Timing relationships between movements of monetary and national income variables

This research paper, prepared in the Bank's Economic Section, is largely the work of A. D. Crockett.

One way of viewing the impact of monetary policy on the economy is to see it as affecting the private sector's holdings of real and financial assets. By their actions in financial markets, changing interest rates and the relative quantities of financial assets, the monetary authorities can bring about a divergence between the private sector's desired portfolio of assets and its actual portfolio. Subsequent attempts by the private sector to restore the desired portfolio balance will involve sales and purchases of assets, both financial and real, which will have repercussions on income flows. Although there is considerable agreement on the value of this insight, there is much less agreement on how best it should be applied to forecasting and policy-making.

On the one hand, it is argued that certain relationships in the economic system are far more dominant and stable than others, and that if these relationships can be established with a substantial degree of statistical significance, the precise nature of the transmission mechanism by which one variable affects another is of secondary importance to the policy-maker. This seems to have been, broadly speaking, the methodological approach adopted in the research undertaken by the Federal Reserve Bank of St. Louis.[1]

On the other hand, it can be argued that this kind of simple single-equation relationship that is not derived as a reduced form from a more complex model is more likely to produce faulty answers should the basic ground-rules of the system change.[2] If the monetary authorities change their open-market tactics, for example, an equation which ignores the route by which open-market operations work through the financial system to the real economy may be unsuccessful in forecasting the consequences of such action. For this reason, it is argued that the transmission mechanism must be explicitly spelt out through the specification of a multi-equation model.

The choice boils down to one of heuristic simplicity against logically rigorous complexity. This paper presents the results of an analysis of correlation undertaken to determine average leads and lags between various monetary and expenditure series.[3] It has two main objectives, corresponding to the two broad approaches described above. In the first place, it provides information about the existence (and perhaps as important, the non-existence) of certain lead/lag relationships that may be useful in forecasting and analysis. Secondly, it provides some basic source material that may be of use in the building of more complex models. Theories must explain facts, and without a greater know-

1 See, for example, L. C. Andersen and J. L. Jordan "Monetary and fiscal actions: a test of their relative importance in economic stabilization" *Federal Reserve Bank of St. Louis Review*, November 1968 pages 11-24; M. W. Keran "Monetary and fiscal influences on economic activity - the historical evidence" *Federal Reserve Bank of St. Louis Review*, November 1969 pages 5-23.

2 See Appendix I to the article "The importance of money" which appeared in the June 1970 *Bulletin*, page 185.

3 Some of these results were referred to in "The importance of money", see footnote 2.

ledge of the facts that have to be explained – such as timing relationships between economic variables – theorising will tend to be a very hit-and-miss affair.

It is widely appreciated and recognised that a simultaneous relationship between two variables can tell nothing about the direction of any causal link. Two series can be highly correlated because changes in the first series are causing changes in the second, or vice versa, or because some third factor is causing changes in both series. It is not, perhaps, so widely appreciated that a lead or lag of one variable over another is also no evidence of causality. The circulation of Christmas cards rises before Christmas; this does not mean that the circulation of cards has caused Christmas.

However, the inability of correlation studies to establish the nature of a causal relationship is not a defect peculiar to them. It is shared by all statistical methods. It is a truism – but not one that is always in the forefront of the mind – to say that, while statistical testing can disprove hypotheses, it cannot prove them. All the same, the finding of a close association whereby changes in one series are followed by changes in another series would at least appear to be evidence consistent with a theory about causal relationships. This evidence will be strengthened if there is a plausible economic explanation for the lag, and no comparably plausible alternative explanation. And although the pattern of linkages will ultimately have to be much more precisely specified, and estimated using regression techniques, correlations can give valuable initial indications of the possible nature of the transmission mechanism.

Furthermore, even if the existence of a regular lead does not allow causality to be imputed without further research, it may enable any such regular relationship to be used as a leading indicator, assuming that the general structure of the system remains unchanged. For these reasons it is of some considerable interest to inspect whether monetary aggregates lead certain income and expenditure variables.

In the present study therefore, the relationships between monetary and national income series have been tested to discover the nature and stability of the timing relationships. Tests have also been performed with disaggregated money stock data. This is of particular interest because an important unresolved area of debate is whether the origin of a change in the money stock is relevant to its impact on expenditure. Monetarist studies[1] generally assume that the particular source of changes in the money stock is immaterial; but others[2] have suggested that the way in which a monetary change is brought about may significantly affect its impact on final demand. Finally expenditure data have also been disaggregated, in a fairly simple way, to see whether different categories of expenditure are similarly related to possible monetary stimuli, particularly in regard to lag structures.

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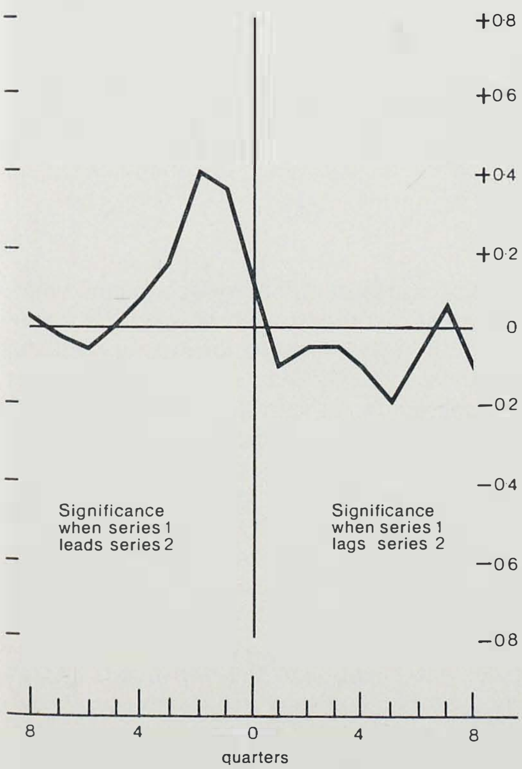
1 See, for example, Milton Friedman and David Meiselman "The relative stability of monetary velocity and the investment multiplier in the United States, 1897-1958" in "Stabilization policies" *C.M.C. Research Papers 1963*, pages 165-268; L. C. Andersen and J. L. Jordan, see footnote *1*, page 459; M. W. Keran, see footnote *1*, page 459.

