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Abstract

This paper seeks to expand the discussion surrounding the dating of UK business cycles. Two different time-series filters are applied to UK output time-series to investigate what they would imply for the creation of any official recession dates. The NBER has a business cycles dating committee that investigates the dates of turning points in US business cycle providing a consolidation of business cycle dates for the US. There is, at present, no analogous committee or consolidation on UK turning points dates. There is a broad definition adopted in the UK that defines a recession as two or more periods with negative growth. The Office of National Statistics (ONS) uses a series of real GDP (deflated GDP) and then observes two or more periods of negative growth to define a recessions. 2020 has certainly provided an interesting critique to this broad definition, whereby the second quarter of 2020 saw a -20% in GDP but then the next quarter recovered by 12% therefore only categorising the first half of 2020 as a recession. However, the decline in economic output within the UK has been more notable than other business cycle recession in recent decades. The aim of this paper is to continue the discussion on defining the turning points of UK business cycles. This study looks at two filters and how they would define UK business cycles. Although the merits of the filters are discussed before being used in estimation. Based on the outcomes of the business cycle dates, the filters that produce the most reasonable results are defined as a better approach. Reasonable approach is defined as one that matches the theory as to how often peaks and troughs can reasonably expected to occur.

Keywords: Business Fluctuations, Cycles, Time-Series Models, Policy Coordination

JEL classification: E32, E61, C32

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1 Introduction

This paper seeks to expand the discussion surrounding the dating of UK business cycles. Two different time-series filters are applied to UK output time-series to investigate what they would imply for the creation of any official recession dates. The NBER has a business cycles dating committee that investigates the dates of turning points in US business cycle providing a consolidation of business cycle dates for the US. There is, at present, no analogous committee or consolidation on UK turning points dates. There is a broad definition adopted in the UK that defines a recession as two or more periods with negative growth. The Office of National Statistics (ONS) uses a series of real GDP (deflated GDP) and then observes two or more periods of negative growth to define a recessions. 2020 has certainly provided an interesting critique to this broad definition, whereby the second quarter of 2020 saw a -20% in GDP but then the next quarter recovered by 12% therefore only categorising the first half of 2020 as a recession. However, the decline in economic output within the UK has been more notable than other business cycle recession in recent decades. The aim of this paper is to continue the discussion on defining the turning points of UK business cycles. This study looks at two filters and how they would define UK business cycles. Although the merits of the filters are discussed before being used in estimation. Based on the outcomes of the business cycle dates, the filters that produce the most reasonable results are defined as a better approach. Reasonable approach is defined as one that matches the theory as to how often peaks and troughs can reasonably expected to occur.

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The choice of filtering method used to extract a business cycle plays a role in drawing conclusions on dates of business cycles. A benchmark filter that is used in Macroeconomics is the Hodrick-Prescott filter (1997). A recent paper by Hamilton (2018) suggests that there could be a better alternative to extract the cyclical component of macroeconomic time-series. An estimation of both filters on UK GDP data obtained is between 1955-2020 is conducted. The impact of two different filters on dating UK business cycles are investigated. The extracted cycles and trends are analysed for their various properties. The Harding-Pagan procedure is then applied to the two extracted cyclical components in order to identify turning points and therefore providing business cycle dates.

Overall, the study finds that the cycles extracted by the Hamilton-filter are more useful for the purposes of exhibiting distinct cycles in the data. Given that the filter is less presumptive about cycles in the first place, the distinct cycles could be interpreted as an artefact of the data itself. In the second half of the sample, the Hodrick-Prescott filter picks up a lot more cycles particularly post 2010, where has the Hamilton-filter does not identify any further cycles post-2014.

In order to understand how the cyclical properties of the filters are impacting the cycle, Spectral analysis is conducted. Spectral analysis moves the analysis of the extracted cycles from the time domain to the frequency domain allowing for more understanding about via the analysis of wavelengths and amplitudes of the cycles in the series. This analysis will be done by the calculation of periodograms. To gain some insight to the impact that the filters themselves are having an impact on the properties of the extracted cycle. The calculated periodograms show that the dominant points of the cycle occurs at roughly the same period which is at almost 9 years.

2 Measuring the UK Business Cycle.

There is a broad definition adopted in the UK that defines a recession as two or more periods with negative growth. Unlike the US and the NBER dates, there is no official database of UK recessions but given the broadly adopted definition, it is straight forward to identify recessions. The ONS national statistical office uses a series of real GDP (deflated GDP) and then observes two or more periods of negative growth to define a recessions. The only possibility of discrepancy occurs if different series are used, i.e. real vs nominal but the impact of this would be minimal on identifying recessions by quarters.

