Measuring variability in economic time series

Introduction

Most economic time series are now published in a seasonally adjusted form. This means that estimates are made of the effect, during the course of a year, of regularly recurring fluctuations (e.g. due to the weather, holidays, seasonal fluctuations in demand) and that they are removed from the basic figures. Series in this form can be used readily for comparing different points in monthly or quarterly series with seasonal influences abstracted. They avoid the misleading impression often given by comparisons of unadjusted figures with corresponding periods in earlier years. An important use of seasonally adjusted series is to assess as reliably as possible current trends or, more strictly, the underlying movement implied by the latest figures. Unfortunately for achieving this aim, seasonally adjusted series are seldom smooth and are affected by irregular factors which overlay and mask the underlying movement.

The extent of this irregular movement in period to period changes as well as varying over time also varies from series to series. It would go some way to help the user, therefore, to provide some measure of the average extent of this irregular movement in each series and to indicate how the smoothing of seasonally adjusted figures would help in assessing changes in the underlying movement. The amount of smoothing to be suggested to the user depends on the magnitude of the average irregular movement in relation to the average movement in the trend; the larger the former compared with the latter the greater will be the smoothing needed. However, the more drastic is the smoothing the less up-to-date can be the assessment of the underlying movement and the user may have to compromise between his needs for topicality and reliability in judging the extent of smoothing. Some guidance as to the likely cost in terms of reliability of different degrees of smoothing is needed.

In the presentation of a number of monthly economic series, the convention has been adopted of showing the percentage change between the average of the most recent three months' figures and that of the previous three months. This can be a useful device for ironing out major month to month variations but it does not necessarily indicate the extent to which this change can be regarded as the most recent trend. The primary purpose of this article is to examine a method of analysing and measuring variability which has been pioneered in the United States and adopted by a number of countries, and to introduce the regular publication for major United Kingdom economic series of the resulting measures. The article also describes another way of looking at the problem which has been developed by the Department of Trade and Industry to measure variability in the monthly figures of exports and imports, and which it is proposed to examine further with a view to its wider application to United Kingdom economic series.

The $\bar{I}/\bar{C}$ ratio

As part of the process of seasonal adjustment, the original series (O) is decomposed into the components accounted for by seasonality (S), the underlying trend cycle (C) and irregular factors (I); so that where (as is normally the case) the process of decomposition is multiplicative, $O=S \times C \times I$(¹). A measure of the variability between successive observations in a seasonally adjusted series in relation to movements in the trend component can be obtained by analysing the estimates of the last two elements, and is called the $\bar{I}/\bar{C}$ ratio. $\bar{I}$ represents the average month to month (or quarter to quarter) percentage change, without regard to sign, in the irregular component as defined above (obtained by dividing the trend component into the seasonally adjusted series). $\bar{C}$ is the average month to month (or quarter to quarter) percentage change without regard to sign in the trend component(²). (If the process of seasonal adjustment is done using an additive decomposition, $O=S+C+I$, percentage changes are usually replaced by differences in the calculation of I and C.) If the $\bar{I}/\bar{C}$ ratio is greater than 1, (for example it is 2·1 for the monthly index of industrial production), the period to period changes *on average* will be mainly reflecting irregular movements; correspondingly, when the ratio is less than 1, (for example it is 0·5 for the average quarterly estimate of GDP at constant prices) the period to period changes mainly reflect the trend. The larger, therefore, is the $\bar{I}/\bar{C}$ ratio, the greater is the extent of smoothing likely to be necessary, and the greater the loss of topicality, to achieve an assessment of a trend of a given reliability.

It is worth at this stage making it clear that the $\bar{I}/\bar{C}$ ratio is not a unique measure of variability. In particular it is dependent on the method of seasonal adjustment that is used. Many official economic series are adjusted using the United States Bureau of the Census Method II Variant X-11 programme, or an adaptation of it; other programmes are used for some series. The seasonally adjusted series and estimates of the $\bar{I}$ and $\bar{C}$ elements will depend on what programme is used; and, indeed, for all given programmes there is normally a range of options that can be taken at various stages in the seasonal adjustment process, the choice of which will affect the resulting

---

(¹) In some economic series the long-term underlying movement is overlaid by shorter term cycles of growth, stability and decline. The component (C) is, strictly speaking, a combination of both elements, which are not separately identified by the seasonal adjustment process. In such series the cyclical element normally predominates over the long-term underlying trend, and in this article references to the 'trend' are to the short-term rather than to the long-term underlying movements in a series.