2 James Tobin "The monetary interpretation of history" *American Economic Review*, June 1965 pages 464-85; W. L. Silber, "Velocity and bank portfolio composition" *Southern Economic Journal*, October 1969 pages 147-52.

Example



Methods used

In what follows, the principal method used to analyse the stability of relationships between two series is the cross-correlogram.[1] A cross-correlogram is a series of coefficients of correlation which can range between +1·0 and −1·0 and which measure the closeness of association (positive or negative) between two series with a given lead or lag.[2] With quarterly observations for about fifteen years (as used in most of the charts in this paper), the correlation is significant if it is greater than ±0·25;[3] but it is perhaps more informative to look at the general shape of the correlogram rather than individual correlations, which will themselves be intercorrelated.[4] If, for example, two variables tend to move cyclically, the peak of the cross-correlogram would show the average lead or lag of one series over the other. Thus, in the example chart, the most significant relationship is a positive one where the first series leads the second by two quarters; but there is also a rather less strong negative relationship where the first series lags the second by five quarters.

In dealing with series which have strong common trends, a high correlation will be observed, without there necessarily being any close causal connection. For example, employment in the computer industry would probably be closely linked, in a purely statistical sense, with the number of tourists visiting Britain, but the link would be a coincidental one. During the past twenty-five years, monetary and national income statistics have both tended to rise over time, and a simple correlation between the two might tend to exaggerate the strength of the link between the two phenomena. It might also obscure the precise timing of any lead or lag relationship. To reduce this possibility, the relationships plotted in this paper are based on data with trends removed.[5]

Although the removal of trends is desirable in order to reduce the possibility of spurious correlation, it is nevertheless possible – indeed, with money and income series,

1 The spectral analysis was also used, and J. P. Burman provided statistical guidance in the application of both methods. The results of the spectral analysis, together with a description of the technique are given in the appendix.

2 Mathematically, for N observations of two variables, x_t and y_t, each with zero mean, the cross-correlogram is the series:

$$r_k = C_k / \sqrt{V_1 V_2}, \quad \text{where } C_k = \frac{1}{N-k} \sum_t x_t y_{t-k}$$

$$\text{and } V_1 = \sum_t x_t^2 / N, \quad V_2 = \sum_t y_t^2 / N$$

for $k = -m, \ldots, 0, \ldots, m$, where m is small compared with N. (The range of summation in the expression for C_k is truncated at one end if k is positive and the other end if k is negative.)

3 The 'significance' of a correlation is measured by the probability of such a correlation arising by chance between two unrelated series: in this paper, significance is measured at the 5% level *i.e.* there is only a 1 in 20 possibility of getting a correlation of ±0·25, or above, by chance.

4 This is because the individual correlations, though estimated separately, are based on the same run of data. If there is autocorrelation in the original series the separate observations will not be independent.

5 The technique of trend removal was to convert data to log form, to remove a linear trend, and then to use an autoregressive transformation, as described by Marc Nerlove in *Spectral Comparisons of two seasonal adjustment procedures*, Technical Report No. 2, 1964, Institute for Mathematical Studies in Social Sciences, Stanford University. Other methods of trend removal were tried: straightforward first differences produced series with somewhat larger residual variance; the autoregressive transformation without removing a linear trend gave much larger residual variance and series that were obviously not stationary. The results using these alternative methods are not reported here. However, it should be noted that the correlograms did prove rather sensitive to the particular method of trend removal used: where straightforward first differences were employed, the observed correlations tended to be lower than those reported in this paper, although the general shape of the correlograms was similar.

probable – that the existence of common trends *does* owe something to a causal link. This being the case, the correlations will tend to underestimate the strength of any such causal link between the two series.

One final point is worthy of mention. An objective of trend removal is to make series more 'stationary' in the statistical sense. One possible consequence of achieving this is that there may be some counterbalancing of positive and negative correlations between 'stationary' series.[1] If two series move in regular cycles which, even though out of phase, are of approximately the same length, then more or less equal positive and negative correlations between them would tend to occur at timings determined by the cycle itself. Simple correlation analysis would not enable one to distinguish which series was leading and which was lagging. Indeed, it would serve as a reminder that a regression analysis which *did* specify a particular direction of causation between such cyclically dominated series would risk imputing causality that did not necessarily exist. However, cycles are not perfectly regular and, this being so, it is quite possible that there will be a unique point of maximum correlation. It will then be easier to say which series leads the other, though the nature of any causal connection will still be a matter of speculation, based on the most plausible economic explanation.

Results

There are a number of issues in monetary economics where theory suggests a clear hypothesis which may be tested against the data (*e.g.* that the demand for money responds negatively to a change in interest rates). In other cases, theory gives much less guidance, and these issues can only be resolved pragmatically. For example, the question of whether the money supply should be confined to currency and demand deposits, or extended to include time deposits, is generally acknowledged to be an empirical matter. It could also be argued that, as there is quite wide agreement that changes in the money stock affect expenditure through an interest rate type mechanism,[2] it is an empirical matter whether interest rates or money stocks most accurately measure that effect. On the one hand, interest rates have the disadvantage that they can be measured only in particular markets, and do not take account of changes in inflationary expectations; on the other, the money stock/ expenditure relationship is subject to unforeseen changes in the demand-for-money function.

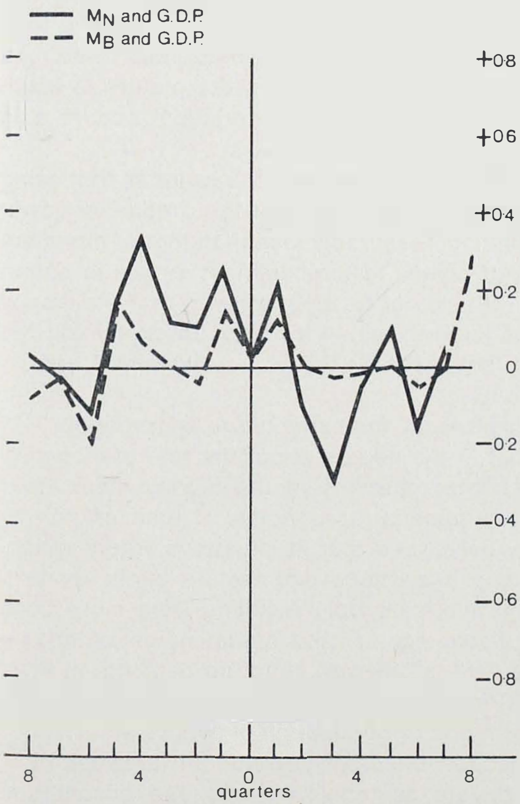
Another important issue is whether monetary policy should be concerned simply with the volume of banks' liabilities (the money stock) or whether it should attempt also to influence the structure of banks' asset portfolios (bank lending). It is sometimes claimed that market imperfections, such as rationing of bank loans, make control

1 This may be seen intuitively by considering that, when one observation in a series is above trend, the average of all other observations must, by definition, be below trend. Thus if there is a positive correlation between one series and synchronous observations in another series, the sum of the correlations with *non-synchronous* observations must be negative.