The table below shows a list of the of UK recession dates as defined by the broad convention of two quarters of negative GDP growth. The sample covers the same window that is being assessed in this study.

1950's	1960's	1970's	1980's	1990's	2000's
1956Q3	1961Q4	1973Q4	1980Q2	1990Q4	2008Q3
		1974Q1	1980Q3	1991Q1	2008Q4
		1975Q3	1980Q4	1991Q2	2009Q1
			1981Q1	1991Q3	2009Q2

The table above shows that that there has been one recession every decade, implying a business cycle length of around 10 years, and furthermore, implies no recession occurred during the last 10 years.

It is of consensus to real business cycle economists that GDP data contains of two kinds of movements, one of which is structural growth which is mainly linear in nature and represents economic growth that is long-term in its additive benefit to the economy. The second movement contained in GDP series are the transitory movements that have short-run impacts on the economy.

GDP series can also contain information about white noise, seasonality as well as other irregular components. As information about cycles are mixed in with information about the trend along with white noise, an econometric approach is needed to extract the business cycle. The debate around which is the most accurate measure to use focuses on beliefs about the assumptions of the underlying DGP of the extraction method. The addition that has been made in recent years to the debate comes from Hamilton (2017) entitled: *Why you should never use the Hodrick-Prescott filter.* This paper unsurprisingly, lays out principle flaws with the Hodrick-Prescott filter and proposes an alternative filter that will not produce spurious cycles and can be confidently assumed to be I(4).

This study investigates the features of the filter suggested in Hamilton (2017) by estimating the filter on countries in the EU and comparing the filtered series to those the Hodrick-Prescott filter. The cross-comparison is to investigate the business cycle theory in an international economy context. In looking at the filter in this context, it exposes the filters to question about data availability and how this could possible impact the choice of filter.

3 Use of Hodrick-Prescott filter

The Hodrick-Prescott filter is widely used as a benchmark filter to extract cyclical components from macroeconomic time-series. The filter decomposes the time-series in to a trend component τ_t and a cyclical component c_t. The cyclical component is the residual of the second difference of the calculated trend component. The aim of the Hodrick-Prescott filter is to minimise the following function:

$$y_t = \tau_t + c_t \tag{1}$$

$$\min_{\left\{\tau_t\right\}_{t=-1}^{T}} \sum_{T}^{t=1} (y_t - \tau_t)^2 + \lambda[(\tau_t - \tau_{t-1}) - (\tau_{t-1} - \tau_{t-2})]^2 \tag{2}$$

The choice of λ plays a role in dictating the relationship between the original series and the extracted cycle. It in essence defines the degree to which the cyclical component of the series is penalised. The user chooses λ, and the higher the λ, the closer the equation is to a pure regression of the series on the time vector. One of the major critiques of the Hodrick-Prescott filter is that the parameter λ is very influential on the end result on the degree of cyclicality estimated in the series.

4 Hamilton's critique

Hamilton proposes an alternative method of filtering that overcomes some of the issues raised with other filters. The method is the t+h ahead observation of the time-series regressed on the t,t-1,t-2....t-ρ observations using a linear estimation. The residuals will form the cyclical component.

$$y_{t+h} = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4} + \varepsilon_{t+h} \tag{3}$$

$$\hat{\varepsilon}_{t+h} = y_{t+h} - \hat{\beta}_0 - \hat{\beta}_1 y_{t-1} - \hat{\beta}_2 y_{t-2} - \hat{\beta}_3 y_{t-3} - \hat{\beta}_4 y_{t-4} \tag{4}$$

The extracted cycle of the series is $\hat{\varepsilon}_{t+h}$ and the fitted values are denoted by the $\beta's$. It is argued that this method guarantees a stations I(4) process and is not subject to end point problems whereby the filter performs consistently even as it moves towards the end of the time-series.

Hamilton argues that this method has a benefit in that the user does not have to make any specifications about lambda.

Other critiques of the Hodrick-Prescott filter are the problems at the end point as the number of observations in the future reduce which reduces the accuracy of the estimation towards the end of the filter.

Some questions arise with Hamilton's approach, as there are still sensitivities of the filter depending on the values adopted for ρ and h. Furthermore, Schuler(2018) argues that the Hamilton-filter is subject to the same critique of creating spurious business cycles. Hamilton(2017) argues that the Hodrick-Prescott filter does not guarantee stationarity. The Hamilton-filter by construction creates a series that is stationary and is I(4).

