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ESCoE Discussion Paper 2022-04

March 2022

ISSN 2515-4664

DISCUSSION PAPER

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Keywords: Regional data, Mixed frequency, Nowcasting, Bayesian methods, Real-time data, Vector autoregressions

JEL classification: C32, C53, E37

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Published by:
Economic Statistics Centre of Excellence
National Institute of Economic and Social Research
2 Dean Trench St
London SW1P 3HE
United Kingdom
www.escoe.ac.uk

Using stochastic hierarchical aggregation constraints to nowcast regional economic aggregates*

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*We thank an associate editor and two anonymous referees for helpful comments. This research has been funded by the Office for National Statistics as part of the research program of the Economic Statistics Centre of Excellence (ESCoE). The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Cleveland or the Federal Reserve System.

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1 Introduction

A key barrier to tracking the performance of regional economies lies in the lack of timely official economic data on key aggregates like economic output and productivity. With increasing trends of decentralization and localism, and the associated growing role of sub-national leaders in responding to economic events and fluctuations, there is a need to improve the timeliness of sub-national economic statistics to support more effective policymaking. In the UK, for example, official quarterly regional output data are released with a delay of around 6 months after the end of the quarter for most regions. It is typically the case that estimates of sub–national economic output are released with a longer lag than equivalent national level data, as is the case for example with US state-level GDP estimates and International Territorial Level (ITL) 1 data for the European Union. This means that local policymakers, who are often on the frontline when responding to the immediate needs created by economic downturns, do not know the severity of any downturn in economic activity for some time after it occurs.

At a national-level the situation is often much better, with GDP data typically released between 4 and 6 weeks after the end of the relevant quarter in most advanced economies. Responding to a demand for more timely and higher frequency economic data, one is even seeing the development and release of monthly GDP data for countries such as the UK (where it is released with a delay of around 6 weeks after the end of the month). Yet this is still not as timely as policymakers would like, which is why a large literature has explored ways of leveraging more timely data on economic activity including both 'hard' (e.g., unemployment claims) and 'soft' (e.g., qualitative business survey data) indicators to produce more timely estimates or 'nowcasts' of national economies. However, relatively little research has explored the challenge of sub-national nowcasting (notable exceptions which we draw on below being Koop et al. (2020b,c)).

Nowcasting regional economies is a different challenge in a number of ways to nowcasting at the national level. Firstly, longer release delays for regional data mean a longer forecast horizon; secondly, regional data typically have much shorter historical samples; thirdly, regions typically have a more limited set of potential predictors that can be used when nowcasting; and fourthly, smaller regional economies tend to exhibit greater economic volatility than national economies. At the same time, however, the hierarchical relationship between regional and national economic aggregates ought to be able to help address some of these issues. Indeed, given accounting identities, it is surely an essential feature of any regional nowcast or forecast that they are cross-sectionally consistent with, and informed by, the corresponding national estimates. But data features, like those observed for the UK which we elaborate on below and bundle together under the banner 'measurement errors', mean that we should not always expect the regional data to 'add up' (or average) exactly to the national data, even when modeling in growth-rates, as we do in this paper. To date, however, there exists no systematic investigation of these issues and the extent to which this hierarchical or cross-sectional relationship can be used to produce more accurate and well-calibrated regional nowcasts and forecasts. Our paper seeks to fill this gap by considering how, within our proposed Bayesian estimation strategy, one can, in effect, 'shrink' towards aggregation constraints rather than impose them exactly.

In doing so, we focus on nowcasting productivity growth at a regional level in the UK. Central banks and other policymakers consult the latest productivity estimates when assessing how future output growth might evolve. Productivity is conventionally measured using some measure of economic output (Q) divided by some measure of labor (L) used to produce that output. In this paper, we utilize official estimates from the UK Office for National Statistics (ONS) on gross value added (GVA) and hours worked at the ITL1 (formerly the NUTS1) regions of the UK of which there are 12 (plus extra-regio [1]), alongside the equivalent national level versions of these data. These measures of Q and L that we use are produced with delay. The exact timing of release delays differs for Q and L, differs between the regions and the UK as a whole, and has changed over time. But the general pattern is that the UK data are released more quickly than the regional data. We exploit this fact to produce, and subsequently evaluate, timely nowcasts of regional productivity.

In order to produce more frequent nowcasts of regional productivity, we use a high-dimensional Mixed Frequency Vector Autoregressive (MF-VAR). MF-VARs have become a popular nowcasting tool; see, among many others, Eraker et al. (2015), Schorfheide and Song (2015), Brave et al. (2019), Koop et al. (2020b,c), and McCracken et al. (2021) But none of these econometric models can be used directly for nowcasting UK regional productivity due to the specific characteristics of our data set. We have two variables, Q and L. At the UK level, we use quarterly observations on these variables going back to 1966Q1. However, regional output data are available at the annual frequency for the first part of the sample (from 1966 to 2011 for Q and 1966 to 1981 for L), but then become quarterly (from 2012Q1 for Q and from 1982Q1 for L).

Thus, the timing of the switch from annual to quarterly differs for Q and L. This makes it desirable to create a high dimensional MF-VAR where Q and L (for every region and for the UK) are the dependent variables, instead of the simpler strategy of working with productivity (Q/L) directly and building a lower-dimensional MF-VAR. Our model is designed to deal with this complicated data set up, where the variables have different release delays and the

[1]For a definition of 'extra-regio' see p22 of: https://ec.europa.eu/eurostat/documents/3859598/5937641/KS-GQ-13-001-EN.PDF/7114fba9-1a3f-43df-b028-e97232b6bac5

frequency mis-match changes over time. This mis-match also helps explain why the assembled historical databases, in particular for Q, are in reality a 'mix-and-match' of estimates measured in different ways and at different points in time. This means that our data are not always perfectly consistent, in the sense of respecting accounting constraints. Our modeling approach therefore incorporates the hierarchical relationship between the national and regional data in stochastic rather than exact form. Importantly, this still enables us to exploit the information in the more rapidly released UK data on Q and L to improve our regional estimates. Our work thus relates to the hierarchical forecasting and forecast reconciliation literatures, for example, the recent contributions from Athanasopoulos et al. (2020), Panagiotelis et al. (2021), and Eckert et al. (2021). We also build on the temporal-and-spatial disaggregation literature, where the focus has been on imposing the cross-sectional aggregation constraints in exact rather than approximate form. This is appropriate if and when the accounting identities are binding. Leading examples of this literature are Di Fonzo (1990), Guerrero and Nieto (1999), Proietti (2011a), and Frale et al. (2011). Proietti (2011b) is the closest paper to ours, as he sets out a frequentist way of imposing cross-sectional aggregation constraints subject to errors. Our point of departure is to consider Bayesian methods appropriate for large dimensional VAR models. VARs are an attractive option for modeling regional data sets such as ours since they allow for information in one region to spill over into others, both statically (via the error covariance matrix) and dynamically (via the lagged dependent variables). But, with 2 variables of interest for each of 12 regions (as well as UK quantities) the resulting VARs are very high-dimensional. This high dimensional setting combined with so many latent states and the extremely unbalanced nature of our (panel) data raises overfitting concerns. Bayesian prior shrinkage can be used to overcome these.

In our empirical application, we focus primarily upon evaluating the accuracy of our regional forecasts produced in pseudo real–time at different forecast horizons. A fully real-time data analysis is not possible. Firstly, historical regional data vintages for Q do not exist. Secondly, some of the regional Q data that we exploit have only been published by ONS since 2017, precluding meaningful analysis of regional data revisions. Given release delays, we produce 3 estimates of growth for a given quarter in each region (i.e., a forecast, a nowcast, and a backcast). We utilize 5 different versions of our model, reflecting different combinations of our hierarchical (cross-sectional) aggregation constraint over the Q and L variables, as well as exploring the contribution of additional regional-level predictors. Our goal in this exercise is to document the contribution that each of these different model features makes to the accuracy of our point and density nowcasts. We show that these hierarchical restrictions deliver significant improvements in nowcast accuracy in cases where the UK aggregate has been published and is known, but not when the corresponding UK value is unknown. Furthermore, these gains are much larger than those produced by incorporating additional regional predictors.

The rest of this paper proceeds as follows. The next section sets out the data and context for this paper, in particular the evolving data release calendar and the construction of our regional database. The nature of these data - and the reality that they are subject to measurement error(s) - helps explain why it is not appropriate to impose the hierarchical aggregation constraints in exact form. Section 3 introduces the notation used in this paper, while Section 4 sets out our econometric model with stochastic aggregation constraints appropriate for modeling the data in growth rates. Section 5 presents our results and Section 6 concludes. Technical details on the variational Bayes estimation algorithm, and prior and hyperparameter choices can be found in online Appendix A. Online Appendix B presents robustness checks on the empirical results in the main paper.

2 The Evolving Regional Output and Employment Data Landscape in the UK

The goal of this paper is to develop a model which produces accurate nowcasts of regional labor productivity in the UK. To do this, we must model both economic output and labor input at the regional level. Given our focus in this paper, we must also model the equivalent national (UK) aggregates. In this section, we explain the UK data landscape and the construction of the database that we use.

Regional economic output data in the UK share a number of features common to sub-national data internationally. The available time series of economic output data is quite short (1998 - 2019 for annual data and 2012Q1 - 2020Q3 for quarterly data, at the time of writing) compared to national level data. It is also much less timely than national level data; the typical delay in releasing annual regional output data in the UK is about a year, and for the quarterly data is 6 months (with slightly more timely data – produced using a different methodology – for Scotland and Northern Ireland). In the European Union annual regional data is released with a similar lag. In the US, quarterly state–level GDP is available, but is released with a delay of around 3 months compared to less than one month for US GDP data. The UK context, therefore, provides a good case-study to explore the efficacy of regional nowcasting models.

Quarterly real–terms gross value added (GVA) data are readily available for the UK as a whole: these data are measured on the output-side, and we consider data from 1966Q1 through 2020Q3.[2]. We use chain-linked volumes. In chained volume terms the growth rates

[2]Recall that GVA plus taxes (less subsidies) on products is GDP: see https://www.ons.gov.uk/ons/

of real GVA and real GDP should be the same and in practice are very close (they have a correlation coefficient 0.99). The UK-level GVA data are released about 45 days after the end of the quarter. In the absence, as we shall explain, of real–time data vintages for the regional GVA data we consider only latest-vintage UK GVA data (at the time of writing, May 2021). It will be interesting for future research, once a sufficiently large real-time regional dataset does build up, to extend our analysis to consider if and how data revisions affect the accuracy of regional nowcasts.

The first element of our corresponding regional output database is real–terms annual regional GVA data for the 12 ITL1 regions of the UK for 1998 - 2019.[3] We use the ONS's 'preferred' single measure of regional output (GVA(B)), that balances the income and production measures of GVA.[4] These real–terms regional GVA data are chain-linked volumes. Chain-linked volume estimates are not additive, and so we should not expect the regional GVA data to sum in levels, across regions, to the UK total as we move away from the base year.[5] Estimates back to 1998 were first published in 2017. This recent history means that sufficient real-time data vintages to allow a meaningful analysis of regional data revisions do not yet exist. We therefore use the latest-vintage estimates (at the time of writing this was the May 2021 vintage). To extend these data back beyond 1998, we make use of historical (revised) vintage data for regional nominal GVA data for 1966 – 1996, released by the ONS[6], which in the absence of regional inflation data we deflate by the UK deflator.[7] This means that the real–terms regional GVA data prior to 1998 are constant-price. These historical GVA data are also measured on the income-side only, rather than being balanced estimates on the income and production sides. This switch from constant-price income-based to chain-linked balanced estimates of regional real GVA is a necessary practical step in constructing an historical

rel/elmr/economic-trends--discontinued-/no--627--february-2006/methodology-notes--links-between-gross-domestic-product--gdp--and-gross-value-added--gva-.pdf2

[3]https://www.ons.gov.uk/economy/grossvalueaddedgva/datasets/nominalandrealregionalgrossvalueaddedbalancedbyindustry

[4]https://consultations.ons.gov.uk/national-accounts/consultation-on-balanced-estimates-of-regional-gva/supporting_documents/Development%20of%20a%20balanced%20measure%20of%20regional%20gross%20value%20added.pdf

[5]See p. 192 in the most recent QNA manual available at https://www.imf.org/external/pubs/ft/qna/pdf/2017/ chapter8.pdf.

[6]https://www.ons.gov.uk/economy/regionalaccounts/grossdisposablehouseholdincome/adhocs/006226historiceconomicdataforregionsoftheuk1966to1996

[7]One further data issue in combining the historical data (1966-1996) with the more recent data (1997-2019) is that, as identified in Koop et al. (2020a), these two datasets in nominal levels evidence a level shift in the value of regional output between 1996 and 1997. Such a shift is not present in the equivalent UK data, and does not appear to reflect more than differences in how regional output was calculated historically with methods used now. We therefore elected to smooth out this spike in the 1996-1997 annual growth rate. As our regional econometric models are estimated in annual growth rates, rather than (log) levels, our solution is to proxy the 1997 growth rate with the average of the growth rates in 1996 and 1998.

regional database. But these data features contribute to why econometrically we propose to impose the hierarchical constraints relating the regional disaggregates to the UK aggregate in stochastic form. We should not expect the constraints to hold exactly, given that our real-terms regional GVA data and the UK GVA data to which the regional data are related are a mix-and-match of constant-price income-based and chain-linked balanced estimates. But we might hope that the constraints remain informative in practice when nowcasting, when imposed in stochastic form.