2 "The crucial issue that corresponds to the distinction between the 'credit' and 'monetary' effects of monetary policy is not whether changes in the stock of money operate through interest rates, but rather the range of interest rates considered . . ." Friedman and Meiselman page 217 – see footnote *1*, page 460.

Chart 1

Money and G.D.P.



Note: In the charts, the correlation coefficient for a synchronous association between the two series is shown in the centre of each diagram. Points to the left of the centre show the correlation coefficient when the first series mentioned leads the second by the number of quarters indicated. Similarly, points to the right of the centre denote the first series lagging the second by the period indicated amount.

over bank lending to the private sector a strategic variable in counter-cyclical policy.[1]

Finally, there is debate whether monetary changes affect the overall level of demand without having any particular systematic effect on the *distribution* of output (a 'monetarist' view), or whether such changes affect first of all the demand for investment goods, and only rather more indirectly, consumption expenditures.

The above questions suggest that it would be interesting to analyse the following correlations:

(a) Monetary aggregates and expenditure aggregates.

(*b*) Monetary components and expenditure aggregates.

(c) Monetary aggregates and expenditure components.

(*d*) Monetary components and expenditure components.

(a) Monetary aggregates and expenditure aggregates

Chart 1 plots the cross-correlogram for two definitions of money against gross domestic product.

The two definitions of money used are:[2]

(i) M_N: a 'narrow' definition, including currency in circulation plus net current account deposits at London clearing banks, and

(ii) M_B: a 'broader' definition, comprising currency plus all net deposits at London clearing banks.

G.D.P. is measured as the arithmetic average of G.D.P. estimates based on income and expenditure data.

Both definitions of money are positively related to movements in G.D.P. when changes in the quantity of money precede changes in G.D.P. by several quarters. Using a narrow definition of money, the peak relationship is r=·34 with money leading G.D.P. by four quarters. With a broad definition, the peak is r=·18, with a five-quarter lead. The 'narrow' definition of money shows a more pronounced downward slope from left to right, suggesting that the lead of changes in the stock of money over changes in money incomes is clearer when money is narrowly defined. On the basis of this evidence it would seem that, at least as an indicator, it is the stock of 'money-as-a-medium-of-exchange' that is the most important to look at.

As noted earlier, correlation analysis does not provide any direct evidence about the direction of causality; but if the causal link was merely one of the stock of money passively accommodating to changes in incomes, it is hard to see why cycles in money should precede cycles in G.D.P. by such a large interval. It seems more likely that there are other cyclical factors at work, which may or may not include a causal link from money to G.D.P. The negative correlation between changes in G.D.P. and changes in the money supply some three quarters later seems unlikely to be due to

1 See, for example, Radcliffe Report, *Committee on the Working of the Monetary System* Cmnd. 827, August 1959, Chapter VI.

2 The official definitions of money supply (see Table 12 of the annex) were not used because of the considerably shorter run of data available.

Chart 2

Interest rates and G.D.P.

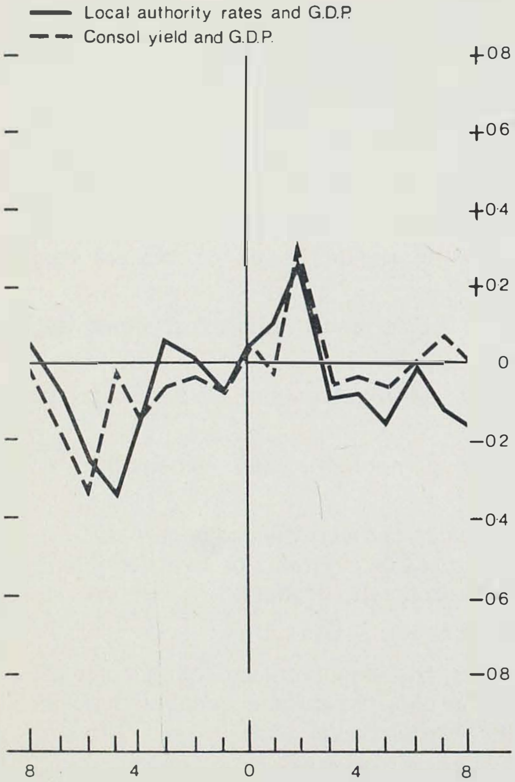
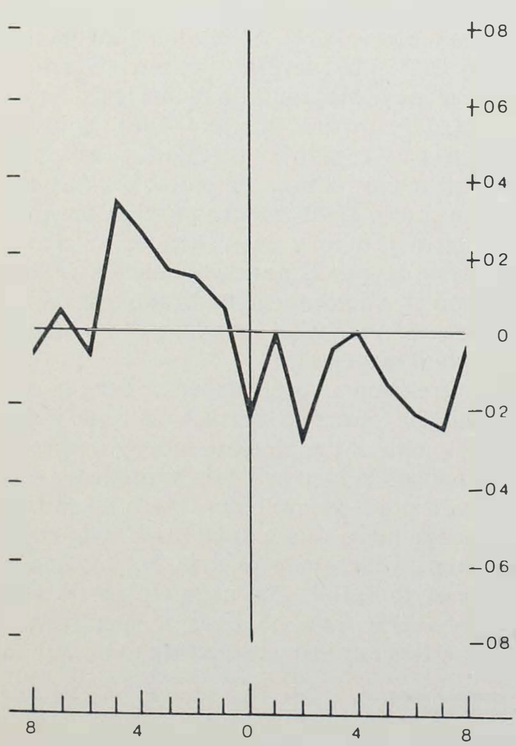


Chart 3

Bank lending and G.D.P.



a direct causal link, and more consistent with the theory that both money and output are responding to other cyclical forces.

