Macroeconomic Data Transformation Matters

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Content

- Accuracy gains from machine learning are well documented. **Some recent examples** : Kim and Swanson (2018), Goulet-Coulombe *et al.* (2019) and Meideiros *et al.* (2019), among others ;
- Typical data transformations in time series used to remove low frequency movements may be suboptimal for machine learning methods ;
- Deep neural networks may automate data transformations, but macroeconomic samples are short and noisy making manual feature engineering advisable (Kuhn and Johnson, 2019) ;
- Moreover, careful feature engineering can encode **prior knowledge** and help improve forecasting accuracy.

Goals :

- Propose rotations of original data which helps encode "time series friendly" priors into machine learning models ;
- We compare the performances of several transformations of **predictors** and the combinations thereof to predict $y_{t+h}^{(h)} = h^{-1}\left(lnY_{t+h} - lnY_t\right)$;
- We compare predicting **directly** $y_{t+h|t}^{(h)}$ and the **path average** of predictions $y_{t+1|t}^{(1)}, \ldots, y_{t+h|t}^{(1)}$.

- **Model** : $y_{t+h} = g\left(f_Z(H_t)\right) + \epsilon_{t+h}$
- **Objective** : $\min_{g \in \mathcal{G}} \left\{ \sum_{t=1}^{T} \left(\left(y_{t+h} - g\left(f_Z(H_t)\right)\right)^2 + \text{pen}(g, \tau) \right) \right\}$
- H_t is the data, f_Z is the feature engineering step, g is the model and pen is a penalty function with hyperparameter vector τ. Hence $Z_t := f_Z(H_t)$ would be the feature matrix.
- **Forecast error decomposition** :

$$y_{t+h} - \hat{y}_{t+h} = \underbrace{g^*(f_Z^*(H_t)) - g(f_Z(H_t))}_{\text{approximation error}} + \underbrace{g(Z_t) - \hat{g}(Z_t)}_{\text{estimation error}} + e_{t+h}.$$

- We focus on how our choice of f_Z (transformations or combinations thereof) impacts forecast accuracy.

We consider "older" or more common candidates :

- **X** : Differentiate the data in levels or logarithms.
- **F** : PCA estimates of linear latent factors of **X** as in Stock and Watson (2002a,b) and Bai and Ng (2008).
- **H** : (Log-)Level of the series.

We consider "newer" or less common candidates :

- **MARX** (Moving average rotation of $\mathbf{X}$) : We use order $p = 1, ..., P_M$ moving averages of each variable in X. This is motivated by Shiller (1973). The following model :

$$y_t = \sum_{p=1}^{p=P} X_{t-p}\beta_p + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

$$\beta_p = \beta_{p-1} + u_p, \quad u_p \sim N(0, \sigma_u^2 I_K).$$

  This can be estimated by a Ridge regression which may be parametrized using inputs $Z := XC$ as inputs and where $C = I_K \otimes c$ with c, a lower triangular matrix of ones.

- This transformation implicitly shrinks β_p to β_{p-1}.

- **MAF** (Moving average factors) : Let $\tilde{X}_{t,k}$ be the k-th variable lag matrix defined as
$$\tilde{X}_{t,k} = \left[X_{t,k}, LX_{t,k}, \ldots, L^{P_{MAF}} X_{t,k}\right]$$
$$\tilde{X}_{t,k} = M_t \Gamma'_k + \tilde{\epsilon}_{k,t}.$$

  We estimate M_t by PCA and use the same number of factors for all variables.

- This is related to Singular Spectrum Analysis – except that SSA would use the whole common component instead of focusing on latent factors.

