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Keywords: Deflation, Distributional national accounts, welfare indicators

JEL classification: D30, D63, I30, E21

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Abstract

In this note we discuss suitable deflators for distributional national accounts. Measures such as the top 10% income share, or the 90/10 income ratio do not require deflation. Other measures such as the income or income growth of particular percentiles of the income distribution do require deflation. We argue that democratic deflators are generally more appropriate than plutocratic deflators, and the ONS now produces democratic price indices for equivalised income quintiles and deciles, in addition to the aggregate democratic CPIH.

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1 Introduction

The OECD is encouraging member states to produce distributional national accounts.¹ In contrast to other data on income distribution, these data are structured so as to be consistent with the corresponding measures in the national accounts, taking into account a wider range of income sources than is typically found in household survey data, for example. This raises the question of what deflators are appropriate to draw inferences

¹See for example Fesseau and Mattonetti (2013) and Zwiijnenburg *et al.* (2017) for details on the methodology developed by the OECD-Eurostat Expert Group on Disparities in a National Accounts framework (EG DNA). Separately, there is also the DINA project developed by the World Wealth and Income Database (WID.world) who have developed annual estimates of the distribution of income and wealth consistent with national accounts (Alvaredo *et al.*, 2017).

about changes in the circumstances of households and household groups in real terms. In this note it is assumed that household incomes as represented in the distributional accounts will be adjusted for household size, using an appropriate equivalence scale and the issue of equivalence scales is not discussed further.

The deflators are likely to be specific to the questions being asked. For example, to know how the circumstances of households in the lowest decile of the income distribution have changed, one should ideally use a deflator appropriate to these households. Such a deflator is likely to give more weight to food and fuel than is a deflator for the highest decile of the income distribution. At the same time, it has to be recognised that, without panel data, it is not possible to say how the living standards of those currently in the lowest decile have changed over some interval. But we can answer the question of how the living standards of those in the bottom decile compare with those of the households that had been in the bottom decile in some previous period.

The weights used in the calculation of deflators such as the CPI are normally derived from the shares in the total spending of the group under consideration. Prais (1958) referred to such deflators as plutocratic because the expenditure weights are biased towards the spending patterns of those with high overall consumption. He contrasted these with democratic deflators which were calculated from the averages of the expenditure shares of each household in the group in question.

Aitken and Weale (2020) show that plutocratic deflators are appropriate when one is concerned with measuring the experience of an average household, i.e. one whose income or consumption is the average value for the group in question. Democratic deflators are appropriate if one is concerned about the average of the experiences of the individual households in the group in question, and these should be applied to geometric means. This is explained fully in appendix A. The application of democratic deflators to aggregates or arithmetic averages results in variables in real terms which cannot obviously be interpreted either as the experience of the average household or the average of the experiences of individual households.

A particular issue arises with the deflation of the median household income. Given the variability of spending patterns across households, it would hardly be satisfactory to derive weights from the expenditure pattern of the median household at the date in question. The question is how best to represent this given available deflators.

2 The Arithmetic Mean, the Median and the Geometric Mean

The distributional accounts make it possible to calculate a number of measures of central tendency and there is perhaps the risk of an *embarras de richesse*. The three obvious measures of central tendency are the arithmetic mean, the geometric mean and the median and, as argued below, each has its own deflator. If these measures were calculated with no adjustment for household size, the arithmetic mean could be calculated simply by dividing aggregate household income by the number of households.² But once the focus changes to equivalised household income, this is no longer true.

The mean is, of course, sensitive to extreme values; growth in the mean will not be a good indicator of changing social welfare if all the growth occurs at the top of the income distribution. Many researchers have therefore instead focused on the median. This describes the experience of a “middling” household but, as a guide to social welfare, suffers from the opposite criticism, that it is insensitive to movements except in the middle of the distribution; for example the median would not be sensitive to a sharp fall in the incomes of the people in the lowest quintile of the distribution.

The geometric mean offers a compromise between these two. Unlike the median, it is sensitive to all the observations. At the same time it gives less weight to extreme observations than does the arithmetic mean. And we show in appendix A that it is the appropriate starting point if one wants to monitor changes in welfare.

3 Distributional Indicators in Current Prices and Deflated Data

The distributional accounts will be used to inform the public about metrics such as the proportion of total income which accrues to the top one per cent of the population, or the ratio of the ninetieth percentile to that of the tenth percentile. Statistics such as these, and other measures of inequality such as the Gini coefficient, do not require any deflation.

Users are, however, also likely to want to know the answers to questions such as how have living standards changed for those households in particular distributional ranges, and to

²Abstracting from the issues raised by people who live in communal establishments.

compare their experiences with that of the population as a whole. If we were looking at a single group of households at two different periods, the growth of the geometric mean of the of household income in the group, deflated by the growth of the democratic price index, would show the average of each household's growth in real income. Abstracting from ageing/mortality and migration, that applies for the population as a whole. But it does not apply to subgroups, because the membership of them changes between the dates in question. The most we can say is that measures structured round both geometric and arithmetic means allow us to infer the changes in welfare of synthetic households. Thus the democratic price deflator applied to the geometric mean of income for the lowest decile would tell us about the growth in real income of a household whose spending patterns in successive years were those of the average of the lowest decile.³ Or, equivalently, it would tell us about the average of the real income growth rates for a synthetic group of households whose spending patterns matched those of the lowest decile.