4.1 Comparing the Filters

Another consideration that needs to be made is whether the filtering methods needs to be able to identify recessions in real time. The two sided HP filters uses observations beyond t in order to calculate τ_t, however , a one sided version of the filter only uses past observations to filter y_t. It is argued that the one-sided filter is sub-par to the two sided filter as the one-sided filter fails to remove low frequency fluctuations and dampens exactly the movements that we are interested in capturing(Wolf et al. 2020). However, Mokinski, Schuler and Wolf (2018) propose some modest adjustments to the one-sided filter in order to overcome these problems.

The Hamilton filter has no difference on the impact on the filtered series depending on whether is is used in real time or retrospectively. This is because future values are not used in the estimation of the filter at all.

In terms of practical considerations, it might be argued that this property of the Hamilton filter, makes it more usable in the context of identifying recessions.

As the two different filters have different data generating processes (DGPs), they make different assumptions about what the underlying cycles should represent. The cycle of the Hodrick-Prescott filter, when applied to a random walk is an artefact of the filter itself and is somewhat constructed. Low frequencies are removed by the Hodrick-Prescott filter. The cycle of the Hamilton-filter is has fewer assumptions underpinning it's data generating process. However, Schuler(2018) argues that the Hamilton filter is subject to the same critique of creating spurious business cycles.