Chart 2 plots the cross-correlogram for interest rates and G.D.P. For short-term interest rates,[1] there is a significant positive relationship with G.D.P. after a lag of two quarters. This probably occurs because in the short run changes in demand tend to pull interest rates in the same direction; the authorities may resist or moderate this change for a while, thus possibly delaying the full effect on interest rates being felt for some months. When interest rate changes lead output changes, however, there appears to be a negative relationship, with a peak value of −·33 when short rates lead G.D.P. by five quarters. For long rates, the picture is much the same, though the strongest correlation occurs one quarter earlier.

The strength of these links is much the same as that using a monetary aggregate, suggesting that the predictive power of interest rates is not measurably less than that of monetary aggregates when it comes to forecasting changes in output around a trend. This is not to say, however, that the money stock may not be superior when it comes to explaining the underlying trend, but that question is not examined here.

(b) Monetary components and expenditure aggregates

An important point in the debate about the role of monetary aggregates is whether changes in the money stock have the same effect on demand, irrespective of their origin. In particular, it may be argued that in situations where rationing of bank lending is an important feature of the system, an increase in the money supply resulting from more bank lending to the private sector (the monetisation of private sector debt) will have a different effect on demand, at least in the short term, from a growth in money that results from open-market operations (monetisation of public sector debt).

To test this question the cross-correlogram between bank lending to the private sector (advances and commercial bills) and G.D.P. is shown in Chart 3. It will be noted that the maximum correlation (r = 0·34) is much the same as that using the narrow definition of money supply, and that the chart has a similar lead/lag pattern.

Why should changes in advances lead changes in national income by a sizable margin? It would probably be widely assumed that under the overdraft system, an advance and the expenditure it financed would take place simultaneously. However, if bank borrowing is a preferred form of debt – and it has certainly been the case that bank credit has been cheaper in recent years than most alternative forms of borrowing – then it is conceivable that easier access to bank credit will initially give rise to repayment of other forms of debt. This might then result in a general easing of the credit situation and possibly a lowering of interest rates, which would combine to induce an increase in effective demand after a lag. However, it could also be the case that Chart 3

1 The local authority three-month deposit rate was used as a proxy for all short-term rates. For the first two years of the period, when these data were not available, changes were assumed to be the same as in Treasury bill rates.

simply reflects time lags in the effect of other policy measures. Economic 'packages' often include measures to stimulate or restrain bank lending; so that if the package takes time to have its full effect on final demand, there might appear to be a lead of bank advances over G.D.P. that did not reflect any direct link.

Chart 4 shows the cross-correlogram between income and bank assets other than advances and commercial bills – a total which could alternatively be described as the component of the money supply based on the monetisation of public sector debt.[1] There is a significant negative relationship between such assets and subsequent changes in G.D.P. which, taken with the results in Chart 3, implies that the assets in which banks invest additional deposits could be of significance for subsequent changes in income. This may well reflect the fact that, in 1955, bank lending was, for historical reasons, abnormally low, so that in expansionary phases, banks have been concerned to add to their advances by running down – or temporarily refraining from adding to – their other investments. In periods of quantitative credit restraint, on the other hand, banks will have been inhibited from making advances and will have tended to employ additional resources in public sector debt to a greater than normal extent.

Chart 5 shows that advances and commercial bills are, in fact, (and as might be expected from inspection of Charts 3

Chart 4

M_B less bank lending and G.D.P.

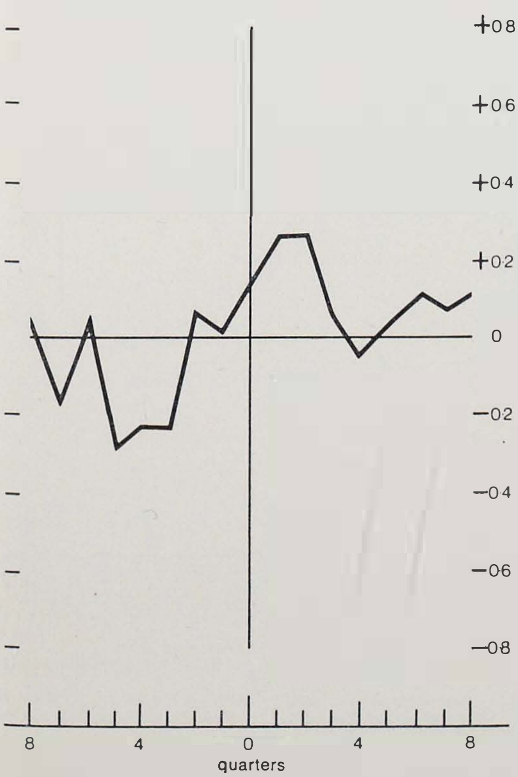
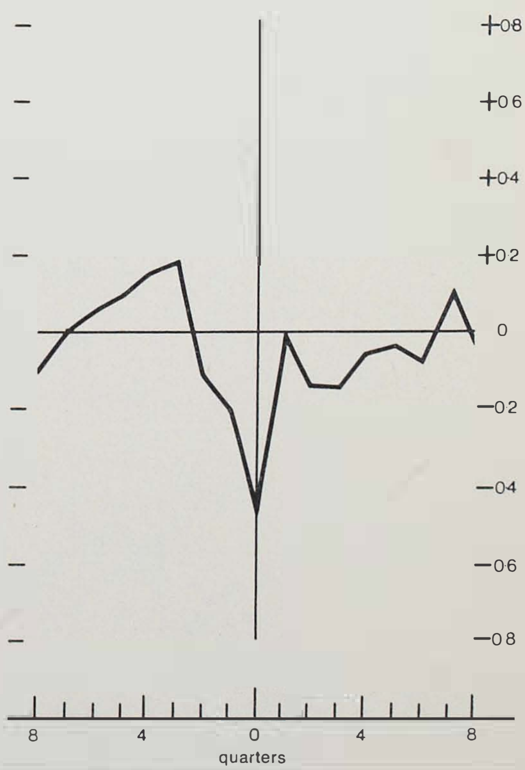


Chart 5

Bank lending and banks' other assets



1 Although not all these other assets are public sector debt.

and 4) negatively associated with simultaneous changes in the banks' holdings of other assets.