Application of the plutocratic deflator to the arithmetic mean of household income would tell us about the growth in the real income of a household with a spending pattern matching that shown by the total expenditure on each good by the group. Equivalently it would show the weighted average of the rates of growth of the real incomes of the synthetic households in the group, weighted by each household's share in the total expenditure of the group.

Statistical offices could coherently take the view that, because of the artificial nature of the constructs in question, they are not going to provide deflators which facilitate the sort of calculations set out above. That would, however, contradict the spirit of the new Household Cost Indices; the experimental releases for those show household cost indices for each income decile. At present they suffer from the fault that, while users are provided with democratic price indices, they are not also provided with the geometric nominal means for which those deflators are appropriate, but that is of course easily remedied. Alternatively statistical offices need to decide whether they will steer users towards the use of the arithmetic mean and plutocratic deflator or geometric mean and democratic deflator. Since the aim of the distributional accounts is to aggregate the experiences of individual households, the second seems the more appropriate.

³The ONS now produces democratic price indices for equivalised income quintiles and deciles, in addition to the aggregate democratic CPIH and CPIH excluding housing. See [CPIH-consistent inflation rate estimates for UK household groups: October to December 2019](#).

4 Deflation of Median Income

Deflation of median income raises particular problems because it, in principle at least, requires the construction of a price index for the median household. Of course, since there is only one household in this category, the consumption pattern of the pseudo-household which lies on the median, is likely to be highly unstable. It should be noted that, were it possible to construct such a deflator, the democratic/plutocratic issue would not arise. For a single household they are the same. But in reality it seems likely that an appropriate deflator might be calculated in one of two ways. Option 1 is to construct a price index for households close to the median, so that it can be interpreted as a price index for households centred on the median. One obvious choice would be to construct a price index for the households in the central quintile. Here there is a choice as to whether it should be democratic or plutocratic. The strong recommendation has to be that it should be democratic; the plutocratic index weights households spending patterns by their expenditure shares. As a result it is likely overall to reflect the spending patterns of households above the median. In contrast the democratic index uses the average of the spending patterns of each household and, for the central quintile of households, one can argue that that is centred on the median.

The alternative approach is, as far as we are aware, untested. Kernel estimators offer a non-parametric approach to modelling the distribution of income from discrete survey data. In the same way multivariate kernels can be used to smooth in multiple dimensions, and these offer a means of smoothing consumption patterns. It is unlikely that this approach could be applied to the eighty-four COICOP categories but it could probably be applied to the twelve broad categories. The consumption shares of these twelve broad categories could be subdivided on the basis of the democratic weights for the central quintile of households.

If this kernel approach proves practical, it could of course, be used to provide weights at other points in the income distribution or, of course, for ranges of the distribution.

5 Deflation of Consumption

Jorgenson and Schreyer (2017) suggested the use of the mean value of log consumption as a welfare indicator. In order to reflect the fact that households at different points in the

consumption distribution have different spending patterns, they advocate constructing (plutocratic) price indices for each consumption quintile. The results at the end of appendix A point to the importance of using the democratic rather than the plutocratic deflator. If we want to measure the change in the mean of the log of real consumption we need to deduct from the mean of the log of nominal consumption, the mean of the log of each household's price index: this is the rate of growth of the democratic price index. The result can, of course, be interpreted only if one looks at the change in the mean of log of real consumption relative to the reference period which provides the base for the household price index.

6 Conclusion

Where deflation of distributional national accounts is required, democratic deflators are generally more appropriate than plutocratic deflators. The only situation where that is not the case arises if utility is linear in consumption (and utility accruing linear in income), as opposed to logarithmic or approximately logarithmic. But that is not generally believed to be the case.

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A Arithmetic and Geometric Means; Plutocratic and Democratic Price Indices

This appendix draws heavily on Aitken and Weale (2020). For an individual household expenditure in money terms C_j can be defined as the product of consumption in real terms, Q_j and a price index P_j . For convenience time subscripts are omitted, but in the base year we assume that $P_j = 1$ for all j . We also make the Divisia assumption that expenditure shares can be monitored continuously. While a range of approximations is needed in practice, that is not material to the distinction between plutocratic and democratic indices. We use the superscript A to represent the arithmetic mean and G to represent the geometric mean. Thus with N households

$$C^A = \sum_j C_j/N = \sum_j P_j Q_j/N$$

and

$$\frac{\dot{C}^A}{C^A} = \frac{\dot{P}^A}{P^A} + \frac{\dot{Q}^A}{Q^A} = \sum_j \frac{C_j}{\sum_j C_j} \left(\frac{\dot{P}_j}{P_j} + \frac{\dot{Q}_j}{Q_j} \right).$$