(²) As $\bar{I}$ is the average of the percentage differences, ignoring sign, between successive I's, and $\bar{C}$ the average of similar differences between successive C's, $\bar{I}$ and $\bar{C}$ should, strictly speaking, be written as $(\overline{100|\Delta I|/I})$ and $(\overline{100|\Delta C|/C})$, respectively. For convenience of presentation, however, the notation $\bar{I}$, $\bar{C}$ is adopted.

decomposition of the series and consequently the $\bar{I}/\bar{C}$ ratios. Of particular importance is the way in which the trend is obtained. In most methods the trend is a smooth flexible moving average of the seasonally adjusted series. It is frequent practice, however, to make prior adjustments to the original data where it is judged that it is preferable to make some rough allowance for exceptional circumstances in particular periods to prevent them distorting the calculation of the seasonal adjustment factors and the trend. For example, prior adjustments are made to the recorded figures of exports and imports to allow for dock strikes. Whenever prior adjustments are made these are excluded from the calculation of $\bar{I}$ and the $\bar{I}/\bar{C}$ ratio which are therefore smaller than if no adjustment had been made. It is also possible in some programmes—e.g. the Census Method II Variant X-11—to adopt an option for further smoothing of the trend; this would increase the $\bar{I}/\bar{C}$ ratio.

Some other related factors need also to be borne in mind in using $\bar{I}/\bar{C}$ ratios. First, as it is, by definition, a ratio of the average of changes in the irregular component to the average of changes in the trend, it will, other things being equal, be high in periods when the trend is virtually flat. Conversely, it will be relatively low in periods when the trend is moving sharply upwards or downwards. For this reason, the $\bar{I}/\bar{C}$ ratio tends to be lower for series at current prices than for those at constant prices because of the upward trend in prices. Taking the monthly series for retail sales at current and constant prices during the period 1961 to 1970 as an example, whereas both series have 0·7 for $\bar{I}$, the $\bar{I}/\bar{C}$ ratios are 1·7 and 3·4 respectively. This reflects the average percentage month to month change in the trend component for the current price series being twice that for the constant price series.

Secondly, the length of the series used in the calculation of $\bar{I}/\bar{C}$ often encompasses periods with both sharp movements and stability in the trend. The measure thus represents an average of experience in the past and care is needed when using it in relation to any particular period. It may, indeed, cover a period during which a number of cycles of growth, stability and decline in the trend are superimposed on the long-term underlying movement in a series. In addition the analysis described in this article is carried out on final figures. The latest figures for any series may be subject to revision and will generally include more irregularity than indicated by the $\bar{I}/\bar{C}$ ratio.

If the factors outlined in the paragraphs above are borne in mind by the user, the $\bar{I}/\bar{C}$ ratios give useful additional information about the series which provides at least a starting point in interpreting the significance of period to period movements in seasonally adjusted series in assessing the underlying trend. No precise guidance can be given for applying the $\bar{I}/\bar{C}$ ratio. It provides a measure, based on past data, of the average extent of movements in the irregular component in relation to the trend. The user must judge for himself, in relation to his needs for topicality and accuracy, and in the light of his assessment of factors that may be influencing a series at a particular time, the amount of smoothing of the seasonally adjusted series that is likely to meet his needs, given the size of the $\bar{I}/\bar{C}$ ratio. It will sometimes help the user in his assessment of variability to look separately at the average movements in the irregular component and the cyclical component, as well as at the relationship between them which is expressed by the $\bar{I}/\bar{C}$ ratio. $\bar{I}$ itself is valuable as a general indicator of variability. For example, in a strongly growing series there is interest not only in the rate of growth but in changes in the rate of growth; in assessing the latter, $\bar{I}$ would be useful as a guide to the size of irregular fluctuations which could then be borne in mind in relation to the changes being observed in the rate of growth. Figures for $\bar{I}$ and $\bar{C}$ are therefore given for each series in the table at the end of the section 'Charts and Tables', and also for $\overline{CI}$ the average period to period change in the seasonally adjusted series.