We then add in higher frequency quarterly regional real–terms output data which have historical coverage back to 2012Q1. The ONS began producing these so-called Regional Short Term Indicators (RSTIs), that include real GVA data, in September 2019; see Koop et al. (2020b) for details. They are referred to by ONS as 'Regional GDP' and are for the 9 ITL1 regions of England plus Wales, with equivalent data for Scotland[8] and Northern Ireland[9] (both ITL1 regions of the UK) provided directly by these two devolved administrations. How do these quarterly and annual output data align? Not perfectly. While, in principle, the quarterly and annual data from ONS should align exactly, with the quarterly data constrained to sum to the published annual data, over the period for which both exist, delays in data release and the timing of data revisions will not always ensure that this is the case. For example, at the time of writing, the annual data only runs to 2019 while the quarterly data are available to Q3 2020. When 2020 annual data are published the quarterly estimates will be revised to constrain to these new annual totals.

The data for Scotland and Northern Ireland do not constrain to the totals published by the ONS, although comparison on a nominal basis suggests quite close alignment. In real–terms, however, differences in the approach to deflation mean that the Scottish Government data align less well to the ONS published annual data (Koop et al. (2020b)).[10] Complicating the data landscape further, the quarterly regional output data in the UK are released with different delays after the end of the relevant quarter. The Scottish Government operate with a release delay of just under 3 months, the data for Northern Ireland are released with a delay of just over 3 months, and the ONS data for the English regions and Wales are released with a delay of around 6 months. Output data for the UK as a whole is released with a delay of around 6 weeks after the end of the relevant quarter.

In short, measurement errors (including differences in methodology) explain why temporal and cross-sectional constraints, based on accounting identities, should not be expected to hold exactly. Another factor contributing to measurement error is the reality that, due to limited

[8]https://www.gov.scot/collections/economy-statistics/

[9]https://www.nisra.gov.uk/statistics/economic-output-statistics/ni-composite-economic-index

[10]For more on this, see the Scottish Government GDP methodology document here: https://www2.gov.scot/Resource/0054/00542708.pdf.

availability of vintage/historical regional data, the regional and UK data have not always been through the same revisions and benchmarking processes. This means the data do not always 'add up' (or average) exactly. This lack of consistency between the RSTI data, the regional GVA(B) data, and the UK GDP data is conveniently summarized in this extract from the ONS[11]: '...[RSTIs] will align with the annual growth rates determined by regional accounts, while fitting a quarterly path based on the underlying [RSTIs] data. Since regional accounts themselves are constrained to national estimates of GDP, this process of benchmarking ensures that [RSTIs] are also broadly in line with the national estimates. However, there may still be inconsistencies between our [RSTIs] data, post regional accounts benchmarking, and our short-term estimates of GDP. This is because there are some clear differences in the data sources and methods used (for example, in the extent to which VAT data is used). This means that while [RSTIs] aims to produce the best estimates at a regional level, the sum of the regions (adding in published estimates for Scotland and Northern Ireland) may not equal the national total in the time period following the regional accounts benchmarking.' Indeed, even in nominal terms, as discussed in the online appendix of Koop et al. (2020a), the historical (pre-1998) regional GVA data in levels do not sum exactly to the UK total. The ONS acknowledge this via explicit publication of a statistical discrepancy in their underlying Regional Trends publications.

To construct a measure of labor productivity we also require a measure of labor input, either in the form of hours worked or jobs to produce a measure of output per hours worked or output per job. Given changes in the structure of the labor market over time, the preferred measure – and the one which is used in this paper – is hours worked. In order to construct this measure we use data on hours worked at a UK level, available at the quarterly frequency for our full sample period.[12] At the regional level the data are not so easily available. From 1997Q2 to 2021Q1 regional hours worked data are available at the quarterly frequency.[13] Prior to this, however, these data are not available, and so we backcast this measure over the earlier part of our sample using regional level data on the number of jobs. The cross-sectional constraint between hours worked in the UK as a whole and in the ITL1 regions of the UK, over the first part of our sample will therefore not hold exactly. In the later part of our sample, however these do aggregate directly. Data on hours worked at the regional level are released 3 months after the release of equivalent national level data (which, in turn, are released with a delay of

[11] Taken from https://www.ons.gov.uk/economy/grossdomesticproductgdp/methodologies/introducinggdpforthecountriesoftheukandtheregionsofengland

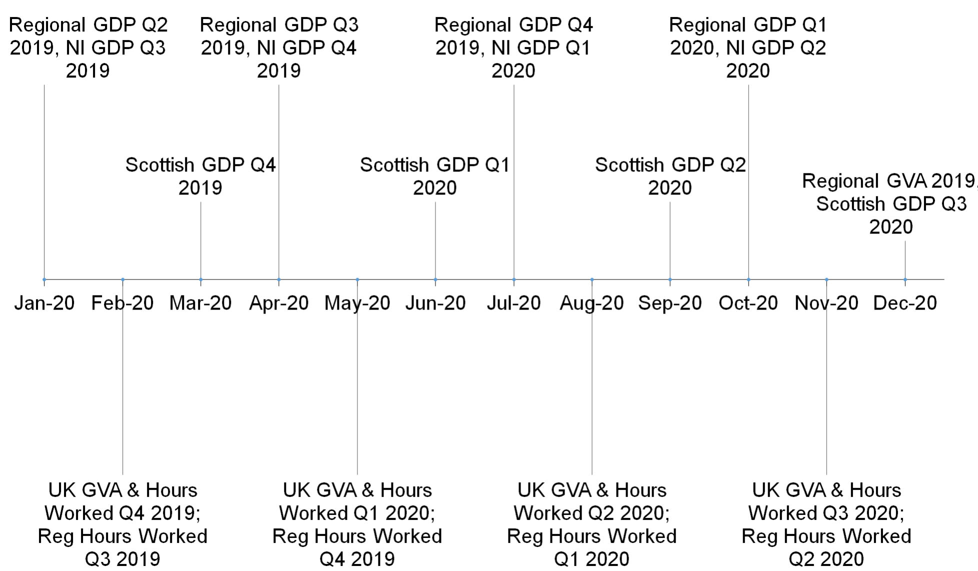
[12] https://www.ons.gov.uk/employmentandlabormarket/peopleinwork/laborproductivity/datasets/laborproductivity

[13] https://www.ons.gov.uk/economy/economicoutputandproductivity/productivitymeasures/datasets/quarterlyregionalproductivityhoursandproductivityjobsnuts1

around 6 weeks). For both Q and L data at a national level are more readily available with a longer time-series than equivalent data at a regional level, motivating our use of stochastic hierarchical aggregation constraints in this paper.

In order to explore the role of adding region-specific information into our model – acknowledging the limited time series of data that is available at a regional level – we include two additional quarterly predictors for each region into our model. The first is the claimant count measure of unemployment, and the other is the Confederation of British Industry's (CBI) Business Optimism index. In our model we also include four UK wide macroeconomic indicators (the Bank rate, CPI inflation, USD:GBP exchange rate, and the Brent oil price). These data are available in real–time with the exception of the CPI Index, which has a release delay of 1 month, and so are omitted from Figure 1 which summarizes the release calendar for our application. We refer to this release calendar below when explaining the timing of our nowcasts and forecasts.

Figure 1: Illustrative release calendar for UK regional data indicating data published through 2020



3 Notation and Key Data Features

We begin by describing some variable definitions, relationships, and notational conventions used in this paper. All changes and growth rates referred to below are exact (not log differenced).

- $t = 1, .., T$ runs at the *quarterly* frequency.

- $r = 1, .., R$ denotes the R regions in the UK.

- $j = Q, L$ indexes output (Q) and labor (L).

- $Y_t^{UK,j}$ are Q and L for the UK in quarter t. $y_t^{UK,j}$ are the corresponding quarterly growth rates. These are observed throughout the sample. $y_t^{UK} = (y_t^{UK,Q}, y_t^{UK,L})'$ is the vector of quarterly growth rates for UK quantities.

- $Y_t^{r,j}$ are Q and L for region r. $y_t^{r,j}$ are the corresponding quarterly growth rates. For $j = Q$ these are not observed before 2012 and for $j = L$ these are not observed before 1997. $y_t^{Q} = \left(y_t^{1,Q}, y_t^{1,L}, .., y_t^{R,L}\right)'$ is the vector of quarterly growth rates for regional quantities.

- $Y_t^{r,j,A} = Y_t^{r,j} + Y_{t-1}^{r,j} + Y_{t-2}^{r,j} + Y_{t-3}^{r,j}$ are annual Q and L for region r.[14] $y_t^{r,j,A}$ are the corresponding annual growth rates. These are observed in quarter 4 of each year throughout the sample, but not in other quarters. $y_t^{A} = \left(y_t^{1,Q,A}, y_t^{1,L,A}, .., y_t^{R,L,A}\right)'$ is the vector of annual growth rates for regional quantities.

The MF-VAR we work with uses $y_t = \left(y_t^{UK}, y_t^{Q\prime}\right)'$ as the vector of dependent variables.[15] Note that the regional variables in our MF-VAR are not observed for some periods of time. These unobserved quarterly quantities are constrained to add up to observed annual quantities via the following temporal constraint (see equation C.1 in the online appendix to Koop et al., 2020a):

$$y_t^{r,j,A} = c^{r,j} + \frac{1}{4}y_t^{r,j} + \frac{1}{2}y_{t-1}^{r,j} + \frac{3}{4}y_{t-2}^{r,j} + y_{t-3}^{r,j} + \frac{3}{4}y_{t-4}^{r,j} + \frac{1}{2}y_{t-5}^{r,j} + \frac{1}{4}y_{t-6}^{r,j} + \eta_{t,j}^{r}. \tag{1}$$

This constraint, familiar in the MF-VAR literature as a Mariano and Murasawa (2003)-type temporal aggregation constraint when applied to log differenced data, is an approximation to avoid the need for a nonlinear measurement equation.[16] The approximation becomes poorer in more volatile periods. For this reason, and reflecting the data inconsistency issues discussed in the preceding section, we impose the constraint stochastically, allowing for an error, $\eta_{t,j}^{r}$. To

[14] This identity in fact holds as the annual average, rather than the annual sum of the quarterly values, when Y is defined as an index rather than in levels, as it is for L and for real-terms (chained-linked volume) regional GVA after 1998.

[15] In our empirical work, as described earlier, we augment this vector with four additional UK quarterly predictors and include two additional regional predictors as exogenous variables.

[16] Nonlinear methods that allow for an exact treatment of the temporal constraint have been developed; e.g., see Proietti (2011a). Our preference in this paper is to stick with linear methods, appropriate for VAR models, and instead consider the utility of stochastic imposition of the constraints.

allow for bias, we include intercepts, $c^{r,j}$, in (1), and for them use relatively non-informative priors centered over 0. For the error variances, $\eta^r_{t,j} \sim N(0, \sigma^2_{j,r})$, we also use relatively non-informative priors. But for robustness, as summarized below (with detailed forecasting results in online Appendix B), we do consider alternative priors that reflect a belief that $\sigma^2_{j,r}$ is small.

Note that the temporal constraint involves seven quarters. This motivates our choice of 7 lags in the VAR model (see online Appendix A). However, as described below, our global-local shrinkage prior can shrink extraneous parameters to zero if, as is likely, our lag length is too long. In essence, the use of this type of shrinkage prior can be thought of as an automatic way of selecting lag length. It has the advantage over conventional methods that it allows different equations in the VAR to have different lag lengths and the included lags need not be sequential (e.g., it could include first and fourth lags, but not the second and third).

In addition, we exploit the constraints that UK growth rates are approximately, due to the data characteristics explained above, the weighted sums of regional growth rates, where the weights are the shares of each region in the national total, and refer to these as cross-sectional aggregation constraints. For $j = Q, L$ these can be shown to be (see equation C.2 in the online appendix to Koop et al. (2020c)):

$$y^{UK,j}_t = c^j + \sum_{r=1}^{R} w^{r,j}_t y^{r,j}_t + \eta_{t,j}, \tag{2}$$

where $w^{r,j}_t = \left(\frac{Y^{r,j}_t}{\sum\limits_{r=1}^{R} Y^r_t} \right)$ is the share of the regional quantity in the aggregate quantity in quarter t. Since this share is not observable for L at the quarterly frequency for much of our sample, while for Q it is never observed quarterly only annually, we proxy $w^{r,j}_t$ based on the observed averages from 1999-2013 (our out-of-sample evaluation period starts in 2014). This use of an average is (yet) another reason not to expect the cross-sectional constraint to hold exactly.

As emphasized above, we again allow for intercepts and errors in the aggregation constraints. We assume $\eta_{t,j} \sim N(0, \sigma^2_j)$.[17] Below we refer to robustness checks undertaken to explore the

[17] Another reason for allowing for an error in this aggregation constraint is because of our modeling choice not to include extra-regio, sometimes referred to in the UK as the 'UK continental shelf' (UKCS) in our vector of regional outputs, given its idiosyncratic time series properties. UKCS mostly reflects oil and gas output from the North Sea. Since both the quantity of oil and gas produced and their price have fluctuated greatly over time it is a very volatile series, with time series properties that are very different from other regions of the UK. Koop et al. (2020a) similarly preferred to omit UKCS from their model, finding that its inclusion did not help deliver improved regional nowcasts (which is not surprising given the global nature of the drivers of

sensitivity of results to how (1) and (2) are specified.

