revalued

Appendix C

	Expenditure at prices obtaining in										
	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1958	1,148·16	1,154·40	1,189·10	1,230·54	1,275·19	1,278·50	1,329·95	1,406·54	1,476·05	1,507·64	1,593·84
1959	1,236·07	1,258·12	1,272·71	1,309·54	1,347·95	1,362·80	1,396·24	1,479·34	1,545·59	1,576·86	1,667·26
1960	1,620·95	1,645·33	1,631·85	1,637·96	1,652·59	1,648·39	1,672·53	1,751·42	1,814·12	1,850·30	1,958·87
1961	1,716·36	1,746·89	1,719·75	1,713·42	1,726·11	1,701·25	1,726·63	1,799·15	1,858·40	1,894·27	2,006·60
1962	1,631·29	1,657·45	1,644·64	1,659·47	1,679·07	1,676·85	1,704·90	1,788·54	1,853·00	1,891·64	2,008·36
1963	2,389·39	2,428·03	2,354·88	2,327·58	2,305·80	2,262·54	2,274·81	2,358·14	2,421·16	2,470·90	2,621·60
1964	2,043·67	2,070·47	2,043·35	2,028·37	2,042·60	2,011·21	2,031·08	2,120·06	2,186·85	2,228·60	2,366·26
1965	2,673·83	2,706·27	2,639·08	2,577·68	2,556·58	2,489·69	2,494·80	2,580·81	2,643·33	2,687·55	2,855·34
1966	2,549·65	2,567·14	2,524·93	2,481·88	2,478·01	2,422·01	2,436·87	2,531·52	2,605·10	2,648·41	2,815·12
1967	2,781·85	2,817·92	2,748·46	2,706·25	2,690·77	2,634·67	2,644·73	2,752·04	2,818·90	2,873·04	3,054·36
1968	3,047·47	3,090·41	2,988·99	2,918·88	2,887·00	2,811·33	2,813·73	2,913·21	2,974·89	3,033·69	3,221·04

d. = old pence (240d. = £1)

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