(c) Monetary aggregates and expenditure components

In so far as financial factors – whether measured as monetary aggregates or as credit aggregates – are measurably related to expenditure, the question arises whether the relationship is the same with all categories of expenditure. Monetarists generally seem to expect that variations in monetary quantities will exercise a pervasive effect on demand,[1] while the distribution of income and expenditure will be largely influenced by other factors, including fiscal policy. The empirical studies,[2] and the theoretical analysis, of those following a Keynesian income-expenditure approach, on the other hand, would suggest that the main impact of financial factors would fall on investment (including investment in durables),[3] while other categories of expenditure would be less directly affected.[4]

Charts 6–9, therefore, plot the relationship between various financial indicators, and, respectively, all private

Chart 6

M_N and all private expenditure

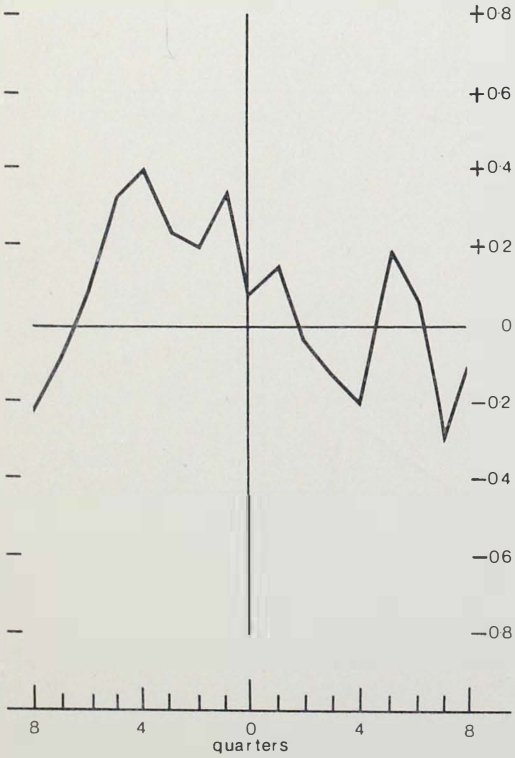
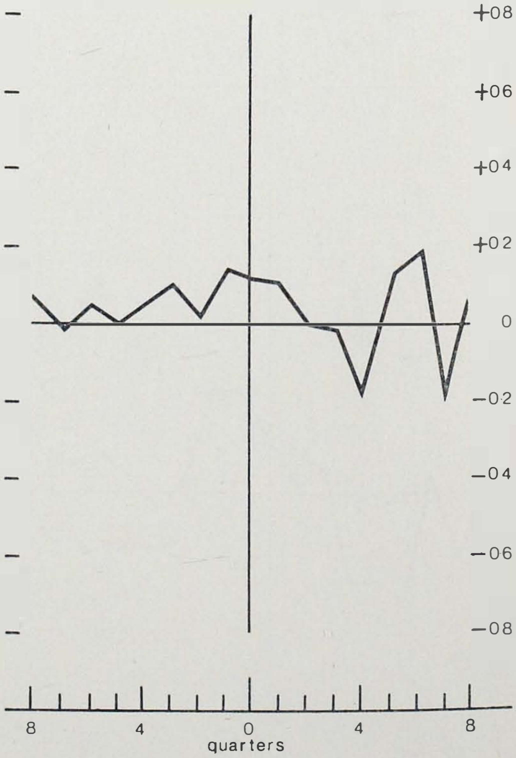


Chart 7

M_N and consumers expenditure



1 See Friedman and Meiselman, footnote *1*, page 460.

2 See, for example, "The Federal Reserve – MIT econometric model" by Frank de Leeuw and Edward Gramlich, *Federal Reserve Bulletin*, January 1968 pages 11–40.

3 Keynesian theory might also suggest that stockbuilding would be responsive to changes in financial conditions. In practice, it has proved very difficult to establish such a relationship empirically.

4 Though in so far as changes in financial conditions gave rise to a change in investment, which then affected consumption (via the multiplier), there could be a *statistical* association between financial factors and other components of demand.

Chart 8

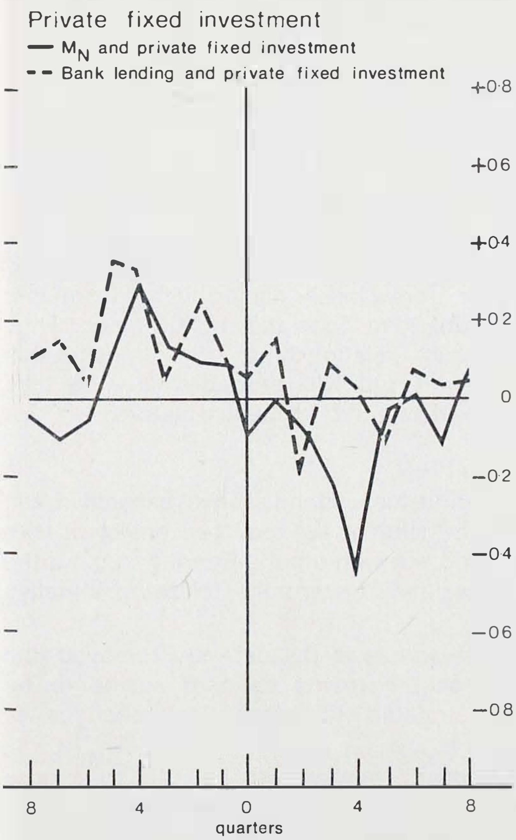
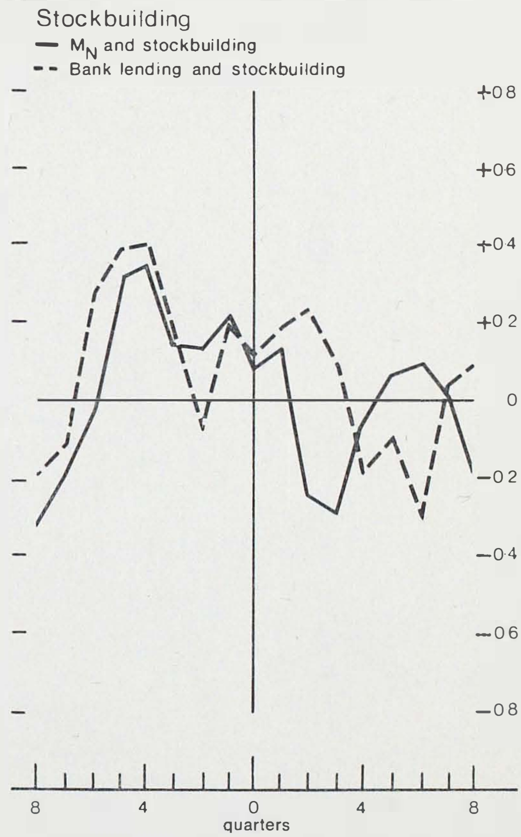


Chart 9



expenditure, and its components: consumers' expenditure, investment and stockbuilding. There is a noticeable 'twin peak' in the relationship between money and all private expenditure, a possible explanation of which may be that the link between money and consumers' expenditure is quite short, while that between monetary factors and investment is somewhat longer. A number of studies[1] have indicated that there are quite substantial delays in the planning and implementation of investment decisions. In principle, stockbuilding ought to react fairly quickly to changes in financial conditions; though empirical work on the matter has so far proved inconclusive.