We remind the reader that we are modeling regional growth rates; and seeking to test, when nowcasting, the empirical utility of imposing stochastic hierarchical constraints. We do not aim to study the underlying levels data nor do we assess to what degree the implied level nowcasts of L and Q are themselves aggregation-consistent. Finally, details of our priors, including prior hyperparameter choice, are given in Appendix A.

4 Overview of Econometric Methods

In this section, we provide an overview of the econometric methods used in this paper. Complete details are given in online Appendix A. We use a MF-VAR, which as noted above is simply a VAR using $y_t = \left(y_t^{UK}, y_t^{Q\prime}\right)'$ as the vector of dependent variables. It differs from a standard VAR in that some of the variables are not observed (at least for some periods). These are treated as unobserved latent states and the MF-VAR is a state space model. The measurement equations in the state space model are given by the temporal constraints, given in (1), which link the observed annual regional data to the unobserved quarterly counterparts. The model just described is a standard MF-VAR as used, inter alia, in Schorfheide and Song (2015). Standard Bayesian methods exist for estimating this model. Our model differs in several ways. First, the frequency mis-match changes at different points in time for different variables. Methods for handling such a change are developed in Koop et al. (2020b) and described for the present case in online Appendix A. Intuitively, at points in time when regional data switch from annual to quarterly, the appropriate blocks of the model switch to becoming VARs instead of MF-VARs.

A second difference from most existing MF-VARs is that our model is much larger, with many more latent variables to estimate (and a long lag length). For instance, Schorfheide and Song (2015) use a MF-VAR with 11 variables, of which 3 are at the lower frequency. We have 24 low frequency variables (Q and L for each of 12 regions) and only six high frequency variables (Q and L for the UK plus four additional UK variables). In such a high dimensional model with so many latent states to estimate, the role of prior information becomes important so as to avoid over-fitting. In the large Bayesian VAR literature, various global-local or variable

oil and gas activity and UKCS activity's relatively small share of UK GDP (0.8% in 2018)). But output from the UKCS is included in the UK GVA total. This means UK output is not the sum of the regions' output in levels (even when we ignore measurement errors). Note that it is not possible to remove UKCS activity from the overall estimates of UK quarterly GVA, because it is not separately identified within quarterly GDP and it includes activity in multiple sectors of the economy. It is common for countries to have an 'extra regio' element within their national accounts (capturing, for example, activities in overseas embassies, military bases, territorial enclaves, and so on. For a more formal definition, see: https://www.ons.gov.uk/aboutus/transparencyandgovernance/freedomofinformationfoi/gvadataforextraregio).

selection priors have been shown to work well, automatically shrinking superfluous coefficients to zero with minimal subjective prior input from the researcher (see, e.g., George et al. (2008), Koop (2013), Korobilis (2013), Gefang (2014), and Kastner and Huber (2020)). In this paper, we use a popular prior in this class: the adaptive Lasso prior.[18]

To explain the adaptive Lasso, consider equation i of the VAR which involves k_i right hand side variables. The adaptive Lasso prior is a conditionally Normal prior with mean zero (thus ensuring shrinkage towards zero) and covariance matrix:

$$\mathbf{V}_i = \text{diag}(\tau_{i,1}, \ldots, \tau_{i,k_i}). \tag{3}$$

The $\tau_{i,j}$, which control the strength of shrinkage of the j^{th} coefficient in the i^{th} equation, are treated as unknown parameters that are estimated in a data-based fashion. These shrinkage parameters have a prior of the form:

$$\tau_{i,j} \sim Exp(\tfrac{\lambda_{i,j}}{2}), \quad \text{for } j = 1, \ldots, k_i \tag{4}$$

with

$$\lambda_{i,j} \sim G(\underline{a}_0, \underline{b}_0), \tag{5}$$

where Exp denotes the exponential distribution and G denotes the Gamma distribution. Note that the only prior hyperparameters to be chosen are $\underline{a}_0$ and $\underline{b}_0$. See online Appendix A for more details on the specification of the hyperparameters. Zou (2006) demonstrates that the adaptive Lasso satisfies so-called oracle properties. These include asymptotically selecting the correct variables and having the optimal estimation rate.

The model we have specified thus far is identified and, in our empirical section, we will show it produces sensible nowcasts of regional productivity growth. However, there are other features we can exploit which are not available with the conventional MF-VAR. These are the aggregation constraints given in (2). Note that there are two of these, one for Q and one for L. One of the issues we explore in our empirical application is whether exploiting either or both of these can help improve the accuracy of our nowcasts. As shown in Koop et al. (2020c), an aggregation constraint can be treated as an additional measurement equation in the state space model that defines the MF-VAR. Bayesian estimation and nowcasting in state space models is achieved using Markov Chain Monte Carlo (MCMC) methods, but these can be computationally slow in MF-VARs of high dimension such as we have. Accordingly, we use Variational Bayes (VB) methods. VB methods are approximate, but are much faster

[18]Results using the Horseshoe prior are available in the online Appendix B. These tend to be very similar to the adaptive Lasso.

than MCMC methods. Gefang et al. (2022) develop VB methods for MF-VARs and we adopt similar methods in this paper. Further computational details are in online Appendix A.

Mixed frequency models are particularly useful for providing timely updates of low frequency variables. Given release delays in the regional data, as described in Section 2, for any quarter τ of the year, we produce 3 estimates of any quarter's regional growth rate as new information is released. We refer to these as: 1. the forecast, 2. the nowcast and 3. the backcast. Take the 2nd quarter of the year as an example (it may be useful for the reader to refer to Figure 1 while reading this explanation). We receive official estimates of Q2 UK GVA growth in August of that year, at which time the latest regional data that we have (for expositional purposes, setting aside complications that arise from the more timely data for Scotland and Northern Ireland) is for Q4 of the previous year.

More formally, our timing convention is that we run the model immediately upon receipt of the latest UK output data for each quarter, which takes place for quarter $\tau - 1$ mid-way through quarter τ, and we incorporate the latest available data on all other indicators. When we do this we have a two quarter gap between the available UK and regional output data given the 6 month release delay which exists for the regional output data. We therefore produce a 'nowcast' for each region for $\tau - 1$, as well as a 'backcast' for each region for $\tau - 2$, and a 'forecast' for each region for τ. For regional labor market data (hours worked in our case), these are released 3 months after the release of equivalent data for the UK as a whole, and so we only have need for a 'nowcast' and a 'forecast' for these data, as at time τ we already have data on hours worked at a regional level for $\tau - 2$ making production of a backcast redundant.

Prediction is carried out using the simulation methods set out in online Appendix A. But, in summary, our VB methods provide us with an approximation of the posterior of all the parameters and states in the model. We simulate draws from this posterior and, conditional on each draw, produce a one-step ahead forecast from the VAR. This procedure is repeated for 1,000 draws from the posterior. This procedure will produce draws of the growth of Q and L (for the UK and for the regions). These draws are then differenced to produce draws of productivity growth.

We also present results from an AR(1) benchmark which uses a relatively non-informative prior, based on taking 10,000 posterior draws with a 5,000 burn-in. The timing convention for the AR(1) model is as follows. Again we produce 'forecasts', 'nowcasts', and 'backcasts' from this model coincident with the release of UK output data (i.e., 4 times a year, mid-quarter τ). At τ, the latest regional output data relate to $\tau - 3$. So to compute forecasts we estimate an AR(1) model of regional Q for τ on $\tau - 3$, using the sample τ= 1 through $\tau - 3$. Then, given these parameter estimates, we use the latest (in real-time) known regional Q value for $\tau - 3$ to forecast τ (i.e., the current quarter).

To compute nowcasts, we estimate an AR(1) regression of regional Q for τ on $\tau-2$, using the sample τ= 1 through $\tau-3$. Then we use the latest (in real-time) known value for $\tau-3$ to estimate $\tau-1$. To compute backcasts, we estimate an AR(1) regression of regional Q for quarter τ on $\tau-1$, using the sample τ= 1 through $\tau-3$. Then we use the latest (in real-time) known value for $\tau-3$ to estimate $\tau-2$.

AR(1) estimation for L proceeds in a similar way, but we only need to produce forecasts and nowcasts, given L is published more rapidly than Q. The forecast now involves estimating L for τ on L at $\tau-2$, τ=1,..., $\tau-2$, and using L at $\tau-2$ to estimate L for τ. The nowcast involves estimating L for τ on $\tau-1$, τ=1,..., $\tau-2$, and using L at $\tau-2$ to estimate L at $\tau-1$.

5 Empirical Results: Quarterly Regional Growth Estimates

5.1 Overview

In this section, we investigate how our mixed frequency methods perform in forecasting, nowcasting, and backcasting regional productivity growth. We consider 5 different variants of the MF-VARs. Four of these differ in the treatment of the stochastic cross-sectional aggregation constraint, (2). In particular, we consider versions of the MF-VAR which impose (2) both on Q and L, just on Q, just on L, and on neither Q nor L. The fifth model imposes the cross-sectional aggregation constraint on both Q and L, but does not include the region-specific exogenous predictors. We include this model so as to investigate whether including additional regional predictors does help improve our estimates of quarterly regional productivity. In each of these 5 models, the temporal aggregation constraint, (1), is imposed, to ensure temporal consistency. Our primary interest is in assessing the utility of the cross-sectional constraint, that is in testing if and when there are accuracy gains to conditioning the regional estimates on those for the UK as a whole - hence our interest in these 5 models. Results also include a benchmark AR(1) model, as described in the previous section. In online Appendix B, we present two sets of results from our MF-VAR models when we impose restricted variants of the two stochastic constraints, (1) and (2). First, we set $c^{r,j}=0$ in (1) and $c^{j}=0$ in (2). Second, in addition to setting $c^{r,j}=0$ in (1) and $c^{j}=0$ in (2), we set $\sigma^2_{j,r}=0$, and thus allow for only the cross-sectional constraint to be stochastic, for the reasons elucidated in Section 2. Results are similar to those presented below. As an additional robustness check, Appendix B also present results using a prior which is more informative to that used in the body of the paper and reflects a belief that $\sigma^2_{j,r}$ is small; and we present results when we use the Horseshoe rather than adaptive Lasso prior. Again, in both cases, results are qualitatively similar to those presented below.

We use root mean squared forecasts errors (RMSEs) and continuous ranked probability scores (CRPSs) to evaluate our point and density estimates. Our out-of-sample evaluation period is 2014Q2 through 2020Q3.[19] Note that this choice is made to cover the time period the RSTI data are available for, allowing for sufficient lags to apply the temporal constraint in (1) throughout the evaluation period.

5.2 Forecasts, Nowcasts, and Backcasts of Regional Productivity Growth

Tables 1 and 2 evaluate our forecasts, nowcasts, and backcasts of regional productivity growth for the regions of the UK. In relation to point forecast performance, as measured by RMSEs, the MF-VAR forecasts with aggregation constraints on Q, and both Q and L at the same time, do not beat the simple AR(1) benchmark. Remember that the forecasts for time t reflect information known at that time, which at a regional level is output growth for $(t-2)$, and data on hours worked for $(t-1)$; while, at a national level, only data for time t are available. Thus, we are still having to forecast the 'aggregate' which the cross-sectional constraint applies to. It is therefore unsurprising that the aggregation constraints do not really add anything in this case, since we do not know the quarter $t+1$ UK quantities yet and, thus, the aggregation constraints are of little benefit. Without any aggregation constraints, and when the aggregation constraint only applies to the hours worked data, we do see some improvements over the AR(1) model.[20].

When we turn to the nowcasts and backcasts the MF-VAR methods are delivering point forecast performance which is substantially better than the AR(1). Why might this be? Recall that at time t we produce regional 'nowcasts' for t conditional on the UK realization for time t, and this appears to help produce more accurate nowcasts. Specifically, we see the utility of our aggregation constraints, (2), that link observed UK growth to our regional estimates. At the same time, when producing our 'backcasts' at time t, we are conditioning on time t and $t-1$ data for the UK (at time t the latter will also incorporate data revisions relative to its first release value). Clearly, this UK information is allowing us to refine our estimates of regional productivity growth, and given release delays, doing so with a significant timing advantage.

[19] Note that, at the time of writing, 2020Q3 is the latest data available and it includes the some of the pandemic period. We have repeated all our empirical work using a data set which ends with 2019Q4 and the main conclusions are very similar.

[20] Our short evaluation period precludes a strong interpretation, but Tables B.25 through B.30 in online Appendix B.5 show that these gains over the AR(1) can be statistically significant when, following Diebold and Mariano (2002) and Giacomini and White (2006), we use a t-statistic, assuming asymptotic normality and serially uncorrelated errors (expected for *optimal* one-step-ahead forecasts, nowcasts, and backcasts), and implement a two-sided test of equal forecast accuracy. These tests assume the impact of decreasing parameter uncertainty due to recursive estimation is negligible.