Tableau – Summary : Feature Matrices and Models

Transformations	Feature Matrix
F	$Z_t^{(F)} := [F_t, LF_t, \ldots, L^{p_f} F_t]$
X	$Z_t^{(X)} := [X_t, LX_t, \ldots, L^{p_X} X_t]$
MARX	$Z_t^{(MARX)} := \left[MARX_{1t}^{(1)}, \ldots, MARX_{1t}^{(p_{MARX})}, \ldots, MARX_{Kt}^{(1)}, \ldots, MARX_{Kt}^{(p_{MARX})} \right]$
MAF	$Z_t^{(MAF)} := \left[MAF_{1t}^{(1)}, \ldots, MAF_{1t}^{(r_1)}, \ldots, MAF_{Kt}^{(1)}, \ldots, MAF_{Kt}^{(r_K)} \right]$
Level	$Z_t^{(Level)} := [H_t, LH_t, \ldots, L^{p_H} H_t]$
Model	**Functional space**
Autoregression (AR)	Linear
Factor Model (FM)	Linear
Adaptive Lasso (AL)	Linear
Elastic Net (EN)	Linear
Linear Boosting (LB)	Linear
Random Forest (RF)	Nonlinear
Boosted Trees (BT)	Nonlinear

- Several combinations of many of those transformations are also considered.

The Horse Race

- **Direct forecasts**
- **Data** : FRED-MD
- **Targets** : Industrial production, non farm employment, unemployment rate, real personal income, real personal consumption, retail and food services sales, housing starts, M2 money stock, consumer and producer price index
- **Horizons** : $h \in [1, 3, 6, 9, 12, 24]$
- **POOS Period** : 1980M1-2017M12
- **Estimation Window** : Expanding from 1960M1

Tableau – Best Specifications in Terms of MSE

	INDPRO	EMP	UNRATE	INCOME	CONS	RETAIL	HOUST
H=1	RF●●●●	RF●●●●	BT●●	RF●	FM●	FM●	EN●●
H=3	RF●	$\underline{\text{RF}}$●●	RF●●●●	RF●●	RF●●	$\underline{\text{BT}}$●●●	$\underline{\text{EN}}$●●
H=6	$\underline{\text{RF}}$●	$\underline{\text{BT}}$●●	$\underline{\text{RF}}$●●	$\underline{\text{RF}}$●●●	$\underline{\text{RF}}$●●	AL●●	$\underline{\text{RF}}$●●●
H=9	$\underline{\text{RF}}$●	$\underline{\text{BT}}$●●	$\underline{\text{LB}}$●●●●	$\underline{\text{RF}}$●●	$\underline{\text{RF}}$●	BT●●●●	BT●●
H=12	$\underline{\text{RF}}$●	$\underline{\text{BT}}$●●	$\underline{\text{LB}}$●●●●	$\underline{\text{RF}}$●●	$\underline{\text{RF}}$●●	BT●●●	RF●
H=24	RF●●	$\underline{\text{BT}}$●	BT●●	$\underline{\text{RF}}$●●●	$\underline{\text{RF}}$●●	BT●●●	RF●

	M2	CPI	PPI
H=1	RF●●	AL●	EN●●
H=3	$\underline{\text{AL}}$●●	$\underline{\text{RF}}$●	$\underline{\text{EN}}$●
H=6	RF●●	RF●	RF●
H=9	RF●●	RF●	RF●
H=12	BT●●	RF●	RF●
H=24	$\underline{\text{RF}}$●●	$\underline{\text{RF}}$●	BT●●

Note : Bullet colors represent data transformations included in the best model specifications : *F*, *MARX*, *X*, *L*, *MAF*. Path average specifications underlined.

- The **marginal contribution** of a transformation to model performance can be evaluated using a panel regression :

$$R^2_{t,h,v,m} = \alpha_f + \psi_{t,v,h} + v_{t,h,v,m}$$

where

- $R^2_{t,h,v,m} := 1 - \frac{\epsilon^2_{t,h,v,m}}{\frac{1}{T}\sum_{t=1}^{T}\left(y^{(h)}_{v,t} - \bar{y}^{(h)}_{v}\right)^2}$
- $\psi_{t,v,h}$ are time, variable and horizon fixed effects
- $\epsilon_{t,h,v,m}$ is the time t, horizon h, variable v and model m forecast error
- α_f is one of $\alpha_{MARX}, \alpha_{MAF}, \alpha_F$ associated with the corresponding transformations. The null hypothesis is $\alpha_f = 0$.