The MCD (and QCD)

The MCD is a statistic closely associated with the $\bar{I}/\bar{C}$ ratio. $\bar{I}$ and $\bar{C}$ as defined above are calculated from changes from one period to another in a seasonally adjusted series. They can also be calculated from changes spanning two or more periods and as the number of periods spanned is increased the average irregular movement generally becomes a decreasing proportion of the average movement in the seasonally adjusted series. The number of periods that need to be spanned to bring the $\bar{I}/\bar{C}$ ratio below 1 is described as the MCD (months for cyclical dominance) or QCD in the case of quarterly series. Changes over the MCD span of months are proportional to changes between adjacent months in an unweighted moving average of MCD length, where the constant of proportionality is the MCD. This relationship holds approximately for percentage changes. The MCD (or QCD) thus indicates the least number of months (or quarters) that need to be taken for calculating from the seasonally adjusted series, a moving average in which the movement in the trend component is likely on average to be at least as large as that in the irregular component. For example, the monthly index of industrial production for manufacturing industry between January 1963 and June 1971 has an $\bar{I}/\bar{C}$ ratio of 2·5; the shortest span for which the average change without regard to sign is less for the irregular component than for the trend is three months, for which $\bar{I}/\bar{C}$ is 0·8. The MCD, therefore, is 3 and month to month changes in a three-month moving average of the seasonally adjusted series will on average be dominated by changes in the trend component. Where the $\bar{I}/\bar{C}$ ratio for one month's span is less than 1, (e.g. in the series for wholly unemployed) the MCD is, of course, 1.

While the minimum span of periods which must be taken to obtain an $\bar{I}/\bar{C}$ ratio below 1 (i.e. the MCD or QCD) may be the same for a number of series, there may still be a substantial variation in the extent to which the average movement in the trend component predominates over the average movement in the irregular component. In order, therefore, to enable the user to make a more precise assessment the $\bar{I}/\bar{C}$ ratio for the span equal to the MCD (or QCD) is also given. This figure is particularly useful when the MCD (or QCD) is 1. For instance if comparing two series both of which have an MCD of 1, but where one has an $\bar{I}/\bar{C}$ ratio of 0·9 and the other an $\bar{I}/\bar{C}$ ratio of 0·3, the trend movements are clearly dominating irregular movements much more for the latter series than for the former. If a high degree of such domination is required by the user (e.g. $\bar{I}/\bar{C}$ ratio of 0·5 or less) movements over at least two months must be considered for the first series.

It may be noted that for some series the span of periods needed to arrive at an $\bar{I}/\bar{C}$ ratio below 1 may be quite lengthy. This may occur for one or two reasons. First, because the seasonally adjusted series is itself extremely variable; for example in the case of engineering export new orders, an $\bar{I}/\bar{C}$ ratio of less than 1 is not achieved even after taking a 12 month span. Second, as mentioned above, when the trend is flat, although the series is not highly variable. In these cases increases in the span of periods taken do not significantly increase the average change in the trend and thus do not much reduce the $\bar{I}/\bar{C}$ ratio. Other things being equal, therefore, the sharper the rise (or fall) in the trend, the lower will be both the $\bar{I}/\bar{C}$ ratio and the MCD.

The factors to be taken into account when using the $\bar{I}/\bar{C}$ ratio, and outlined above, apply with equal force to the MCD (and QCD). The user also needs to bear in mind that it indicates only one aspect of variability. It simply shows how many periods need to be combined so that, on average, the trend element predominates over the irregular element; this may or may not be a sufficiently stringent requirement for the user's purpose.

Finally, two other facets of the MCD (or QCD) should be mentioned. First, it is frequent practice to quote a value of 6 in all cases where the figure is 6 or more, in which case no $\bar{I}/\bar{C}$ ratio for MCD span is given; secondly while the MCD (or QCD) can frequently be obtained by rounding the $\bar{I}/\bar{C}$ ratio for one months span up to the nearest whole number, this is not invariably the case.

Standard deviations: an alternative approach

Another approach to assessing variability which has been applied to the monthly figures of exports and imports, is to calculate standard deviations(3). Two which are particularly useful are that of seasonally adjusted values about the trend line, and that of differences between seasonally adjusted values. These standard deviations can be calculated either for single observations or for moving averages. This approach to measuring variability is especially helpful in judging the closeness of seasonally adjusted values to the trend line and also, as an alternative to $\bar{I}/\bar{C}$ ratios, as a means of assessing the significance of *movements* in seasonally adjusted series.