When we consider density forecast performance, as measured by CRPS, we obtain a similar story, but one even more favorable to the MF-VARs. That is, with some exceptions to be discussed below, all three of our estimates (i.e., forecasts, nowcasts, and backcasts) now beat the AR(1) benchmark, often by a substantial margin. With regards to the cross-sectional aggregation constraints, which are one of the main links in the model between regional Q and L and UK Q and L, the overall picture is that it is beneficial to impose both of them. However, there are some subtle differences between backcasts, nowcasts, and forecasts which are worth exploring. For the backcasts, we have strong evidence, from both RMSEs and CRPSs, that imposing both cross-sectional constraints leads to substantial improvements. For instance, looking at Table 1 (Table 2), the cross-region average of RMSE (CRPS) is 0.72 (0.26) when they are imposed, but rises to 1.00 (0.34) when they are not imposed.

It is worth stressing that the MF-VAR without the aggregation constraints does allow for newly released UK information to update the regional backcasts, since UK quantities are included as dependent variables in the VAR. However, the MF-VAR with aggregation constraints has this property with the additional link between the UK and its regions provided by these constraints. This additional link clearly helps improve the backcasts. This same pattern holds to a lesser extent with the nowcasts. For the density forecasts, as for the point forecasts, there is little or no benefit to imposing the aggregation constraints. That is, the MF-VAR with no aggregation constraints imposed is forecasting better than other alternatives.

Having established the usefulness of the aggregation constraints, at least in improving nowcasts and backcasts, we assess whether including exogenous regional-level predictors is similarly useful. If we compare the MF-VAR with aggregation constrains to this model with these exogenous predictors added, it can be seen that results are quite similar. In other words, these predictors are adding only very small improvements to our forecasts, nowcasts, and backcasts.

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	2.35	3.11	4.00	2.16	1.91	3.18	2.51	2.48	2.12	3.80	2.04	1.81	**2.62**
Nowcast	1.63	1.86	1.96	2.10	1.78	1.86	1.85	2.03	1.51	1.66	1.51	1.03	**1.73**
Backcast	1.66	1.91	1.89	1.85	1.73	1.70	1.69	1.87	1.35	1.71	1.32	1.24	**1.66**
MF-VAR model - (with both aggregation constraints)													
Forecast	4.21	2.74	1.73	3.02	3.63	2.99	1.89	1.59	3.51	3.82	2.54	1.84	**2.79**
Nowcast	2.50	2.20	2.15	1.22	1.87	1.55	0.82	0.58	2.54	2.06	0.83	0.74	**1.59**
Backcast	1.07	1.17	1.15	0.24	0.94	0.92	0.45	0.33	1.19	0.81	0.20	0.22	**0.72**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	4.22	2.79	1.73	3.12	3.72	3.18	1.98	1.73	3.53	3.83	2.59	1.86	**2.86**
Nowcast	2.05	1.64	1.97	1.16	1.70	1.63	0.68	0.57	2.04	1.81	0.84	0.72	**1.40**
Backcast	1.04	1.16	1.31	0.22	0.88	1.02	0.45	0.32	1.23	0.82	0.21	0.22	**0.74**
MF-VAR model - (aggregation constraint only in L)													
Forecast	2.06	1.43	1.56	1.13	1.32	1.35	1.20	1.25	1.32	1.75	0.72	1.71	**1.40**
Nowcast	1.61	1.57	2.12	2.09	1.08	1.56	1.54	1.92	1.47	1.29	1.51	1.24	**1.58**
Backcast	0.91	1.05	1.23	1.17	0.87	1.05	1.06	1.15	1.10	0.78	0.93	0.65	**1.00**
MF-VAR model - (No aggregation constraints)													
Forecast	2.01	1.46	1.55	1.10	1.31	1.35	1.20	1.27	1.17	1.72	0.72	1.57	**1.37**
Nowcast	2.01	1.90	2.50	2.49	1.50	1.95	1.82	2.35	1.85	1.55	1.81	1.43	**1.93**
Backcast	0.98	1.05	1.23	1.16	0.86	1.04	1.06	1.17	1.13	0.79	0.92	0.64	**1.00**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	4.18	2.76	1.88	3.09	3.70	3.08	1.96	1.89	3.35	3.88	2.44	1.93	**2.85**
Nowcast	2.23	2.18	2.40	1.26	1.79	1.59	0.85	0.59	2.38	1.96	0.80	0.74	**1.56**
Backcast	1.00	1.15	1.31	0.22	0.88	0.96	0.44	0.33	1.16	0.78	0.16	0.23	**0.72**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table 1: RMSFE (Multiplied by 100) for Productivity Growth, from an AR(1) and 5 MF-VAR Models Differing in Whether They Impose the Cross-Sectional Aggregation Constraint, (2), on Output (Q) and/or Labor (L)

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.44	2.04	2.65	1.31	1.21	2.18	1.66	1.56	1.41	2.36	1.31	1.13	**1.69**
Nowcast	0.76	0.80	0.95	0.78	0.76	0.82	0.79	0.85	0.62	0.77	0.59	0.50	**0.75**
Backcast	0.70	0.71	0.80	0.67	0.71	0.70	0.64	0.72	0.49	0.63	0.43	0.49	**0.64**
MF-VAR model - (with both aggregation constraints)													
Forecast	1.39	1.20	0.96	1.07	1.30	1.22	0.89	0.77	1.10	1.56	0.88	0.81	**1.10**
Nowcast	0.99	0.86	0.80	0.57	0.81	0.72	0.44	0.33	0.87	0.93	0.34	0.39	**0.67**
Backcast	0.39	0.34	0.36	0.14	0.30	0.32	0.20	0.17	0.39	0.30	0.09	0.11	**0.26**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	1.40	1.22	0.95	1.09	1.30	1.28	0.93	0.84	1.15	1.58	0.89	0.81	**1.12**
Nowcast	0.84	0.74	0.76	0.52	0.77	0.76	0.39	0.33	0.76	0.85	0.34	0.37	**0.62**
Backcast	0.38	0.34	0.39	0.13	0.29	0.34	0.20	0.16	0.40	0.30	0.09	0.11	**0.26**
MF-VAR model - (aggregation constraint only in L)													
Forecast	1.07	0.80	0.87	0.64	0.76	0.77	0.68	0.69	0.74	1.04	0.45	0.84	**0.78**
Nowcast	0.75	0.68	0.85	0.69	0.54	0.68	0.66	0.71	0.60	0.66	0.52	0.53	**0.66**
Backcast	0.35	0.32	0.40	0.34	0.30	0.33	0.34	0.37	0.39	0.25	0.26	0.25	**0.33**
MF-VAR model - (No aggregation constraints)													
Forecast	1.08	0.83	0.86	0.63	0.76	0.78	0.69	0.71	0.69	1.01	0.45	0.80	**0.77**
Nowcast	0.90	0.76	0.92	0.78	0.65	0.77	0.71	0.82	0.70	0.72	0.58	0.58	**0.74**
Backcast	0.40	0.32	0.40	0.36	0.31	0.34	0.34	0.38	0.40	0.26	0.26	0.27	**0.34**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.38	1.26	1.04	1.11	1.31	1.25	0.95	0.85	1.07	1.68	0.85	0.83	**1.13**
Nowcast	1.04	0.98	0.99	0.68	0.88	0.84	0.60	0.41	0.95	1.04	0.38	0.46	**0.77**
Backcast	0.43	0.37	0.41	0.16	0.32	0.36	0.23	0.21	0.41	0.32	0.07	0.14	**0.29**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table 2: CRPS (Multiplied by 100) for Productivity Growth, from an AR(1) and 5 MF-VAR Models Differing in Whether They Impose the Cross-Sectional Aggregation Constraint, (2), on Output (Q) and/or Labor (L)

5.3 Forecasts, Nowcasts, and Backcasts of Output and Employment Growth

The previous sub-section discussed results for regional productivity (Q/L) growth. But, of course, our MF-VARs also produce results for Q and L growth individually. By examining these, we can gain a better understanding of where the improvements in the productivity estimates provided by the MF-VAR come from. Tables 3 through 6 present results for these variables individually using exactly the same models as in the previous sub-section. Note that regional L data are released more quickly than regional Q data and at the time we make the backcast the initial release for L has already occurred. For this reason, we do not provide a backcast for L.

The results for output growth exhibit a similar pattern to those for productivity growth. That is, the MF-VARs offer substantial improvements over the AR(1) benchmark, particularly for the nowcasts and backcasts. In addition, the benefits of imposing the cross-sectional aggregation constraints can be clearly seen. However, for the growth in hours worked, results are weaker. Our MF-VARs are not clearly beating the AR(1) benchmark and there is little benefit to imposing the aggregation constraints. This may well reflect greater persistence of changes in hours worked compared to growth in output – or, put differently, the time series properties of growth in hours worked relative to output growth make an AR(1) model harder to beat (as indeed we see from the results' tables). We are finding evidence that the improvement in the quality of the estimates of productivity growth produced by the MF-VAR is mostly due to improvements in the quality of the output growth estimates.

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	3.10	3.03	4.35	2.63	2.59	3.38	2.51	3.93	2.39	3.11	2.25	1.91	**2.93**
Nowcast	1.96	2.23	2.46	2.29	1.93	2.12	2.14	2.42	1.91	1.95	1.79	1.33	**2.04**
Backcast	1.66	1.91	1.89	1.85	1.73	1.70	1.69	1.87	1.35	1.71	1.32	1.24	**1.66**
MF-VAR model - (with both aggregation constraints)													
Forecast	3.18	1.95	2.01	2.13	2.64	2.21	1.38	1.83	2.34	2.81	1.53	1.07	**2.09**
Nowcast	1.84	1.71	1.87	0.53	1.40	1.65	0.85	0.97	1.96	1.33	0.45	0.71	**1.27**
Backcast	1.07	1.17	1.15	0.24	0.94	0.92	0.45	0.33	1.19	0.81	0.20	0.22	**0.72**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	3.15	1.94	1.99	2.18	2.71	2.33	1.42	1.85	2.33	2.80	1.55	1.08	**2.11**
Nowcast	1.51	1.44	2.02	0.47	1.23	1.58	0.86	0.97	1.63	1.16	0.41	0.67	**1.16**
Backcast	1.04	1.16	1.31	0.22	0.88	1.02	0.45	0.32	1.23	0.82	0.21	0.22	**0.74**
MF-VAR model - (aggregation constraint only in L)													
Forecast	2.79	2.38	2.55	2.38	2.04	2.42	2.23	2.74	2.25	1.95	1.96	2.05	**2.31**
Nowcast	1.64	1.78	2.13	1.89	1.49	1.77	1.79	2.08	1.72	1.27	1.56	1.23	**1.69**
Backcast	0.91	1.05	1.23	1.17	0.87	1.05	1.06	1.15	1.10	0.78	0.93	0.65	**1.00**
MF-VAR model - (No aggregation constraints)													
Forecast	2.76	2.38	2.56	2.41	2.07	2.43	2.24	2.76	2.22	1.92	1.97	1.90	**2.30**
Nowcast	1.75	1.79	2.12	1.91	1.52	1.78	1.79	2.11	1.76	1.29	1.57	1.23	**1.72**
Backcast	0.98	1.05	1.23	1.16	0.86	1.04	1.06	1.17	1.13	0.79	0.92	0.64	**1.00**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	3.14	2.01	1.90	2.12	2.62	2.23	1.38	1.84	2.19	2.74	1.47	1.12	**2.06**
Nowcast	1.61	1.67	2.10	0.51	1.26	1.66	0.84	0.95	1.79	1.20	0.38	0.76	**1.23**
Backcast	1.00	1.15	1.31	0.22	0.88	0.96	0.44	0.33	1.16	0.78	0.16	0.23	**0.72**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table 3: RMSFE (Multiplied by 100) for Output Growth, from an AR(1) and 5 MF-VAR Models Differing in Whether They Impose the Cross-Sectional Aggregation Constraint, (2), on Output (Q) and/or Labor (L)

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.61	1.41	2.07	1.40	1.67	1.68	1.36	1.83	1.08	1.37	0.95	0.91	**1.45**
Nowcast	0.85	0.89	1.06	0.92	0.88	0.91	0.88	1.01	0.76	0.81	0.68	0.58	**0.85**
Backcast	0.70	0.71	0.80	0.67	0.71	0.70	0.64	0.72	0.49	0.63	0.43	0.49	**0.64**
MF-VAR model - (with both aggregation constraints)													
Forecast	1.22	0.86	0.95	0.89	1.03	1.00	0.66	0.75	0.91	1.15	0.60	0.55	**0.88**
Nowcast	0.74	0.58	0.80	0.30	0.58	0.59	0.39	0.43	0.61	0.57	0.19	0.34	**0.51**
Backcast	0.39	0.34	0.36	0.14	0.30	0.32	0.20	0.17	0.39	0.30	0.09	0.11	**0.26**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	1.20	0.85	0.94	0.90	1.05	1.04	0.68	0.77	0.93	1.14	0.62	0.55	**0.89**
Nowcast	0.65	0.52	0.83	0.28	0.54	0.57	0.40	0.44	0.55	0.52	0.19	0.32	**0.48**
Backcast	0.38	0.34	0.39	0.13	0.29	0.34	0.20	0.16	0.40	0.30	0.09	0.11	**0.26**
MF-VAR model - (aggregation constraint only in L)													
Forecast	1.43	1.02	1.19	1.11	1.10	1.09	0.98	1.12	1.12	0.97	0.76	0.99	**1.07**
Nowcast	0.81	0.71	0.88	0.78	0.71	0.74	0.70	0.81	0.72	0.59	0.54	0.54	**0.71**
Backcast	0.35	0.32	0.40	0.34	0.30	0.33	0.34	0.37	0.39	0.25	0.26	0.25	**0.33**
MF-VAR model - (No aggregation constraints)													
Forecast	1.43	1.03	1.19	1.12	1.11	1.08	0.98	1.13	1.08	0.94	0.77	0.94	**1.07**
Nowcast	0.87	0.71	0.88	0.80	0.73	0.73	0.71	0.82	0.75	0.59	0.55	0.55	**0.72**
Backcast	0.40	0.32	0.40	0.36	0.31	0.34	0.34	0.38	0.40	0.26	0.26	0.27	**0.34**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.30	0.90	0.94	0.91	1.04	0.99	0.68	0.77	0.87	1.10	0.58	0.57	**0.89**
Nowcast	0.82	0.64	0.93	0.35	0.63	0.66	0.49	0.52	0.63	0.59	0.17	0.42	**0.57**
Backcast	0.43	0.37	0.41	0.16	0.32	0.36	0.23	0.21	0.41	0.32	0.07	0.14	**0.29**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table 4: CRPS (Multiplied by 100) for Output Growth, from an AR(1) and 5 MF-VAR Models Differing in Whether They Impose the Cross-Sectional Aggregation Constraint, (2), on Output (Q) and/or Labor (L)