Main Findings

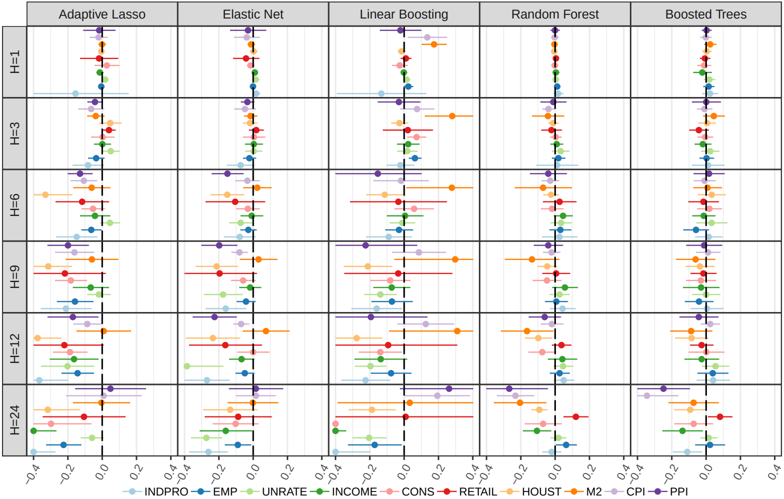
- MARX is especially potent when used in combination with nonlinear models to forecast measures of real activity. It's especially true around recessions Cumulative Squred Errors (Real).
- Factors helps a lot with Random Forest and Boosted Trees, especially at longer horizons as in Goulet-Coulombe *et al.* (2020) and big gains for Random Forests with factors at 12 months Cumulative Squred Errors (CPI).
- MAF regressions focus on models which includes $\mathbf{X}$. Results are more muted, but it seems to help Random Forests and Linear Boosting for horizons of 6 and 9 months.
- Target Transformation $\hat{y}_{t+h}^{direct}$ can prove largely suboptimal

Why Path Averaging Works ?

- Speculation : If useful signals change across horizons, the direct learning problem is a lot harder than each of learning problems used in path averaging. It is ”denser.”
- We have some suggestive evidence that this speculation is correct using variable importance and random forest : path averaging works best when horizon-wise model composition heterogeneity is high.

Conclusion

- At shorter horizons, combining non-standard and standard data transformations helps reduce RMSE.
- MARX is especially potent when used in combination with nonlinear models to forecast measures of real activity, especially around recessions.
- Factors remain one of the most effective feature engineering tool available for macroeconomic forecasting, even for inflation.
- For many cases, RMSE reductions can obtain from producing entire paths even when we target an average.



Back **Figure – MAF** Average Treatment Effects ((h,v) Subsets)