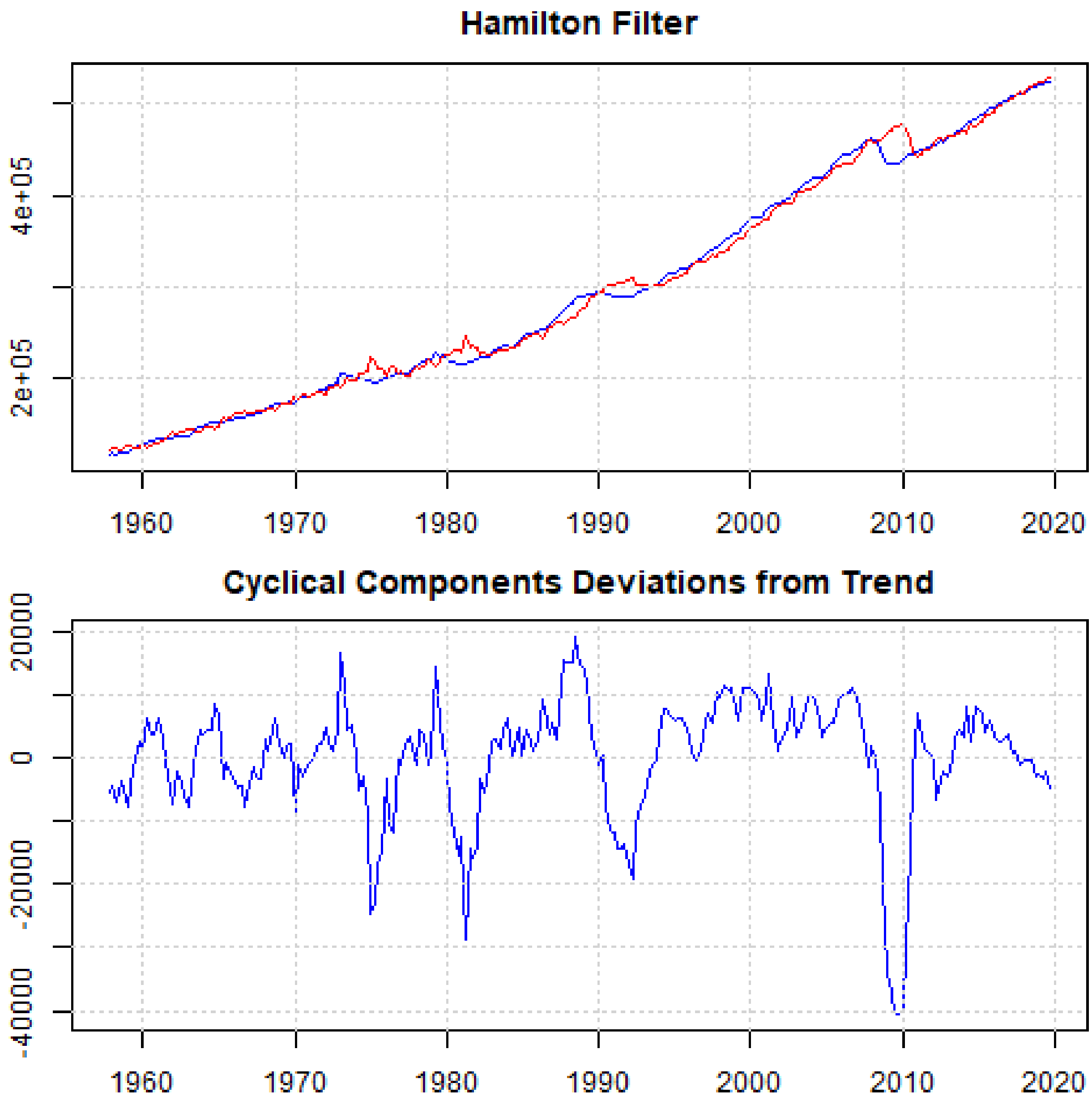
5 Estimation

The data is real quarterly GDP for the UK from 1955 to 2020. The GDP series are pre-seasonally adjusted and logged. The Hamilton-filter is applied with h=8 and $\rho = 4$ where h refers to the number of periods behind that is being used to predict the current value. So observations from two years ago are being used to predict y_t. ρ refers to the number of lags being used in the estimation. h=8 and $\rho = 4$ as specified as appropriate values when dealing with quarterly data (Hamilton 2018). The second filter that is applied is the Hodrick-Prescott filter with a $\lambda = 1600$. As mentioned already, the choice of λ, plays a pivotal role in the subsequent estimation. The broad convention for choice of λ for quarterly data is 1600 (Ravn & Uhlig 2001).

Once the cycles have been extracted, they are compared for wave-length, amplitude, and measured along a number of dimensions. The comparison with cycles extracted with the Hodrick-Prescott filter and other papers that have extracted business cycles will be compared.

6 Results

The graphs below plot the raw GDP series against the extracted cycle series of the relative filters. The series goes from 1957q1 - 2019q4



Hamilton Filter
2e+05
4e+05
1960
1970
1980
1990
2000
2010
2020
Cyclical Components Deviations from Trend
-40000
-20000
0
20000
1960
1970
1980
1990
2000
2010
2020

Figure 1:

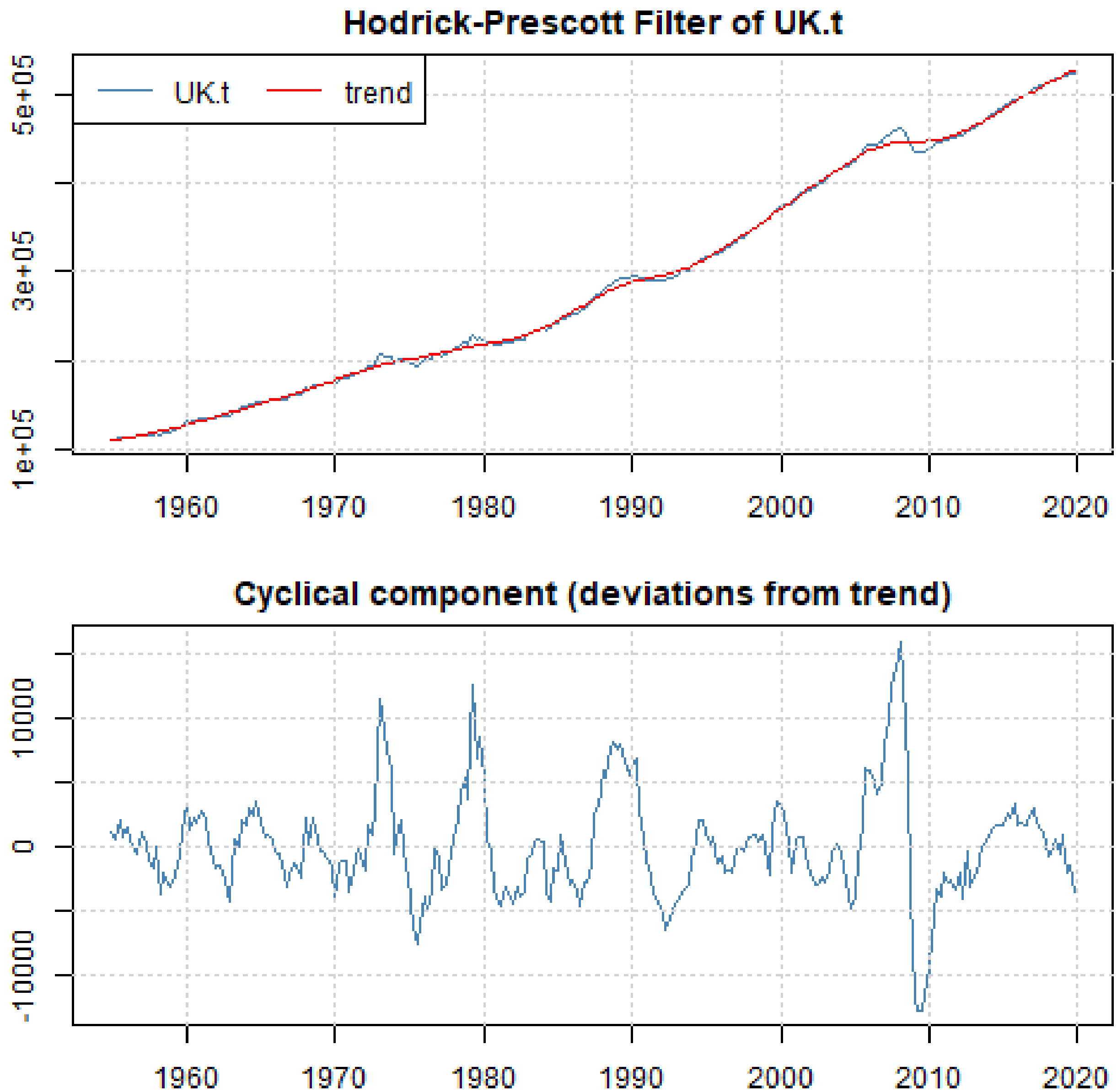


Figure 2:

7 Comparison of Extracted Cycles

7.1 Dates of Troughs and Peaks

Troughs and peaks are identified using the standard Harding Pagan method. The criteria for a business cycle as outline by the following criteria:

- A peak is defined as a peak if $y_{t-2} < yt > y_{t+2}$. Vica-versa to define a trough.

- Peaks and troughs must occur at alternate.
- creating specifications for“adequate” criteria of peaks and troughs such as amplitude, duration and frequency of occurrence.

The Hamilton-filter has more pronounced cycles than the Hodrick-Prescott filter. The average cycle duration for the Hamilton-filter is just under 4 years where as the Hodrick-Prescott filter, the average duration of the cycle is around 2.5 years. The amplitude of the cycles generated by the Hamilton-filter are two to three times larger than the cycles generated by the Hodrick-Prescott filter.