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	2.10	3.22	2.39	2.91	2.28	2.98	2.88	2.36	3.77	2.68	2.34	2.51	**2.70**
Nowcast	0.97	1.01	1.10	1.20	0.80	0.93	0.94	1.06	1.07	0.96	1.05	1.09	**1.01**
MF-VAR model - (with both aggregation constraints)													
Forecast	1.92	1.85	1.79	1.90	1.59	1.84	1.75	1.89	2.00	1.69	1.82	1.87	**1.82**
Nowcast	0.94	1.05	1.16	1.35	0.77	0.98	0.97	1.09	1.05	0.94	1.10	1.21	**1.05**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	1.93	1.87	1.80	1.91	1.56	1.82	1.77	1.89	2.00	1.71	1.83	1.87	**1.83**
Nowcast	0.95	1.02	1.06	1.18	0.76	0.95	0.93	1.04	1.03	0.92	1.03	1.07	**0.99**
MF-VAR model - (aggregation constraint only in L)													
Forecast	2.06	1.80	1.85	1.99	1.70	1.88	1.78	1.90	2.03	1.71	1.85	1.95	**1.87**
Nowcast	1.12	1.03	1.18	1.33	0.79	0.98	0.96	1.10	1.06	0.96	1.11	1.23	**1.07**
MF-VAR model - (No aggregation constraints)													
Forecast	2.09	1.86	1.87	2.02	1.74	1.93	1.85	1.92	2.08	1.74	1.89	1.97	**1.91**
Nowcast	1.35	1.23	1.44	1.62	0.92	1.23	1.14	1.33	1.24	1.10	1.29	1.35	**1.27**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.94	1.87	1.83	1.97	1.74	1.94	1.77	1.96	2.04	1.76	1.81	1.89	**1.88**
Nowcast	0.93	1.07	1.15	1.39	0.87	1.04	0.98	1.12	1.08	0.98	1.09	1.17	**1.07**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table 5: RMSFE (Multiplied by 100) for Growth in Hours Worked, from an AR(1) and 5 MF-VAR Models Differing in Whether They Impose the Cross-Sectional Aggregation Constraint, (2), on Output (Q) and/or Labor (L)

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.06	2.10	1.53	1.33	1.19	1.83	1.74	1.29	2.17	1.70	1.49	1.43	**1.57**
Nowcast	0.39	0.41	0.41	0.43	0.33	0.38	0.36	0.38	0.41	0.41	0.40	0.41	**0.39**
MF-VAR model - (with both aggregation constraints)													
Forecast	0.87	0.90	0.80	0.77	0.66	0.82	0.74	0.78	0.91	0.92	0.82	0.84	**0.82**
Nowcast	0.42	0.46	0.46	0.51	0.29	0.41	0.37	0.41	0.43	0.47	0.46	0.51	**0.43**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	0.88	0.92	0.79	0.77	0.63	0.82	0.75	0.79	0.91	0.94	0.82	0.83	**0.82**
Nowcast	0.43	0.43	0.39	0.42	0.28	0.38	0.33	0.36	0.40	0.45	0.41	0.42	**0.39**
MF-VAR model - (aggregation constraint only in L)													
Forecast	0.92	0.88	0.82	0.80	0.70	0.85	0.75	0.79	0.94	0.94	0.83	0.86	**0.84**
Nowcast	0.52	0.46	0.47	0.50	0.29	0.41	0.37	0.42	0.45	0.48	0.47	0.51	**0.45**
MF-VAR model - (No aggregation constraints)													
Forecast	0.94	0.92	0.81	0.81	0.69	0.86	0.78	0.80	0.95	0.96	0.85	0.86	**0.85**
Nowcast	0.60	0.54	0.55	0.58	0.36	0.50	0.44	0.50	0.53	0.55	0.53	0.54	**0.52**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	0.89	0.90	0.82	0.80	0.72	0.90	0.76	0.83	0.94	0.98	0.83	0.85	**0.85**
Nowcast	0.45	0.52	0.50	0.60	0.42	0.50	0.43	0.48	0.50	0.53	0.51	0.55	**0.50**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table 6: CRPS (Multiplied by 100) for Growth in Hours Worked, from an AR(1) and 5 MF-VAR Models Differing in Whether They Impose the Cross-Sectional Aggregation Constraint, (2), on Output (Q) and/or Labor (L)

5.4 Calibration of the Forecasts, Nowcasts, and Backcasts

Our preceding results indicated that the MF-VAR performed well relative to a simple benchmark and established that the cross-sectional aggregation constraints, particularly the one associated with output growth, led to substantial improvements in nowcast performance. In this subsection, we use probability integral transforms (PITs) to investigate the calibration of the best-performing model: the MF-VAR with both cross-sectional aggregation constraints imposed. Figures 2 through 4 plot the cumulative distribution functions of the PITs for our backcasts, nowcasts, and forecasts, respectively. If the forecasts were perfectly calibrated, we would see the empirical distribution function for the PITs on the 45-degree line. We have only 25 observations in our backcast evaluation period, and 26 in our nowcast and forecast evaluation period, and so with such a small sample size we can expect deviations from the 45-degree line even from a well-calibrated model. Following Rossi and Sekhposyan (2019), we plot 90% confidence bands around the 45-degree line to account for sample uncertainty.[21] With the small-sample qualification in mind, visually the PITs plots indicate that calibration is quite good for most of the regions. However, for a number of regions we see that the empirical distribution function is somewhat flatter than the 45-degree line, particularly for the backcasts and nowcasts, indicating that we are underestimating uncertainty. It is well-established that VB methods can often underestimate posterior and, thus, predictive variances; e.g., see Giordano et al. (2018) for a general discussion, and Gefang et al. (2022) for an investigation of this issue in VARs. This could be partially explaining some of these findings. However, this feature of the PITs plots does not happen universally, and so it is hard to extract a general message. For example, looking at the backcasts for Scotland (see Figure 2) we see a tendency for our model to overestimate uncertainty. The data for Scotland are constructed on a slightly different basis to those for the English regions, which might help explain these apparent differences.

[21] These bands should be interpreted as providing general guidance, since they are derived assuming a rolling window of estimation, while we use an expanding estimation window.

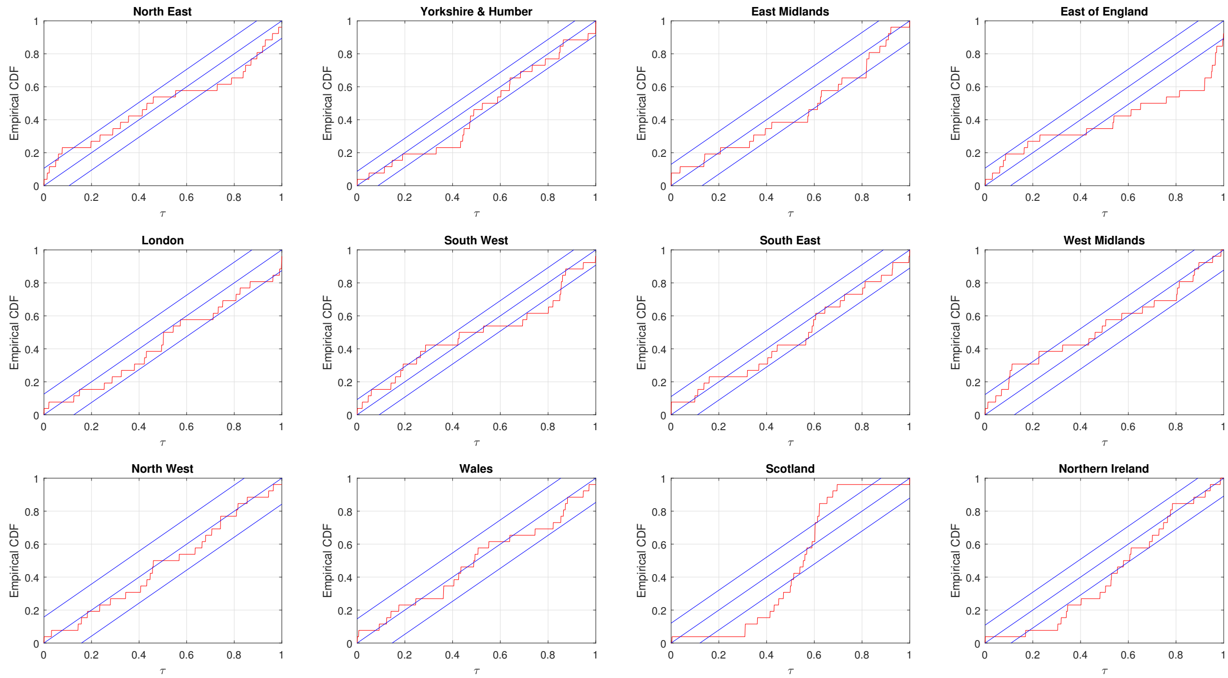


Figure 2: Empirical Cumulative Distribution Function of the PITs for the Backcasts and Rossi and Sekhposyan (2019) 90% Bands for the Test of Correct Specification

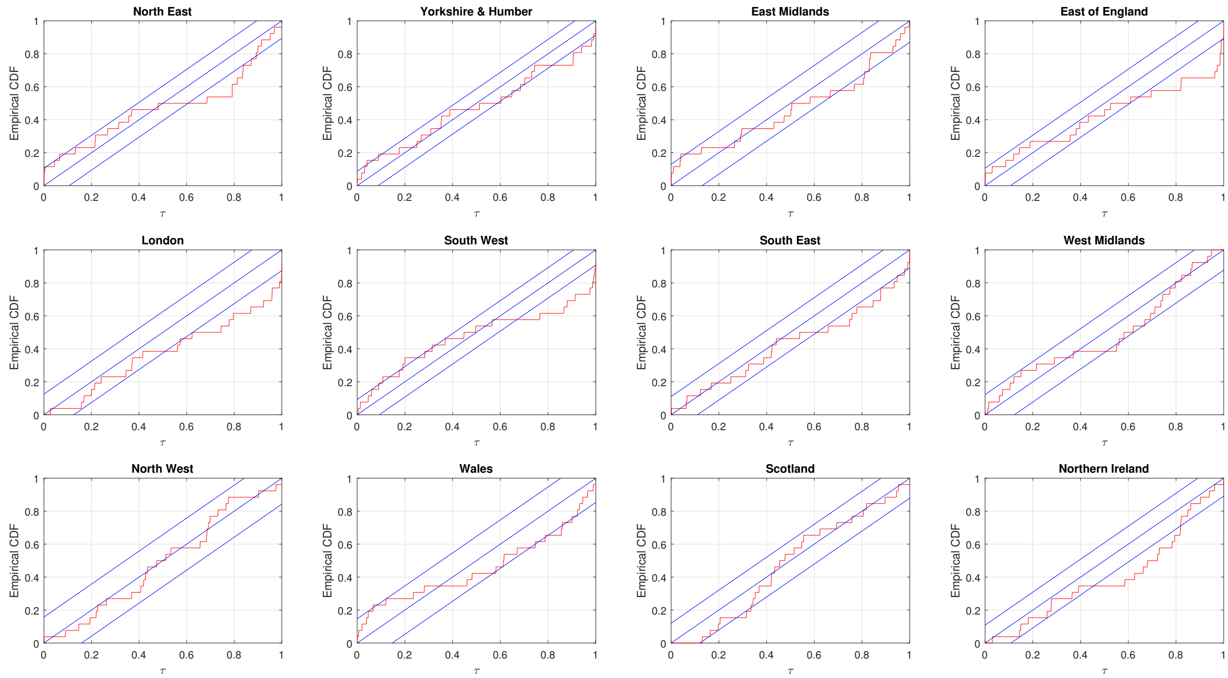


Figure 3: Empirical Cumulative Distribution Function of the PITs for the Nowcasts and Rossi and Sekhposyan (2019) 90% Bands for the Test of Correct Specification