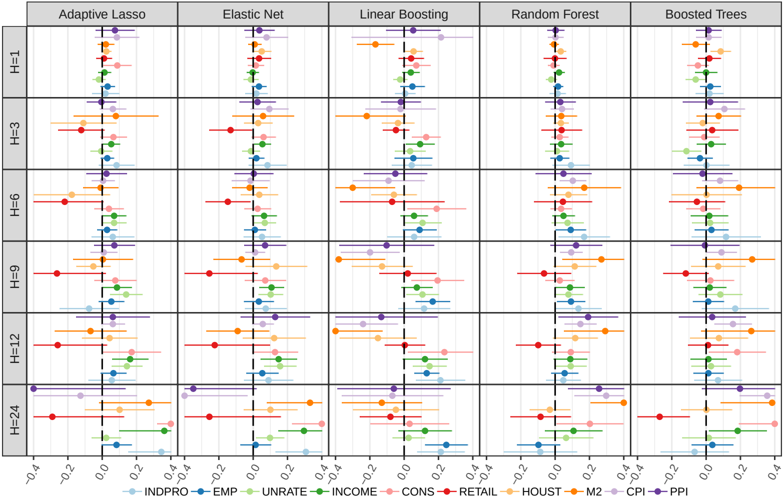


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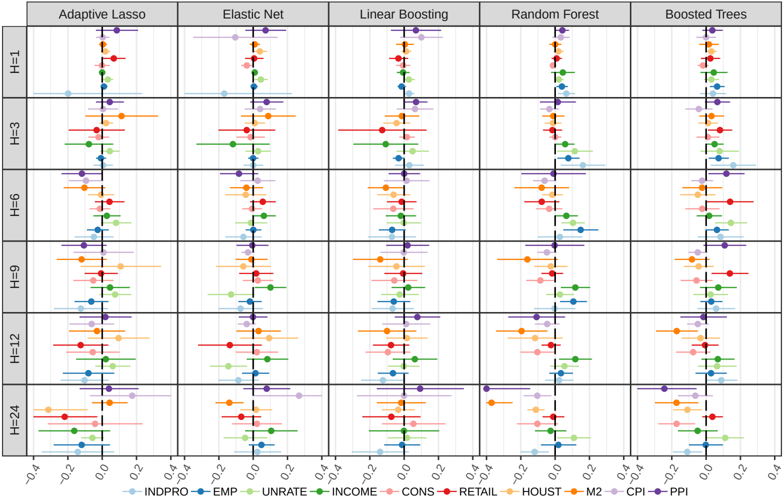
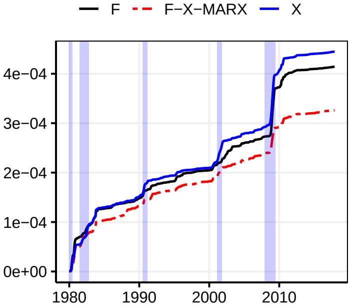
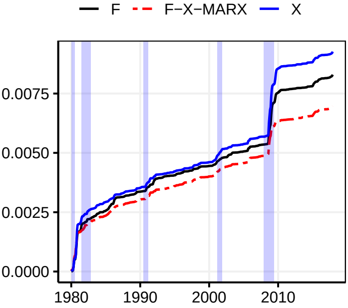
FIGURE – F Average Treatment Effects ((h,v) Subsets)



Back **Figure – MARX** Average Treatment Effects ((h,v) Subsets)



Employment (3 months)



Industrial Production (3 months)

FIGURE – Cumulative Squared Errors (Random Forest)

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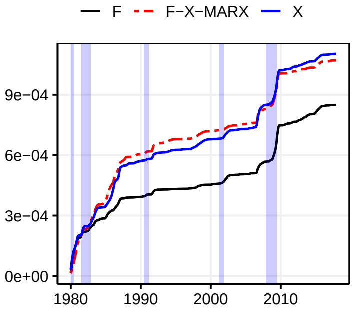

CPI Inflation (12 months)

FIGURE – Cumulative Squared Errors (Random Forest)

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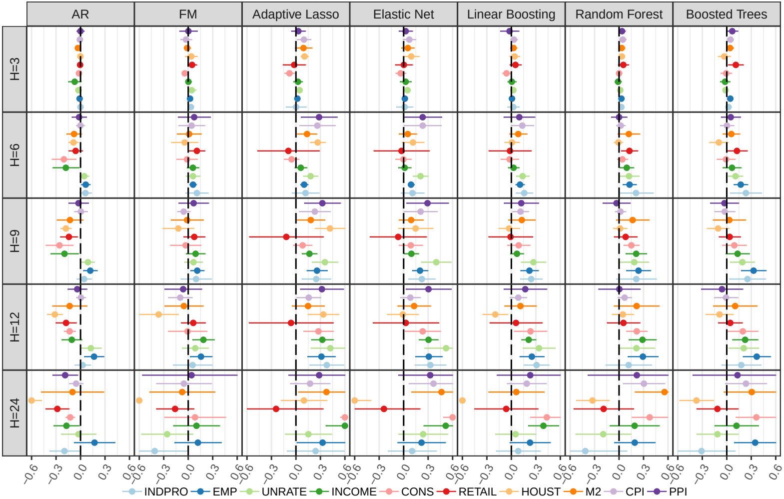

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FIGURE – From Direct to Path Average ((h,v) Subsets)