From Chart 7 it will be seen that there is very little correlation between M_N and consumers' expenditure.[2]

Charts 8 and 9, however, contain rather more meaningful correlations. With both investment[3] and stockbuilding, there is a strong positive association both with money and with bank lending after a lag of some 4–5 quarters. For investment, this lag is consistent with the delay in planning and implementing investment indicated by other studies.[4] With stockbuilding, however, the lag is rather longer than might

1 See, for example, articles by J. P. Burman and A. G. Hines and G. Catephores in *The econometric study of the United Kingdom* (edited by Kenneth Hilton and David F. Heathfield); and Shirley Almon "The distributed lag between capital appropriations and expenditures" *Econometrica*, January 1965.

2 The correlation was rather higher (though not quite significant at the 5% level) when the broad definition of money was used.

3 The national income definition of investment, used here, does not include investment in consumer durables.

4 See, for example, Burman, footnote 1 above.

be expected on a *priori* grounds, since in principle one would probably expect companies to be able to adjust their holdings of stocks faster than their fixed capital.

(*d*) *Monetary components and expenditure components*
Precisely how monetary factors affect expenditure decisions is something that must await a more detailed model embodying the relationships which operate in the system. But it seems likely from Charts 8 and 9 that some measure of bank credit to the private sector may be a fruitful variable to use in regressions. In each of these charts, bank advances appear as closely, or more closely, linked with subsequent changes in expenditure than does the money stock. Thus whatever the long-run relationships, the transmission mechanism in the short run will probably have to take account of credit as well as purely monetary factors.

Conclusion
Although timing relationships cannot prove causality,[1] the length of the observed lead or lag may be consistent with certain hypotheses and not with others. Bearing in mind the necessary caveats, we may hazard the following tentative conclusions:

(i) Even when trends in the data are removed, the money stock, narrowly defined, seems to be positively related to subsequent changes in expenditure.

(ii) There appears, however, to be little to choose between monetary, credit and interest rate variables, as indicators of subsequent changes in G.D.P.

(iii) Investment appears to be more strongly related to changes in financial conditions than are the other components of expenditure.

In general it seems doubtful whether the strength of the observed association between monetary and real variables is great enough to support the view that control of monetary aggregates should be the main weapon of counter-cyclical macro-economic policy.

1 See J. L. Tobin and W. C. Brainard "Pitfalls in financial model building" *Papers and proceedings of the American Economic Association*, May 1968, pages 99–122 for a forcible demonstration of this.

Appendix
Cross-spectral analysis of monetary and income time series

A drawback of the technique of cross-correlograms, used in the body of the paper, is that the estimates of correlations at different lags are themselves correlated. To get around this problem the technique of spectral analysis can be used. A detailed description of this technique may be found in Granger and Hatanaka (*Spectral analysis of economic time series*, Princeton 1964) but it is possible to provide an intuitive understanding quite briefly.

It is well known that economic time series can be regarded as composed of a trend, a seasonal component (cycles repeating every twelve months) and an irregular component. The trend in turn may be decomposed into long-term growth and a business cycle (which in the United Kingdom has recently been repeating itself roughly every 4-5 years). If the business cycle is not completely constant in size, shape, or period, this implies the existence of other cycles interacting with the basic one. The irregular component is often regarded as not containing any cyclical (regularly recurring) influences; but in a formal mathematical sense it is possible to think of it as being composed of a large number of short cycles of different periods. Of course, as the number of cycles increases, the proportion of the fluctuations that each one explains becomes very small. The process can be extended until the variation of the time series about its average level has been *completely decomposed* into an infinite series of cycles, each explaining an infinitesimal amount of the total variation.[1]

Having broken a series down into its constitutent cycles, it is possible to determine which are the most important in explaining fluctuations in the series. For this we need a measure of the *length* of cycles, and for convenience 'frequencies' are used. The frequency of a cycle is simply the number of times it occurs in one time-interval (for example, with monthly observations, the annual cycle has a frequency of $\frac{1}{12}$, and the quarterly cycle has a frequency of $\frac{1}{3}$). It is convenient in tables and charts to use angular frequencies, which are obtained by multiplying actual frequencies by 360 and interpreting the result in degrees – so that the annual cycle would then have a frequency of $360/12 = 30°$.

The 'spectrum' is a measure of the amount of the overall variation in a series which can be explained by cycles of a particular frequency.[2] Thus, in a series strongly subject to seasonal influences (*e.g.* sales of turkeys), the value of the spectrum (known as its 'power') would certainly be highest in the frequency corresponding to a twelve-month cycle.[3] In general, the power of the spectrum for most economic series tends to be high at low frequencies (*i.e.* the longer cycles such as the business cycle, and seasonal variations) and much lower at high frequencies (corresponding to the irregular random component in the series).

To compare leads and lags in two series, one may use 'cross-spectral analysis'. Broadly speaking this involves comparing the relative position of cycles of the same frequency (length) in each series. For example, if there was a persistent lead of three months of one series over another, one would expect a lead of one quarter of a cycle in twelve-month cycles, half a cycle in six-month cycles, a whole cycle in three-month cycles, and so on.