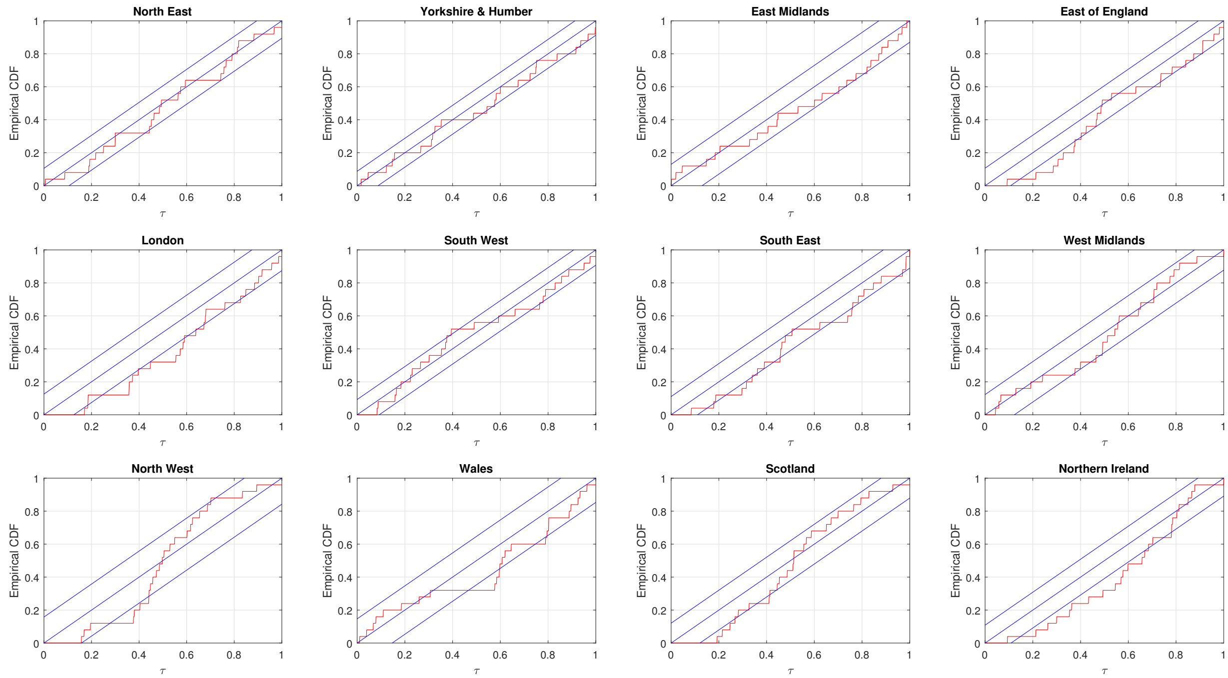


Figure 4: Empirical Cumulative Distribution Function of the PITs for the Forecasts and Rossi and Sekhposyan (2019) 90% Bands for the Test of Correct Specification

6 Conclusions

This paper has taken on the challenge of nowcasting regional productivity growth in the UK at the quarterly frequency. Productivity is not something that is directly measured, but is the ratio of a measure of output (Q) to a measure of labor (L), and it is Q and L that are directly measured. The characteristics of UK data for these two variables (i.e., that they change in frequency over time and that this change occurs at different times and that they have different release schedules and are measured subject to various errors) necessitates the development of a high-dimensional mixed frequency econometric model that accommodates these features. In this paper, we have developed such a model, derived a VB algorithm which allows for estimating and forecasting with it, and taken it to the data.

We have explored the contribution that different features of our modeling approach contribute in a pseudo real–time empirical exercise, comparing backcast, nowcast, and forecast accuracy across 5 different versions of our model, each reflecting different combinations of our hierarchical (cross-sectional) aggregation constraints, as well as the contribution of additional regional-level predictors. We demonstrated the importance that hierarchical aggregation constraints, imposed in stochastic form, play in producing more accurate estimates of sub–national output, hours worked, and productivity growth, when the national aggregate was known (as it is in our 'nowcasts' and 'backcasts') but not as clearly when the aggregate is also being forecast.

For sub–national policymakers seeking to close the information gap that exists between national level understanding of economic fluctuations and changes in the local economy, the methods set out in this paper provide a good way of producing timely regional growth estimates consistent with the national level aggregate. Given our focus in this paper on producing more timely sub–national estimates of productivity growth, and exploring the role of national level data in improving these estimates, we have not explored many features that might improve national level forecasting and which might be expected, via the cross-sectional aggregation constraint, to improve our forecast of the national aggregate. We leave this to future work, but note that the methods set out in this paper should help translate improvements in forecasting national growth to improved regional estimates via the aggregation constraint in the same way as we saw happen in our empirical exercise when the aggregate was known.

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Online Appendix for: Using Stochastic Hierarchical Aggregation Constraints to Nowcast Regional Economic Aggregates

Gary Koop, Stuart McIntyre, James Mitchell, and Aubrey Poon

A Technical Appendix

A.1 The Econometric Model

Our econometric model is a mixed-frequency state-space VAR (MF-VAR) with seven lags:

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{b}_0 + \mathbf{B}_1\mathbf{y}_{t-1} + \ldots + \mathbf{B}_7\mathbf{y}_{t-7} + \gamma_1\mathbf{z}_t^1 + \gamma_2\mathbf{z}_t^2 + \epsilon_t, \epsilon_t \sim N(0, \Sigma), \tag{A.1}$$

where $\mathbf{y}_t = (y_t^{UK}, y_t^{Q'})'$ is the $n \times 1$ vector of endogenous variables, and $\mathbf{z}_t^1$ and $\mathbf{z}_t^2$ are the two region specific exogenous variables. The latter are only included in the equations for the corresponding region and not other regions nor for the UK as a whole and, thus, γ_1 and γ_2 will contain zeros in the appropriate places. $\mathbf{A}_0$ is a lower triangular matrix with ones on the diagonal, $\mathbf{b}_0$ is a $n \times 1$ vector of intercepts, $\mathbf{B}_p$ is the $n \times n$ VAR coefficient matrix on the lag p, and Σ is a diagonal matrix.

The MF-VAR is a state space model with state equations being those for the unobserved regional quarterly quantities in (A.1), and with measurement equations being the observed UK quarterly quantities along with the stochastic constraints given in (1) and (2).

A.2 Priors and Variational Bayesian (VB) Estimation of Model Parameters

In this sub-section, we describe VB methods for estimating the model's parameters assuming the dependent variables in the MF-VAR are observed. In practice, the unobserved values are replaced by the estimates produced using methods described in the next subsection. We use the VB methods for VARs with global-local shrinkage priors developed in Gefang et al. (2022).

We can rewrite (A.1) as:

$$\mathbf{y}_t = \mathbf{X}_t\beta + \mathbf{W}_t\mathbf{a} + \epsilon_t, \tag{A.2}$$

where $\mathbf{X}_t = \mathrm{I}_n \otimes [1, \mathbf{y}_{t-1}', \ldots, \mathbf{y}_{t-7}', \mathbf{z}_t^{1'}, \mathbf{z}_t^{2'}]$ is an $n \times K$ matrix, $\beta = vec([\mathbf{b}_0, \mathbf{B}_1, \ldots, \mathbf{B}_7, \gamma_1, \gamma_2]')$ is $K \times 1$ vector of coefficients, $\mathbf{a}$ consists of the free elements of $\mathbf{A}_0$ stacked by rows, with $\mathbf{W}_t$ being the $n \times m$ matrix containing the appropriate contemporaneous elements of $\mathbf{y}_t$. Equation

(A.2) can be written in terms of n independent equations, with the i^{th} equation being:

$$y_{i,t} = \tilde{\mathbf{x}}_{i,t}\theta_i + \epsilon_{i,t}, \epsilon_{i,t} \sim N(0, \sigma_i^2). \tag{A.3}$$

where $\tilde{\mathbf{x}}_{i,t}$ is a row vector with k_i elements and θ_i is a vector containing the elements of β and $\mathbf{a}$ pertaining to the i^{th} equation. Below we also use notation where $\tilde{\mathbf{X}}_i = (\tilde{\mathbf{x}}_{i,1}, \ldots, \tilde{\mathbf{x}}_{i,T})'$, $\mathbf{y}_i = (y_{i,1}, \ldots, y_{i,T})'$ and $\epsilon_i = (\epsilon_{i,1}, \ldots, \epsilon_{i,T})'$. Following Gefang et al. (2022), we specify the prior distributions for (A.3):

$$\theta_i \sim N(0, \mathbf{V}_i), \tag{A.4}$$

$$\sigma_i^{-2} \sim G(\underline{\nu}, \underline{s}), \tag{A.5}$$

and thus the VB approximating densities can be derived as:

$$\bar{\mathbf{V}}_i = [(\frac{\underline{\nu} + \frac{T}{2}}{\bar{s}_i})\tilde{\mathbf{X}}_i'\tilde{\mathbf{X}}_i + \mathbf{V}_i^{-1})]^{-1}, \tag{A.6}$$

$$\bar{\theta}_i = (\frac{\underline{\nu} + \frac{T}{2}}{\bar{s}_i})\bar{\mathbf{V}}_i\tilde{\mathbf{X}}_i'\mathbf{y}_i, \tag{A.7}$$

$$\bar{s}_i = \underline{s} + \frac{1}{2}\left\|\mathbf{y}_i - \tilde{\mathbf{X}}_i\bar{\theta}_i\right\|^2 + \frac{1}{2}tr(\tilde{\mathbf{X}}_i'\tilde{\mathbf{X}}_i\bar{\mathbf{V}}_i). \tag{A.8}$$

Note that the VB approximating densities depend on three arguments: $\bar{\theta}_i, \bar{\mathbf{V}}_i$, and $\bar{s}_i$. These are optimized in an iterative process. Beginning with an initialization of any two of these, the algorithm iterates using the preceding formulae. After each iteration, the evidence lower bound or $ELBO_i$ is calculated. Iteration continues until the increase in $ELBO_i$ between the j^{th} and $(j-1)^{th}$ iteration is less than some convergence criterion. The formula for the evidence lower bounds are given in Gefang et al. (2022).

A.2.1 Adaptive Lasso Prior

The adaptive Lasso assumes that equation i has a prior variance covariance matrix of:

$$\mathbf{V}_i = \text{diag}(\tau_{i,1}, \ldots, \tau_{i,k_i}). \tag{A.9}$$

Note that this allows for the different equations to have different prior shrinkage. The adaptive Lasso is a hierarchical prior which assumes:

$$\tau_{i,j} \sim Exp(\tfrac{\lambda_{i,j}}{2}), \quad \text{for } j = 1, \ldots, k_i \tag{A.10}$$

with:

$$\lambda_{i,j} \sim G(\underline{a}_0, \underline{b}_0). \tag{A.11}$$

With this hierarchical shrinkage prior, the optimal VB approximating densities for $q(\theta_i)$ and $q(\sigma_i^{-2})$ are as above, but we add approximating densities for $\tau_{i,j}$ and λ_i where $\lambda_i = (\lambda_{i,1}, \ldots, \lambda_{i,k_i})'$. These are:

$$q(\tau_{i,j}^{-1}) \sim iG(\sqrt{\frac{\bar{\lambda}_{i,j}}{\bar{\theta}_{i,j}^2 + \bar{\mathbf{V}}_i^{jj}}}, \bar{\lambda}_{i,j}), \tag{A.12}$$

where iG denotes the inverse Gaussian distribution and:

$$q(\lambda_{i,j}) \sim G(\underline{a}_0 + 1, 0.5\bar{\tau}_{i,j} + \underline{b}_0). \tag{A.13}$$

These involve the following terms to be iterated in the VB algorithm:

$$\overline{\tau_{i,j}^{-1}} = \sqrt{\frac{\bar{\lambda}_{i,j}}{\bar{\theta}_{i,j}^2 + \bar{\mathbf{V}}_i^{jj}}}, \tag{A.14}$$

$$\bar{\lambda}_{i,j} = \frac{\underline{a}_0 + 1}{0.5\bar{\tau}_{i,j} + \underline{b}_0}, \tag{A.15}$$

and $\bar{\tau}_{i,j}$ denotes $\frac{1}{\overline{\tau_{i,j}^{-1}}}$.

The evidence lower bound can be found in Gefang et al. (2022).

A.2.2 Horseshoe Prior

The main results in the paper use the adaptive Lasso prior. As a prior robustness check, we have also produced results using the Horseshoe prior. The horseshoe prior assumes a prior variance covariance matrix of:

$$\mathbf{V}_i = \text{diag}(\lambda_{i,1}\tau_i, \ldots, \lambda_{i,k_i}\tau_i), \tag{A.16}$$

where the priors for the new parameters are:

$$\lambda_{i,j}^{-1} | \nu_{i,j} \sim G(\frac{1}{2}, \frac{1}{\nu_{i,j}}), \tag{A.17}$$

$$\tau_i^{-1}|\xi_i \sim G(\frac{1}{2}, \frac{1}{\xi_i}), \tag{A.18}$$

$$\nu_{i,1}^{-1}, \ldots, \nu_{i,k_i}^{-1}, \xi_i^{-1} \sim G(\frac{1}{2}, 1) \tag{A.19}$$

and i indexes equations and j indexes coefficients.