Looking first at the standard deviation of individual seasonally adjusted values about the trend line, to calculate this, the percentage differences between the seasonally adjusted values and the trend line are squared, the average of these squares calculated and then the square root taken of this average. (It is of interest to note that these percentage differences between the seasonally adjusted values and the trend line are virtually identical to the deviations of the irregular component about its mean so that the calculation is virtually equivalent to that of the standard deviation of the irregular component.) The standard deviation about the trend line for exports, for example, is about 2 per cent. Empirical evidence from export data over a period of about 15 years suggests that the distribution of the irregular component, excluding extreme values is approximately normal. This means that, apart from extreme fluctuations caused by serious disturbances, on average about two-thirds of the monthly figures of exports will fall within a range of 2 per cent above or below the trend line. About one-third of the monthly values, however, will fall outside this range and about one in twenty will fall beyond about 4 per cent on either side of the trend line. As a refinement, of importance in some series, the standard deviation and corresponding ranges can be calculated separately for each of the twelve months; these show whether the seasonally adjusted values are a less reliable guide to the trend values in some months than in others.

Averages of three months, often used to smooth out some of the variability in seasonally adjusted series, may be expected to lie nearer the trend line. This happens particularly in the case of series where successive values tend to be inversely correlated. The standard deviation of moving averages about the trend line can be calculated in the same way as for single observations. In the case of exports, the standard deviation for three-month moving averages is about $\frac{3}{4}$ per cent. Thus about two-thirds of these averages (again, apart from extreme fluctuations caused by serious disturbances) may be expected to fall within a band of about $\frac{3}{4}$ per cent on either side of the trend line, leaving about one-third outside it; about one in twenty may fall beyond about $1\frac{1}{2}$ per cent.

In assessing the significance of movements shown by seasonally adjusted figures, interest lies in the probability that movements of a certain size can occur as part of the normal fluctuations of the series. The movements might be between single observations or, for example, between non-overlapping three-monthly averages. The standard deviations of these movements can also be calculated(4). In the case of exports they are about $3\frac{1}{3}$ per cent and $1\frac{1}{3}$ per cent respectively. This means that about one-third of movements between individual observations (apart from extreme fluctuations caused by serious disturbances) can be expected to be greater than about $3\frac{1}{3}$ per cent on account of normal fluctuations in the series and about one in twenty of movements greater than about $6\frac{2}{3}$ per cent. In the case of non-overlapping three-monthly averages of the export series, about one-third of movements between such averages can be expected to be greater than about $1\frac{1}{3}$ per cent on account of normal fluctuations of the series and one in twenty greater than about $2\frac{2}{3}$ per cent.

Conclusion

There can be no fully satisfactory way of providing the user with a ready made means of determining the current trend from the last few observations in a seasonally adjusted series. All methods must necessarily be based on analyses of past experience and relationships which may or may not be appropriate to the period under review. At best they can provide only a 'band' within which the trend is generally likely to fall.

(3) See article 'The seasonal adjustment of United Kingdom overseas trade figures' in *Trade and Industry*, 17 February 1971.

(4) For differences between single observations, the standard deviation is calculated by squaring the differences between adjacent values of the irregular component, taking the average of these and then the square root of this average; for differences between non-overlapping three-month averages, the standard deviation is calculated from differences between adjacent non-overlapping values of three-month moving averages of the irregular component. The standard deviations calculated in this way from adjacent values are also broadly appropriate for comparisons of non-adjacent values, of either single or non-overlapping three-month averages.

The analysis by way of the $\bar{I}/\bar{C}$ ratio has its limitations, but it is a method used in a number of other countries and enables the user to set a series in perspective and gives him a starting point in examining variability. Starting this month, $\bar{I}/\bar{C}$ ratios and the MCD (or QCD) for a number of major economic indicators, including the main components of the national accounts, will be given at the end of the section 'Charts and Tables' of each issue of *Economic Trends*. The symbol § against the heading of the appropriate series in the section 'Charts and Tables' will indicate that these measures of variability are given. (Some monthly series are given as quarterly averages in the sub-section 'Main economic series' as well as in monthly form later in these sections: the measures of variability relate to the monthly series.) The period over which the measures have been calculated are shown for each series. The measures will be updated when the seasonally adjusted series are themselves updated (normally annually), and the range of series included will be kept under review.

Further work in this area will continue, and in particular consideration will be given to developing, and giving wider application to, the analysis of standard deviations.

Central Statistical Office