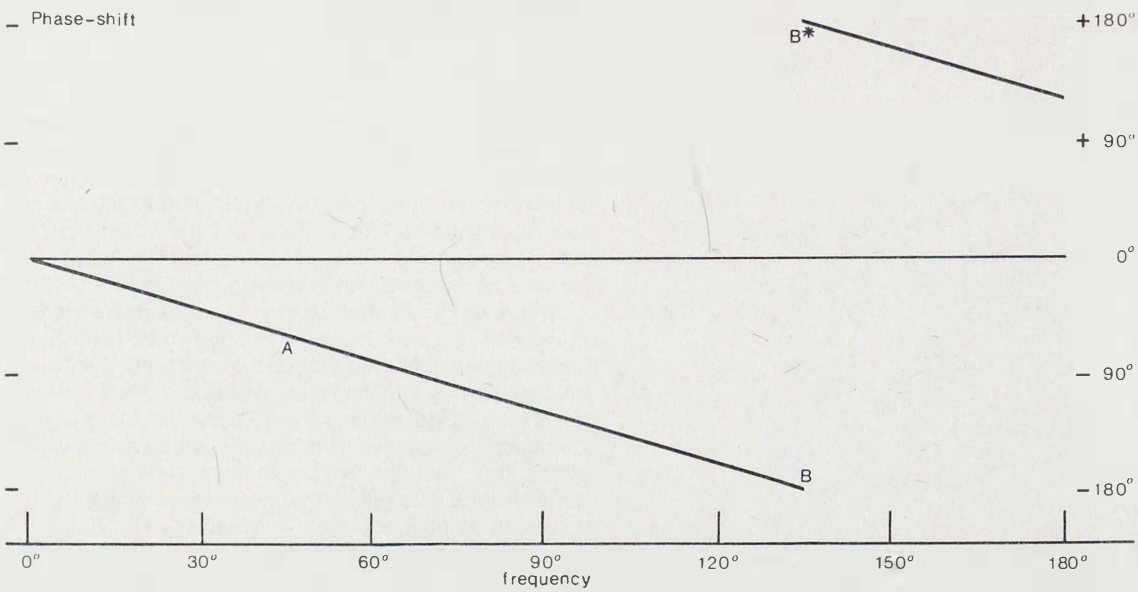
This moving lead in cycles of different lengths is called the 'phase-shift' and may be plotted on a diagram as follows: let the horizontal axis represent the frequency of the cycle concerned, measured in degrees. Let the vertical axis represent the lead of one series over another in each frequency band, expressed as a proportion of 360°

1 Strictly speaking this can only be done if the series is 'stationary' *i.e.* there is no trend in either the mean, or the variance, or in the correlation between observations equal distances apart. Many economic series are not 'stationary', but can be made approximately so *e.g.* by performing the operation described in footnote 5 on page 461.

2 There is, of course, theoretically an infinity of cycles; but we are concerned in practice with the average strength of the cycles in a finite *band* of frequencies.

3 Though possibly with subsidiary peaks in cycles corresponding to sub-divisions of twelve months (*i.e.* 6, 4, 3, 2·4 and 2-month cycles).

Example



(*e.g.* a phase shift of 90° would represent a lead of one quarter of a cycle).

As an example, let us assume that we are comparing two series with monthly observations. Take the point A on the example diagram. It corresponds to −60° on the vertical axis and 45° on the horizontal axis. This means that in cycles of length equal to eight months (360°/45°), series 1 leads series 2 by $\frac{1}{6}$ of a cycle (60°/360°); *i.e.* by $1\frac{1}{3}$ months. At point B, the cycle length is $2\frac{2}{3}$ months (360°/135°) and the lead of series 1 over series 2 is half a cycle (again $1\frac{1}{3}$ months).

Another way of making the same calculation is to divide the phase shift by the cycle frequency. At points A and B this gives −60°/45° and −180°/135° respectively, or $-1\frac{1}{3}$. This figure is the number of time periods (in this case months) by which one series leads the other. By working out the values of intermediate points it may readily be discovered that a constant lead of one series over another is represented as a straight line passing through the origin of the diagram.[1] It is, of course, extremely unlikely (because of estimation problems) that in practice all the plotted points would lie on a straight line: the average lead or lag in a particular frequency band can, therefore, only be approximated by the average slope of the phase-shift line over that band.

Naturally, a half-cycle *lead* can equally be interpreted as a half-cycle *lag*, and so point B reappears at the top of the diagram (B*). The break in the line is, then, merely a graphical convenience; conceptually points B* and B are the same.

The next question that arises is how much confidence can be placed in the average lead or lag discovered in cycles of a particular length. Is there a measure of significance comparable to the R^2 in ordinary regression analysis? There does exist such a measure, called the 'coherence', which, like the R^2 statistic, varies between 0 and 1. At a given frequency we can estimate the amplitude and phase (position) of the wave over successive cycle periods, for each series: they will vary in a random or systematic way. To say that the two series are fully coherent means that their variations in amplitude and phase completely match, so that there is a constant ratio

1 In fact, a line passing through any multiple of 360° at 0° frequency represents a constant lead of one series over another. The observation at 0° frequency, however, cannot itself be meaningfully interpreted, and although it is calculated in the program, it is not shown in the charts.