The optimal $q(\theta_i)$ and $q(\sigma_i^{-2})$ are the same as in preceding sub-sections. The conditional posteriors for the remaining parameters using the horseshoe prior can be found in Makalic and Schmidt (2016). These can be used to derive the VB approximating densities:

$$q(\lambda_{i,j}^{-1}) \sim G(1, \overline{\nu_{i,j}^{-1}} + \frac{\bar{\theta}_{i,j}^2 + \bar{\mathbf{V}}_i^{jj}}{2} \overline{\tau_i^{-1}}), \tag{A.20}$$

$$q(\tau_i^{-1}) \sim G(\frac{k_i+1}{2}, \overline{\xi_i^{-1}} + \frac{1}{2} \overline{\lambda_{i,j}^{-1}} \sum_{j=1}^{k_i} (\bar{\theta}_{i,j}^2 + \bar{\mathbf{V}}_i^{jj})), \tag{A.21}$$

$$q(\nu_{i,j}^{-1}) \sim G(1, 1 + \overline{\lambda_{i,j}^{-1}}), \tag{A.22}$$

and

$$q(\xi_i^{-1}) \sim G(1, 1 + \overline{\tau_i^{-1}}). \tag{A.23}$$

The terms which are updated in the VB iterations are (A.7), (A.8), and:

$$\overline{\lambda_{i,j}^{-1}} = \frac{1}{\overline{\nu_{i,j}^{-1}} + \overline{\tau_i^{-1}} \frac{\bar{\theta}_{i,j}^2 + \bar{\mathbf{V}}_i^{jj}}{2}}, \tag{A.24}$$

$$\overline{\tau_i^{-1}} = \frac{k_i + 1}{2\overline{\xi_i^{-1}} + [\overline{\lambda_{i,j}^{-1}} \sum_{j=1}^{k_i} (\bar{\theta}_{i,j}^2 + \bar{\mathbf{V}}_i^{jj})]}, \tag{A.25}$$

$$\overline{\nu_{i,j}^{-1}} = 1/(1 + \overline{\lambda_{i,j}^{-1}}), \tag{A.26}$$

$$\overline{\xi_i^{-1}} = 1/(1 + \overline{\tau_i^{-1}}). \tag{A.27}$$

These values can be plugged into the formula for $\mathbf{V}_i$ and used to update $\bar{\mathbf{V}}_i$. The evidence lower bound can be found in Gefang et al. (2022).

A.2.3 VB Estimation of the Parameters in the Stochastic Constraints

The other parameters in the model enter the stochastic constraints. These are the intercepts in each constraint: $c^Q, c^L, c^{r,j}$, and their error variances: σ_j^2 and $\sigma_{j,r}^2$. For the intercepts, we assume relatively non-informative Normal priors centered over 0: $c^Q, c^L, c^{r,j} \sim N(0,1)$. For the error variances, we also use relatively non-informative priors: σ_j^2 and $\sigma_{j,r}^2 \sim IG(1000, 0.01)$. Below in Section B.3 we investigate prior sensitivity in relation to these stochastic constraints by considering an alternative prior: σ_j^2 and $\sigma_{j,r}^2 \sim IG(10, 0.01)$, that reflects the belief that the errors in these stochastic constraints are smaller.

Given VB estimates of the unobserved quarterly quantities, the stochastic constraints are simply regression models. Thus, standard VB methods for regression can be used to estimate the parameters of the stochastic constraints. These are available in many places, including Gefang et al. (2022).

A.3 Estimating the Unobserved Regional Quarterly Variables

In this sub-section, we describe how to estimate the unobserved regional quarterly variables given values for the parameters. In practice, the parameter estimates will be those described in the preceding sub-section, and the VB algorithm proceeds by iterating between results in these two sub-sections. Following Schorfheide and Song (2015), we write the MF-VAR in what they call a compact state space form. We differ from their derivations in two main ways. First, our data observability varies over time. For the first part of the sample we only have annual data for both regional Q and L. Then there is a period where we have quarterly regional L, but annual regional Q. Finally, since 2012 we have quarterly values for both regional variables. We refer to this as the frequency change issue. Second, we have more constraints than Schorfheide and Song (2015) and these are imposed stochastically.

We deal with the frequency change issue using methods discussed in Section 2 of Schorfheide and Song (2015), and the reader is referred to our earlier paper (Koop, McIntyre, Mitchell and Poon, 2020b) for exact details. Starting in 1998, these methods are applied to the block of the model for regional L. Starting in 2012, these methods are applied to the blocks of the model for both regional L and Q. VB methods simply require the use of standard Kalman filtering and smoothing techniques on the state space models, as defined in Section 2 of Koop et al. (2020b).

B Empirical Appendix: Robustness

The forecasting results in the main paper impose the temporal and cross-sectional constraints, (1) and (2), in stochastic form, also allowing for a bias. Here, in Section B.1 and B.2 we present results for two special cases. First, we set $c^{r,j} = 0$ in (1) and $c^j = 0$ in (2), and so do not allow for a bias (non-zero intercept) in either constraint. Second, we impose the additional restriction that $\eta^r_{t,j} = 0$, implying that the temporal constraint, (2), is exact. Third, in Section B.3 we undertake some hyperparameter sensitivity analysis. Fourth, in Section B.4 we consider use of the horseshoe prior. Fifth, in Section B.5, to complement Tables 1 to 6 in the main paper, we present p-values testing equality of forecast accuracy between each of the 5 MF-VAR models and the AR(1) model on the basis of RMSFE or CRPS loss. We follow Diebold and Mariano (2002) and Giacomini and White (2006), and use a t-statistic, assuming asymptotic normality and serially uncorrelated errors (expected for *optimal* one-step-ahead forecasts, nowcasts, and backcasts), and implement a two-sided test of equal forecast accuracy.

B.1 Forecasting Results from the MF-VAR Models with No Intercepts in the Stochastic Aggregation Constraints

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	2.35	3.11	4.00	2.16	1.91	3.18	2.51	2.48	2.12	3.80	2.04	1.81	**2.62**
Nowcast	1.63	1.86	1.96	2.10	1.78	1.86	1.85	2.03	1.51	1.66	1.51	1.03	**1.73**
Backcast	1.66	1.91	1.89	1.85	1.73	1.70	1.69	1.87	1.35	1.71	1.32	1.24	**1.66**
MF-VAR model - (with both aggregation constraints)													
Forecast	4.27	2.75	1.70	3.00	3.63	3.04	1.82	1.63	3.57	3.80	2.55	1.77	**2.80**
Nowcast	2.51	2.14	2.20	1.18	2.01	1.67	0.74	0.56	2.51	2.06	0.83	0.70	**1.59**
Backcast	1.06	1.15	1.19	0.23	0.92	0.93	0.46	0.33	1.18	0.81	0.20	0.22	**0.72**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	4.25	2.82	1.74	3.02	3.68	3.14	1.85	1.71	3.60	3.81	2.58	1.81	**2.84**
Nowcast	2.07	1.63	1.94	1.12	1.63	1.60	0.68	0.57	2.04	1.80	0.82	0.68	**1.38**
Backcast	1.04	1.14	1.28	0.20	0.85	1.02	0.47	0.32	1.21	0.82	0.21	0.21	**0.73**
MF-VAR model - (aggregation constraint only in L)													
Forecast	1.74	1.47	1.60	1.00	1.19	1.39	1.20	1.29	1.20	1.67	0.84	1.17	**1.31**
Nowcast	1.57	1.53	2.04	2.04	0.94	1.49	1.51	1.91	1.37	1.25	1.48	1.13	**1.52**
Backcast	0.92	1.04	1.21	1.18	0.88	1.05	1.08	1.16	1.09	0.78	0.94	0.66	**1.00**
MF-VAR model - (No aggregation constraints)													
Forecast	1.44	1.51	1.59	0.98	1.14	1.37	1.22	1.33	1.00	1.68	0.85	1.22	**1.28**
Nowcast	1.91	1.88	2.46	2.52	1.51	1.99	1.88	2.42	1.84	1.55	1.87	1.38	**1.93**
Backcast	0.94	1.03	1.20	1.16	0.87	1.04	1.07	1.19	1.13	0.78	0.93	0.62	**1.00**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	4.34	2.99	2.04	2.96	3.57	2.92	1.88	1.85	3.57	4.07	2.40	1.85	**2.87**
Nowcast	2.26	2.15	2.31	1.18	1.88	1.56	0.78	0.56	2.35	2.02	0.76	0.69	**1.54**
Backcast	0.98	1.12	1.25	0.21	0.88	0.93	0.45	0.32	1.15	0.78	0.15	0.22	**0.70**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.1: RMSFE (Multiplied by 100) for Productivity Growth: When $c^{r,j} = 0$ in (1) and $c^{j} = 0$ in (2) in the MF-VAR Models

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.44	2.04	2.65	1.31	1.21	2.18	1.66	1.56	1.41	2.36	1.31	1.13	**1.69**
Nowcast	0.76	0.80	0.95	0.78	0.76	0.82	0.79	0.85	0.62	0.77	0.59	0.50	**0.75**
Backcast	0.70	0.71	0.80	0.67	0.71	0.70	0.64	0.72	0.49	0.63	0.43	0.49	**0.64**
MF-VAR model - (with both aggregation constraints)													
Forecast	1.40	1.20	0.93	1.05	1.27	1.25	0.87	0.79	1.13	1.54	0.90	0.78	**1.09**
Nowcast	0.99	0.85	0.80	0.55	0.84	0.77	0.42	0.32	0.88	0.93	0.35	0.36	**0.67**
Backcast	0.38	0.33	0.36	0.13	0.29	0.32	0.20	0.17	0.38	0.30	0.09	0.11	**0.26**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	1.40	1.23	0.95	1.05	1.26	1.27	0.88	0.82	1.15	1.55	0.90	0.78	**1.11**
Nowcast	0.85	0.74	0.75	0.50	0.73	0.75	0.39	0.33	0.75	0.84	0.34	0.35	**0.61**
Backcast	0.38	0.34	0.38	0.12	0.28	0.33	0.21	0.17	0.39	0.30	0.09	0.11	**0.26**
MF-VAR model - (aggregation constraint only in L)													
Forecast	0.98	0.83	0.88	0.58	0.69	0.80	0.69	0.70	0.69	0.97	0.50	0.70	**0.75**
Nowcast	0.74	0.69	0.83	0.65	0.48	0.66	0.65	0.72	0.56	0.63	0.54	0.49	**0.64**
Backcast	0.36	0.32	0.39	0.34	0.30	0.33	0.35	0.37	0.36	0.26	0.26	0.26	**0.33**
MF-VAR model - (No aggregation constraints)													
Forecast	0.81	0.86	0.87	0.58	0.67	0.79	0.70	0.74	0.60	0.97	0.51	0.71	**0.73**
Nowcast	0.78	0.76	0.92	0.75	0.62	0.78	0.73	0.84	0.68	0.70	0.62	0.56	**0.73**
Backcast	0.37	0.32	0.39	0.34	0.30	0.33	0.34	0.38	0.40	0.26	0.26	0.25	**0.33**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.43	1.33	1.11	1.07	1.28	1.19	0.93	0.85	1.14	1.71	0.85	0.77	**1.14**
Nowcast	1.05	0.99	0.99	0.64	0.91	0.81	0.56	0.41	0.95	1.05	0.38	0.41	**0.76**
Backcast	0.43	0.37	0.40	0.15	0.32	0.35	0.23	0.21	0.41	0.32	0.07	0.13	**0.28**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.2: CRPS (Multiplied by 100) for Productivity Growth: When $c^{r,j} = 0$ in (1) and $c^j = 0$ in (2) in the MF-VAR Models

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	3.10	3.03	4.35	2.63	2.59	3.38	2.51	3.93	2.39	3.11	2.25	1.91	**2.93**
Nowcast	1.96	2.23	2.46	2.29	1.93	2.12	2.14	2.42	1.91	1.95	1.79	1.33	**2.04**
Backcast	1.66	1.91	1.89	1.85	1.73	1.70	1.69	1.87	1.35	1.71	1.32	1.24	**1.66**
MF-VAR model - (with both aggregation constraints)													
Forecast	3.24	1.94	2.01	2.11	2.62	2.24	1.41	1.87	2.37	2.82	1.57	1.03	**2.10**
Nowcast	1.86	1.69	1.96	0.50	1.36	1.66	0.86	0.98	1.98	1.35	0.45	0.71	**1.28**
Backcast	1.06	1.15	1.19	0.23	0.92	0.93	0.46	0.33	1.18	0.81	0.20	0.22	**0.72**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	3.21	1.96	2.02	2.10	2.65	2.29	1.41	1.86	2.38	2.83	1.57	1.05	**2.11**
Nowcast	1.51	1.42	1.98	0.42	1.15	1.55	0.90	0.98	1.60	1.15	0.40	0.66	**1.14**
Backcast	1.04	1.14	1.28	0.20	0.85	1.02	0.47	0.32	1.21	0.82	0.21	0.21	**0.73**
MF-VAR model - (aggregation constraint only in L)													
Forecast	2.52	2.41	2.61	2.37	2.07	2.50	2.33	2.79	2.19	1.91	2.07	1.65	**2.29**
Nowcast	1.60	1.79	2.12	1.91	1.53	1.80	1.84	2.11	1.69	1.25	1.60	1.14	**1.70**
Backcast	0.92	1.04	1.21	1.18	0.88	1.05	1.08	1.16	1.09	0.78	0.94	0.66	**1.00**
MF-VAR model - (No aggregation constraints)													
Forecast	2.33	2.43	2.60	2.39	2.07	2.50	2.35	2.81	2.08	1.92	2.09	1.72	**2.28**
Nowcast	1.58	1.77	2.10	1.92	1.54	1.81	1.85	2.16	1.71	1.27	1.62	1.16	**1.71**
Backcast	0.94	1.03	1.20	1.16	0.87	1.04	1.07	1.19	1.13	0.78	0.93	0.62	**1.00**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	3.31	2.16	1.93	2.06	2.59	2.16	1.41	1.85	2.40	2.97	1.47	1.05	**2.11**
Nowcast	1.62	1.63	1.99	0.49	1.26	1.59	0.86	0.95	1.78	1.25	0.38	0.72	**1.21**
Backcast	0.98	1.12	1.25	0.21	0.88	0.93	0.45	0.32	1.15	0.78	0.15	0.22	**0.70**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.3: RMSFE (Multiplied by 100) for Output Growth: When $c^{r,j} = 0$ in (1) and $c^j = 0$ in (2) in the MF-VAR Models