	Amplitude	Duration
Expansion	6749.5	5.8
Recession	6847	5

Table 1: Properties of the Hodrick-Prescott Filter

	Amplitude	Duration
Expansion	19780.1	8.6
Recession	18993.9	6.3

Table 2: Properties of the Hamilton Filter

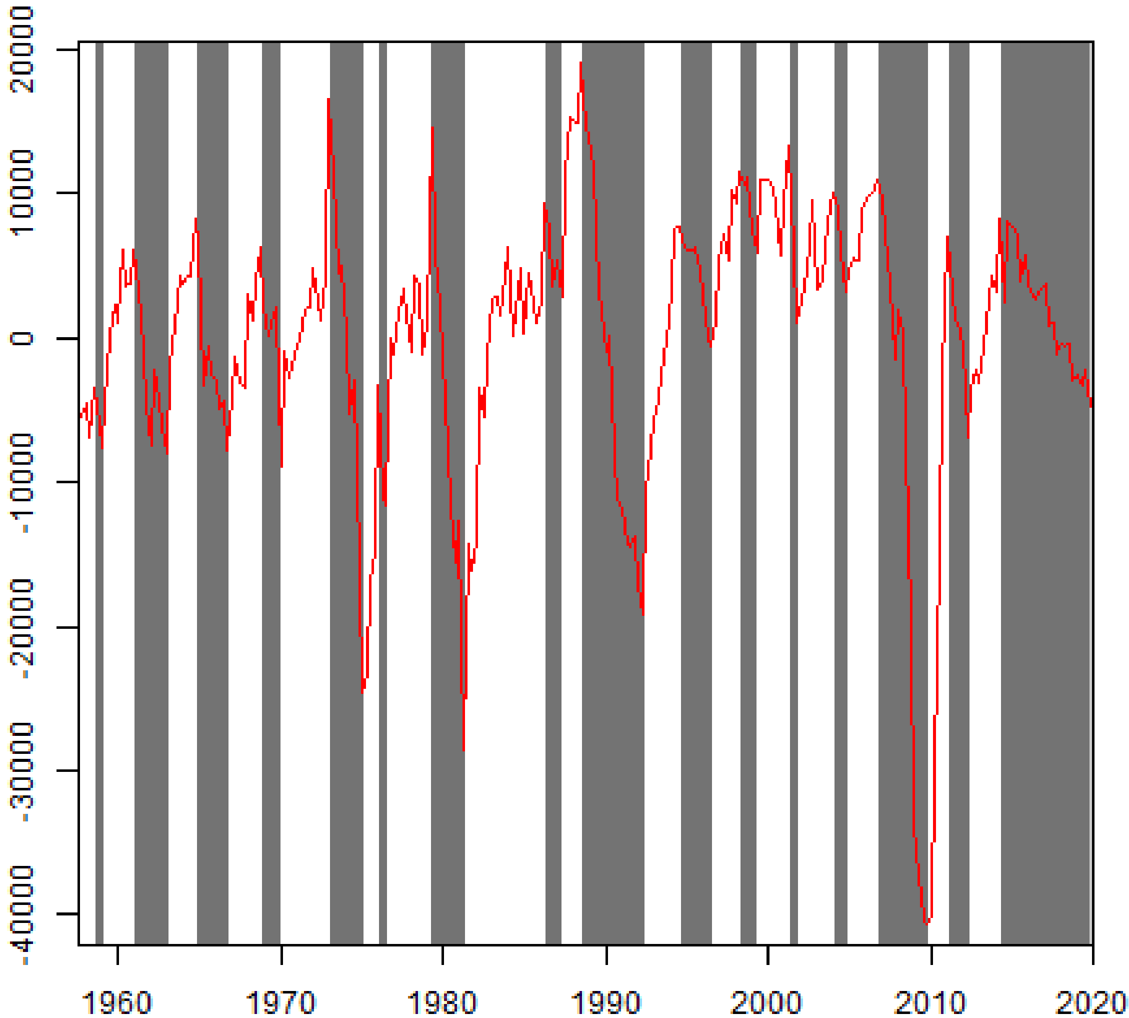

20000
10000
0
-10000
-20000
-30000
-40000
1960
1970
1980
1990
2000
2010
2020

Figure 3:

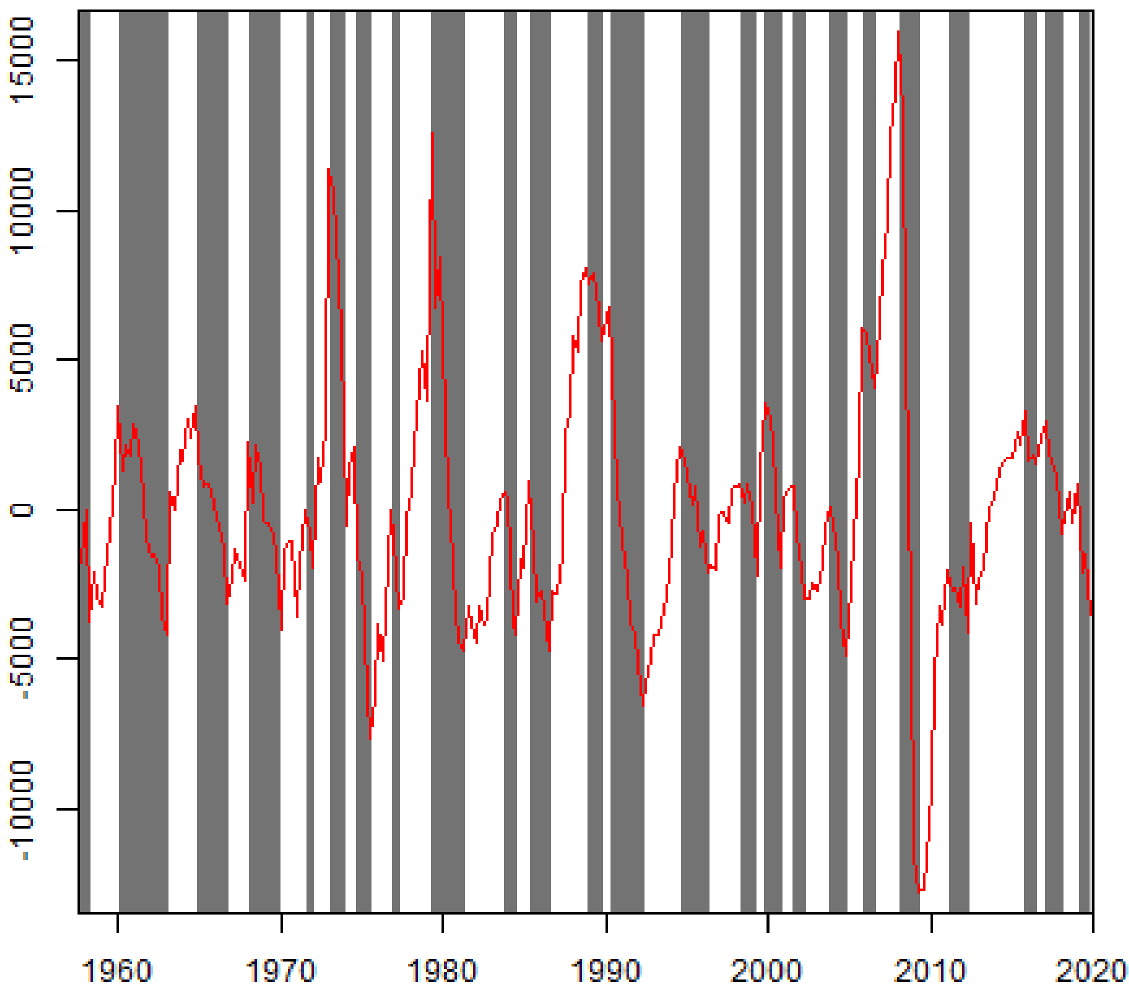


Figure 4:

The table below shows the dates of the turning points of the extracted cycles as decided by the Harding pagan method of dating cycles.

Hamilton Filter				Hodrick Prescott Filter			
	Peaks	Troughs	Duration		Peaks	Troughs	Duration
1	1958Q3	1959Q1	2	1		1958Q2	
2	1961Q1	1963Q1	8	2	1960Q1	1963Q1	12
3	1964Q4	1966Q4	8	3	1964Q4	1966Q4	8
4	1968Q4	1970Q1	5	4	1968Q1	1970Q1	8
5	1973Q1	1975Q1	8	5	1971Q3	1972Q1	2
6	1976Q1	1976Q3	2	6	1973Q1	1974Q1	4
7	1979Q2	1981Q2	8	7	1974Q3	1975Q3	4
8	1986Q2	1987Q2	4	8	1976Q4	1977Q2	2
9	1988Q3	1992Q2	15	9	1979Q2	1981Q2	8
10	1994Q3	1996Q3	8	10	1983Q4	1984Q3	3
11	1998Q2	1999Q2	4	11	1985Q2	1986Q3	5
12	2001Q2	2001Q4	2	12	1988Q4	1989Q4	4
13	2004Q1	2004Q4	3	13	1990Q2	1992Q2	8
14	2006Q4	2009Q4	12	14	1994Q3	1996Q2	7
15	2011Q1	2012Q2	5	15	1998Q2	1999Q2	4
16	2014Q2			16	1999Q4	2000Q4	4
				17	2001Q3	2002Q2	3
				18	2003Q4	2004Q4	4
				19	2005Q4	2006Q3	3
				20	2008Q1	2009Q2	5
				21	2011Q1	2012Q2	5
				22	2015Q4	2016Q3	3
				23	2017Q1	2018Q1	4
				24	2019Q1		

The table shows that there is a more business cycle fluctuations according to the Hodrick-prescott filter which has 23 full cycles as oppose to the Hamilton-filter that only has 15 full cycles. In the first half of the sample, the turning points picked up in the cycles are quite similar. In the second half of the sample, the Hodrick-Prescott filter picks up a lot more cycles particularly post 2010, where has the Hamilton-filter does not identify any further cycles post-2014.

7.2 Spectral Analysis

The analysis conducted in this section involves moving from the time domain to the frequency domain. This allows us to investigate the impact that the filters themselves could be having on the characteristics of the extracted cycle. Each series can be written as a sum of sine and cosine waves. Spectral analysis will allow us to understand which frequencies of the extracted cycle dominate and therefore tell us the average duration of a cycle in the extracted cyclical component.

A periodogram is used to identify the dominant periods (or frequencies) of a time series. This can be a helpful tool for identifying the dominant cyclical behavior in a series, particularly when the cycles are not related to the commonly encountered monthly or quarterly seasonality. The graphs below show the calculated periodograms from the two extracted series. A periodogram is calculated in the following way: The equation below fits x_t with a linear model that uses sine and cosine as explanatory variables.

$$x_t = a_0 + \sum_{k=1}^{\frac{n}{2-1}} a_k cos(\frac{2\pi f_0 kt}{n}) + b_k sin(\frac{2\pi f_0 kt}{n})$$

The periodogram then follows as:

$$P_k = a_k^2 + b_K^2$$

The contributions of frequency k to x_t are measured.