Diagram A

Index of industrial production and M_N

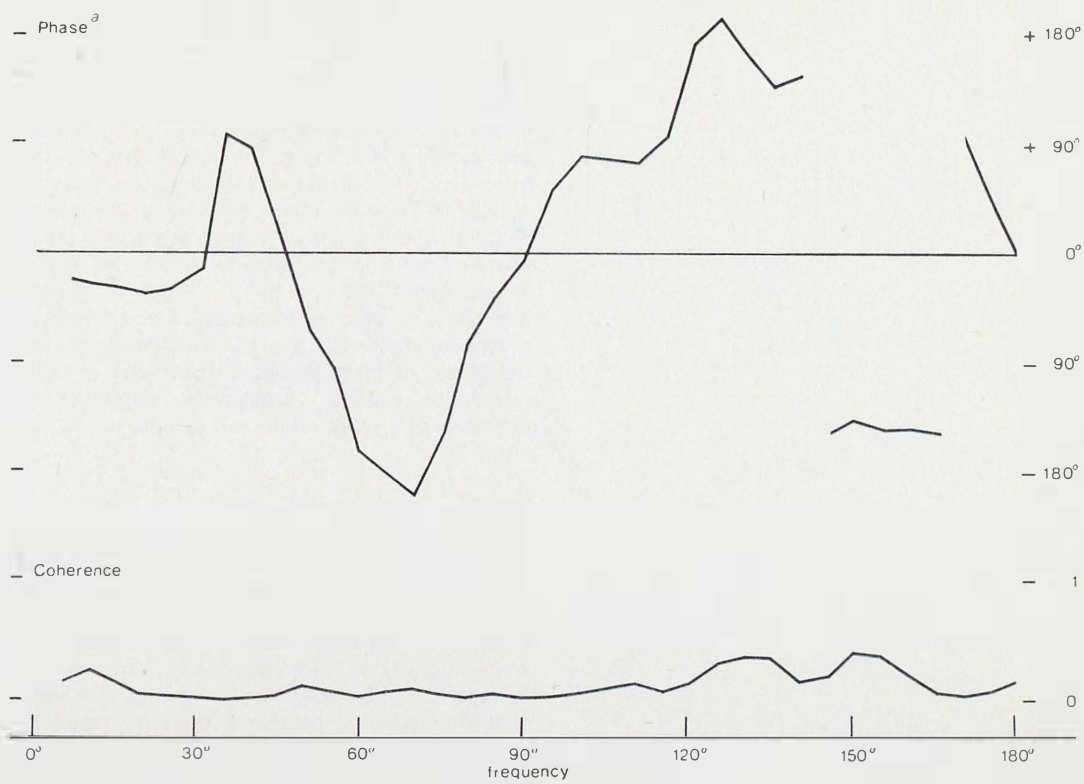
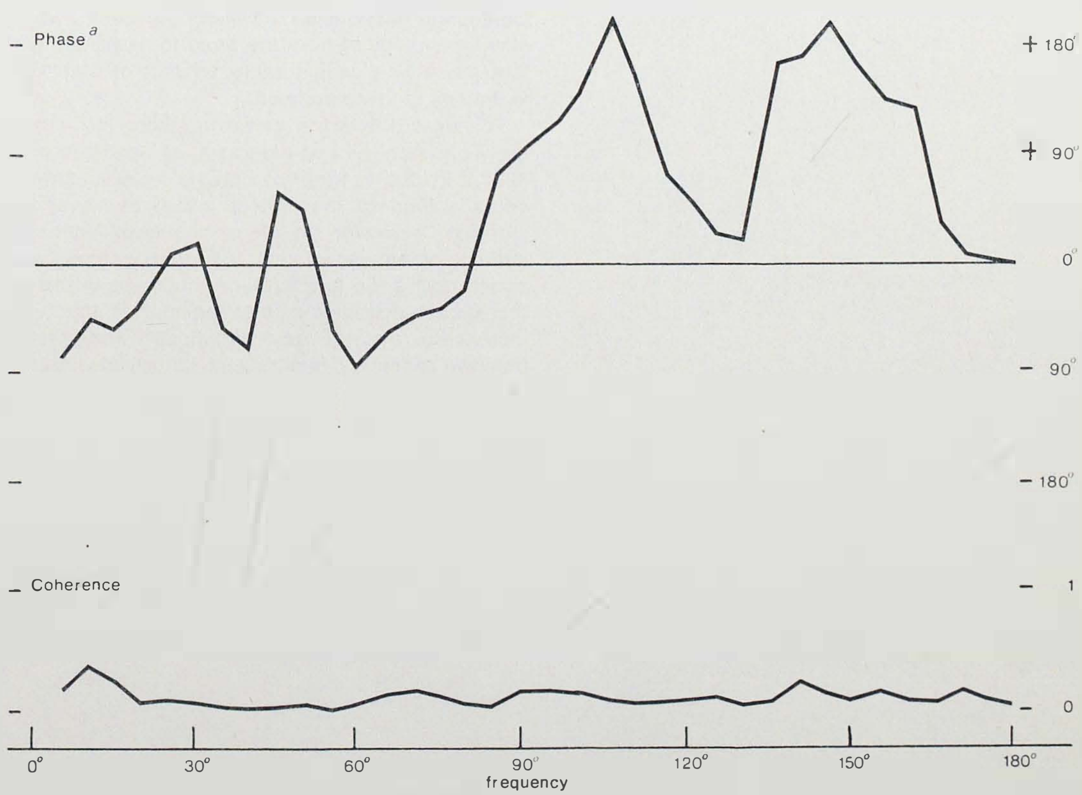


Diagram B

Index of industrial production and bank lending



a For graphical convenience, some plots are outside the band ± 180°.

between the amplitudes and a constant difference between the phases. If one series is, say, a moving average of another, their coherence is 1 at all frequencies. If we now add a completely random series to one of them, the coherence falls away, as the relation between amplitudes and phases becomes weaker.

As in linear regression, the slope of a phase-shift line can be tested for significance: the higher the coherence, the more significance can be attached to a given slope. So a phase diagram should always be looked at in conjunction with a chart of coherence against frequency. For instance, suppose the coherence equals 0·8 at frequency 30° and the phase diagram shows a straight line with phase shift 90° at this frequency (lead/lag of three months): for about 200 observations the 95% confidence interval is about ±13°. In other words, with 19/20 probability the time phase-shift lies between 77° and 103°, or lead/lag between 2·6 and 3·4 months. This would be a very satisfactory range; but if coherences are low, say below 0·5, the phase diagram provides much less reliable information at the relevant frequences. Granger also warns that the confidence-interval estimates are rather crude and depend on the efficient prior removal of the trend.

Results

The chief difficulty with using spectral analysis in empirical work is the large number of observations required. In practice, this has meant that monthly data have had to be used. Most monetary series are available monthly, but output and expenditure data are harder to come by. The index of industrial production, grossed up by the wholesale price index was used, and tested against a narrow definition of the money stock, and against advances and commercial bills. The spectral diagrams are reproduced here as Diagrams A and B; a downward slope from left to right implies a lead of output over money (or advances), an upward slope a lag. The main feature of both diagrams is the extremely low coherence in nearly all frequencies. This implies that there is in fact no statistically significant relationship between the series charted. In part, this may have resulted from the need to use monthly data as it is known that there is a considerable amount of statistical 'noise' (random variation) in these series.

For what it is worth, however, there is a certain superficial similarity between the two diagrams, at least in the frequency band 60°-120°. In this range, both charts have an upward slope that indicates a lead of money (or advances) over output of about six months. However, in other frequency bands, there are no indications of any consistent lag relationships, and in any case the coherence is too low to place much weight on the results. It must therefore be concluded that the tool of spectral analysis has been unable to discern any significant and systematic relationship between monetary factors and industrial output.