This decomposition makes it clear that we should define P^A such that

$$\frac{\dot{P}^A}{P^A} = \sum_j \frac{C_j}{\sum_j C_j} \left(\frac{\dot{P}_j}{P_j} \right)$$

giving

$$\frac{\dot{Q}^A}{Q^A} = \sum_j \frac{C_j}{\sum_j C_j} \left(\frac{\dot{Q}_j}{Q_j} \right)$$

Further, if household j spends a share ω_{ij} on good i with price p_i then its Divisia price index is

$$\frac{\dot{P}_j}{P_j} = \sum_i \omega_{ij} \frac{\dot{p}_i}{p_i} \tag{1}$$

So we can see that

$$\frac{\dot{P}^A}{P^A} = \sum_j \frac{C_j}{\sum_j C_j} \sum_i \omega_{ij} \frac{\dot{p}_i}{p_i} = \sum_i \frac{\dot{p}_i}{p_i} \sum_j \frac{C_j \omega_{ij}}{\sum_j C_j}.$$

But $\sum_j C_j \omega_{ij} = \sum_j c_{ij}$ where c_{ij} is the expenditure on good i by household j and $\sum_j \frac{C_j \omega_{ij}}{\sum_j C_j}$ shows shares in total expenditure. It follows immediately that the rate of growth

of the price index associated with the arithmetic mean of household consumption is calculated by weighting the rate of growth of the price of each good by its share in total expenditure for the group of households in consideration and that this gives greater weight to those households with high household consumption than those with low household consumption.

The geometric mean is written as

$$C^G = \sqrt[N]{\prod_j C_j} = \sqrt[N]{\prod_j P_j} \sqrt[N]{\prod_j Q_j}$$

giving

$$\frac{\dot{C}^G}{C^G} = \frac{1}{N} \sum_j \frac{\dot{C}_j}{C_j} = \frac{1}{N} \sum_j \left(\frac{\dot{P}_j}{P_j} + \frac{\dot{Q}_j}{Q_j} \right).$$

suggesting that the relevant price index is defined as

$$\frac{\dot{P}^G}{P^G} = \frac{1}{N} \sum_j \frac{\dot{P}_j}{P_j} = \frac{1}{N} \sum_i \omega_{ij} \frac{\dot{p}_i}{p_i} = \sum_i \frac{\dot{p}_i}{p_i} \frac{\sum_j \omega_{ij}}{N}$$

and we can see that the price changes of each good are weighted by the arithmetic average of the expenditure shares of the households in question, giving us the rate of growth of the democratic price index. Further, it is clear that the associated quantity measure is the arithmetic average of the volume growth rates of the individual households.

Aitken and Weale (2020) further show that if social welfare, $W^G = \log(Q^G) = \frac{1}{N} \sum_j \log(Q_j)$, the growth of the geometric mean of consumption, deflated by the democratic price index, shows us the rate of growth of social welfare. The same is true of the plutocratic measure if household welfare is linear rather than logarithmic in consumption, $W^P = \sum_j Q_j/N$. We note there is a body of evidence Layard *et al.* (2008) which suggests that the utility function is approximately logarithmic. Aitken and Weale (2020) show that, to a first order approximation, the utility function for consumption can also be applied to income.

Drawing on equation (1) we note that the discrete time the growth of the log of the household price index is given as

$$\Delta \log P_j - \sum_i \omega_{ij} \Delta \log p_i \tag{2}$$

and the increase from a base year, indexed as 0, is approximated as

$$\log P_{j,t} - \log P_{j,0} = \sum_{\tau=1}^t \Delta \log P_{j,\tau} \quad (3)$$

(2) can be averaged across households to give the democratic price index, and it is clear that the average across the whole household population is the arithmetic average of the values of $\Delta \log P_j$ in each quintile. The same therefore applies to the change cumulated across periods in (3). Further the mean of the log of nominal consumption is across quintiles is the mean of the quintile means. Thus, if, following Jorgenson and Schreyer (2017), we denote the social welfare function as $W^G = \log(Q^G) = \frac{1}{N} \sum_j \log(Q_j)$, and measure the change from period 0 to period t , we can write

$$\begin{aligned} W_t^G - W_0^G &= \frac{1}{N} \sum_j \{\log(Q_{j,t}) - \log(Q_{j,0})\} \\ &= \frac{1}{N} \sum_j \{\log(C_{j,t}) - \log(C_{j,0})\} - \frac{1}{N} \sum_j \{\log(P_{j,t}) - \log(P_{j,0})\} \end{aligned} \quad (4)$$

It is immediately clear from (4) that this can be evaluated over a quintile as well as for the population as a whole, and that the figure for the population as a whole can be calculated as the average of the quintile figures. But this result holds only because the democratic price index is calculated by averaging (2) across households.