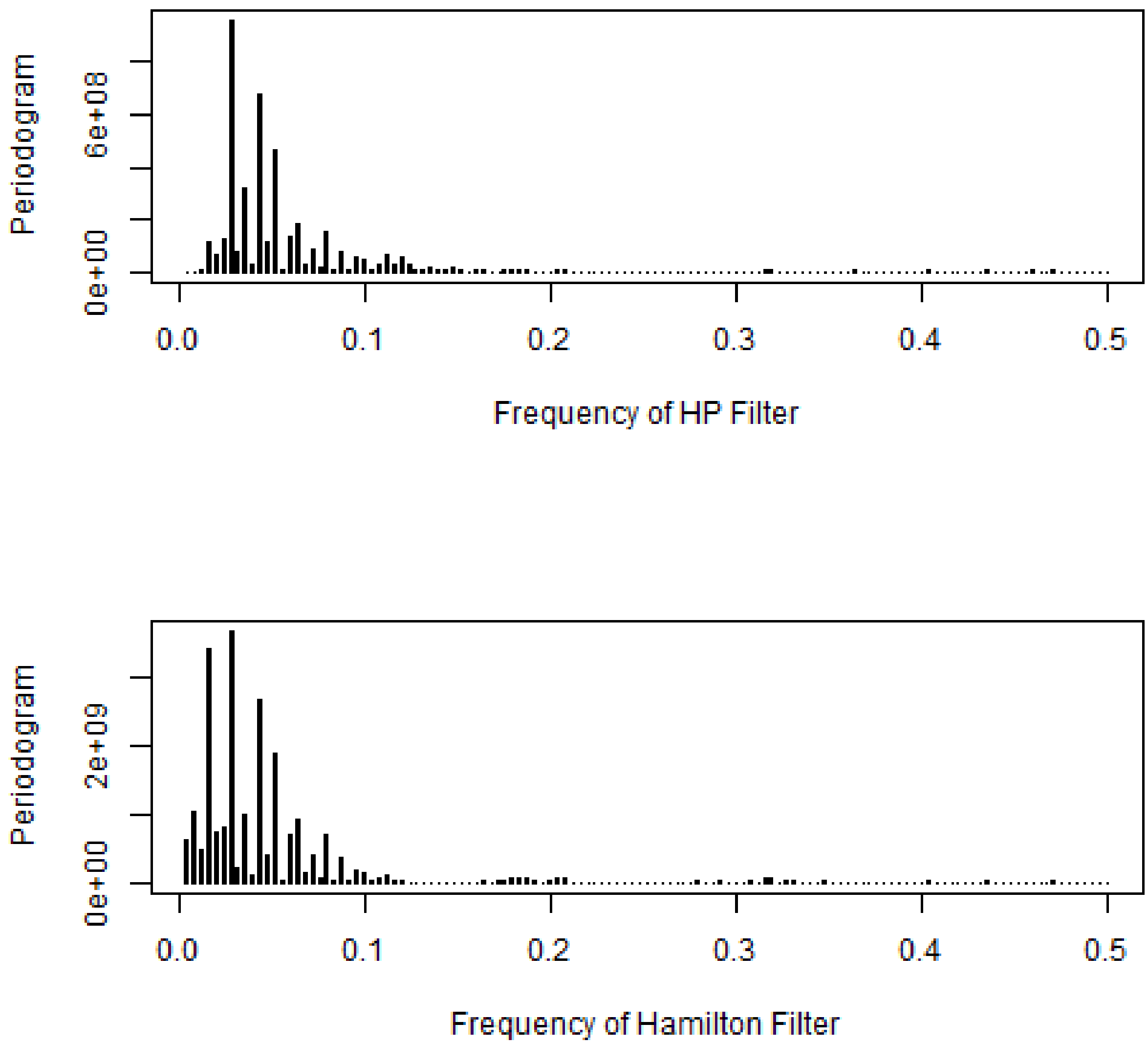


Figure 5:

The two periodograms show that the two filters have dominance at similar frequencies at around 7 years meaning that the cyclical properties of the filters to not have a huge effect on the length of the business cycle.

8 Conclusion

This note extends the discussion surrounding the identification of turning points in the UK business cycle. The implications of the applications of two different filters are analysed for the impact they would have on creating an official set of UK business cycle dates. The two filters compared are the Hodrick-Prescott filter and an proposed alternative by Hamilton (2018) is applied to the same UK time-series. The Harding-Pagan formula is employed to identify turning points in the UK Business cycle. The respective turning point dates indicate how the properties of the filters impact the nature of the cycles extracted. The Hodrick-Prescott filter produces 8 more turning moments than the Hamilton filter does, implying greater volatility in UK business cycles when using this method. The Hamilton-filter implies a slightly longer duration of cycles and lower volatility of cycles. This note provides some initial considerations to bear in mind when thinking about which filter to use when creating an official set of UK business cycle dates. The Hamilton filter allows for more flexibility and seems to provide a more measured interpretation of the UK business cycle owing to the fact that it prescribes less volatility and lengths of cycles that match with theory.

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