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.61	1.41	2.07	1.40	1.67	1.68	1.36	1.83	1.08	1.37	0.95	0.91	**1.45**
Nowcast	0.85	0.89	1.06	0.92	0.88	0.91	0.88	1.01	0.76	0.81	0.68	0.58	**0.85**
Backcast	0.70	0.71	0.80	0.67	0.71	0.70	0.64	0.72	0.49	0.63	0.43	0.49	**0.64**
MF-VAR model - (with both aggregation constraints)													
Forecast	1.24	0.84	0.91	0.87	1.02	1.01	0.65	0.77	0.92	1.14	0.64	0.50	**0.88**
Nowcast	0.74	0.56	0.80	0.29	0.55	0.59	0.39	0.43	0.62	0.57	0.20	0.33	**0.51**
Backcast	0.38	0.33	0.36	0.13	0.29	0.32	0.20	0.17	0.38	0.30	0.09	0.11	**0.26**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	1.22	0.85	0.91	0.85	1.02	1.01	0.66	0.77	0.92	1.15	0.64	0.51	**0.88**
Nowcast	0.65	0.51	0.80	0.25	0.51	0.56	0.42	0.44	0.54	0.52	0.19	0.31	**0.47**
Backcast	0.38	0.34	0.38	0.12	0.28	0.33	0.21	0.17	0.39	0.30	0.09	0.11	**0.26**
MF-VAR model - (aggregation constraint only in L)													
Forecast	1.31	1.04	1.18	1.02	1.06	1.12	1.01	1.16	1.04	0.93	0.85	0.87	**1.05**
Nowcast	0.79	0.71	0.87	0.74	0.71	0.74	0.73	0.83	0.69	0.57	0.59	0.50	**0.71**
Backcast	0.36	0.32	0.39	0.34	0.30	0.33	0.35	0.37	0.36	0.26	0.26	0.26	**0.33**
MF-VAR model - (No aggregation constraints)													
Forecast	1.14	1.04	1.17	1.01	1.05	1.10	1.02	1.16	0.94	0.92	0.86	0.89	**1.03**
Nowcast	0.75	0.71	0.86	0.75	0.71	0.74	0.73	0.85	0.71	0.57	0.59	0.52	**0.71**
Backcast	0.37	0.32	0.39	0.34	0.30	0.33	0.34	0.38	0.40	0.26	0.26	0.25	**0.33**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.36	0.96	0.89	0.87	1.03	0.97	0.68	0.78	0.94	1.16	0.59	0.49	**0.89**
Nowcast	0.82	0.61	0.88	0.35	0.62	0.65	0.48	0.51	0.64	0.62	0.17	0.36	**0.56**
Backcast	0.43	0.37	0.40	0.15	0.32	0.35	0.23	0.21	0.41	0.32	0.07	0.13	**0.28**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.4: CRPS (Multiplied by 100) for Output Growth: When $c^{r,j} = 0$ in (1) and $c^{j} = 0$ in (2)

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	2.10	3.22	2.39	2.91	2.28	2.98	2.88	2.36	3.77	2.68	2.34	2.51	**2.70**
Nowcast	0.97	1.01	1.10	1.20	0.80	0.93	0.94	1.06	1.07	0.96	1.05	1.09	**1.01**
MF-VAR model - (with both aggregation constraints)													
Forecast	1.92	1.85	1.79	1.91	1.64	1.87	1.74	1.92	2.02	1.68	1.84	1.89	**1.84**
Nowcast	0.94	1.04	1.14	1.31	0.86	0.96	0.94	1.08	1.05	0.94	1.08	1.19	**1.04**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	1.91	1.88	1.80	1.89	1.57	1.82	1.74	1.89	2.02	1.68	1.83	1.87	**1.82**
Nowcast	0.94	1.02	1.06	1.17	0.77	0.95	0.92	1.04	1.04	0.91	1.03	1.07	**0.99**
MF-VAR model - (aggregation constraint only in L)													
Forecast	2.05	1.81	1.84	1.96	1.76	1.91	1.77	1.90	2.05	1.70	1.86	1.95	**1.88**
Nowcast	1.09	1.02	1.15	1.29	0.87	0.97	0.94	1.09	1.05	0.95	1.08	1.21	**1.06**
MF-VAR model - (No aggregation constraints)													
Forecast	2.08	1.85	1.85	1.98	1.75	1.93	1.80	1.91	2.08	1.72	1.87	1.95	**1.90**
Nowcast	1.36	1.24	1.44	1.62	0.92	1.24	1.13	1.34	1.25	1.10	1.29	1.35	**1.27**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.91	1.84	1.78	1.92	1.62	1.86	1.73	1.89	2.01	1.70	1.80	1.88	**1.83**
Nowcast	0.93	1.04	1.12	1.32	0.85	0.96	0.94	1.09	1.05	0.96	1.06	1.17	**1.04**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.5: RMSFE (Multiplied by 100) for Growth in Hours Worked: When $c^{r,j} = 0$ in (1) and $c^j = 0$ in (2) in the MF-VAR Models

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.06	2.10	1.53	1.33	1.19	1.83	1.74	1.29	2.17	1.70	1.49	1.43	**1.57**
Nowcast	0.39	0.41	0.41	0.43	0.33	0.38	0.36	0.38	0.41	0.41	0.40	0.41	**0.39**
MF-VAR model - (with both aggregation constraints)													
Forecast	0.87	0.89	0.80	0.76	0.67	0.84	0.74	0.79	0.92	0.92	0.83	0.84	**0.82**
Nowcast	0.42	0.45	0.45	0.49	0.35	0.39	0.36	0.41	0.42	0.47	0.45	0.50	**0.43**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	0.87	0.92	0.79	0.76	0.63	0.82	0.74	0.78	0.92	0.93	0.83	0.83	**0.82**
Nowcast	0.42	0.43	0.39	0.42	0.28	0.38	0.33	0.36	0.41	0.44	0.41	0.42	**0.39**
MF-VAR model - (aggregation constraint only in L)													
Forecast	0.91	0.88	0.82	0.79	0.71	0.86	0.76	0.79	0.93	0.94	0.84	0.86	**0.84**
Nowcast	0.49	0.45	0.45	0.49	0.34	0.39	0.36	0.42	0.43	0.48	0.46	0.50	**0.44**
MF-VAR model - (No aggregation constraints)													
Forecast	0.92	0.91	0.80	0.79	0.69	0.86	0.77	0.79	0.95	0.94	0.84	0.85	**0.84**
Nowcast	0.60	0.55	0.54	0.58	0.35	0.50	0.44	0.50	0.53	0.55	0.53	0.54	**0.52**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	0.89	0.90	0.80	0.78	0.66	0.85	0.74	0.80	0.94	0.96	0.82	0.84	**0.83**
Nowcast	0.46	0.51	0.49	0.55	0.38	0.43	0.40	0.47	0.48	0.53	0.50	0.54	**0.48**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.6: CRPS (Multiplied by 100) for Growth in Hours Worked: When $c^{r,j} = 0$ in (1) and $c^j = 0$ in (2) in the MF-VAR Models

B.2 Forecasting Results from the MF-VAR Models with No Intercepts and when Only the Cross-Sectional Constraint, (2), is Stochastic

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	2.35	3.11	4.00	2.16	1.91	3.18	2.51	2.48	2.12	3.80	2.04	1.81	**2.62**
Nowcast	1.63	1.86	1.96	2.10	1.78	1.86	1.85	2.03	1.51	1.66	1.51	1.03	**1.73**
Backcast	1.66	1.91	1.89	1.85	1.73	1.70	1.69	1.87	1.35	1.71	1.32	1.24	**1.66**
MF-VAR model - (with both aggregation constraints)													
Forecast	4.31	2.76	1.69	3.01	3.61	3.09	1.83	1.67	3.59	3.78	2.59	1.78	**2.81**
Nowcast	2.19	1.94	2.10	1.28	1.57	1.36	0.87	0.63	2.32	1.94	0.86	0.70	**1.48**
Backcast	0.93	1.04	1.16	0.20	0.84	0.88	0.48	0.32	1.07	0.75	0.18	0.21	**0.67**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	4.55	2.96	1.75	3.05	3.69	3.15	1.89	1.76	3.79	3.97	2.61	1.79	**2.91**
Nowcast	2.13	1.65	1.94	1.11	1.63	1.60	0.68	0.58	2.11	1.84	0.81	0.68	**1.40**
Backcast	1.05	1.15	1.30	0.21	0.85	1.04	0.47	0.33	1.23	0.82	0.20	0.22	**0.74**
MF-VAR model - (aggregation constraint only in L)													
Forecast	2.38	1.50	1.63	1.32	1.35	1.50	1.27	1.37	1.76	1.76	0.89	1.61	**1.53**
Nowcast	1.76	1.62	2.14	2.28	1.39	1.82	1.73	2.12	1.69	1.40	1.70	1.33	**1.75**
Backcast	0.91	1.00	1.16	1.18	0.89	1.06	1.08	1.14	1.09	0.78	0.93	0.67	**0.99**
MF-VAR model - (No aggregation constraints)													
Forecast	1.93	1.51	1.61	1.11	1.23	1.49	1.24	1.36	1.42	1.68	0.89	1.31	**1.40**
Nowcast	1.96	1.87	2.47	2.49	1.53	1.99	1.87	2.39	1.87	1.54	1.86	1.40	**1.94**
Backcast	0.93	1.04	1.21	1.17	0.87	1.05	1.07	1.18	1.12	0.79	0.93	0.63	**1.00**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	4.21	3.06	2.12	2.65	3.30	2.54	1.76	1.73	3.55	4.21	2.22	1.78	**2.76**
Nowcast	2.26	2.32	2.45	1.25	1.43	1.19	0.94	0.60	2.56	2.18	0.82	0.72	**1.56**
Backcast	0.98	1.11	1.25	0.21	0.86	0.95	0.46	0.33	1.14	0.79	0.15	0.22	**0.71**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.7: RMSFE (Multiplied by 100) for Productivity Growth: When $c^{r,j} = 0$ in (1), $c^{j} = 0$ in (2), and $\sigma^2_{j,r} = 0$ in the MF-VAR Models

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.44	2.04	2.65	1.31	1.21	2.18	1.66	1.56	1.41	2.36	1.31	1.13	**1.69**
Nowcast	0.76	0.80	0.95	0.78	0.76	0.82	0.79	0.85	0.62	0.77	0.59	0.50	**0.75**
Backcast	0.70	0.71	0.80	0.67	0.71	0.70	0.64	0.72	0.49	0.63	0.43	0.49	**0.64**
MF-VAR model - (with both aggregation constraints)													
Forecast	1.40	1.20	0.94	1.06	1.27	1.25	0.88	0.80	1.14	1.54	0.93	0.77	**1.10**
Nowcast	0.88	0.80	0.78	0.60	0.72	0.68	0.45	0.35	0.83	0.89	0.37	0.36	**0.64**
Backcast	0.35	0.31	0.35	0.12	0.28	0.31	0.21	0.17	0.36	0.29	0.08	0.11	**0.24**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	1.46	1.32	1.00	1.07	1.27	1.30	0.92	0.86	1.17	1.66	0.91	0.74	**1.14**
Nowcast	0.99	0.85	0.88	0.58	0.83	0.86	0.50	0.43	0.86	0.99	0.40	0.41	**0.71**
Backcast	0.42	0.36	0.42	0.15	0.31	0.38	0.23	0.21	0.44	0.34	0.08	0.12	**0.29**
MF-VAR model - (aggregation constraint only in L)													
Forecast	1.24	0.91	0.97	0.72	0.81	0.92	0.77	0.78	0.93	1.09	0.52	0.85	**0.88**
Nowcast	1.03	0.82	1.01	0.84	0.79	0.90	0.81	0.89	0.83	0.81	0.68	0.70	**0.84**
Backcast	0.42	0.34	0.43	0.39	0.35	0.38	0.38	0.42	0.41	0.30	0.28	0.29	**0.37**
MF-VAR model - (No aggregation constraints)													
Forecast	1.08	0.92	0.95	0.64	0.74	0.92	0.75	0.77	0.80	1.04	0.51	0.74	**0.82**
Nowcast	1.03	0.87	1.07	0.86	0.78	0.94	0.84	0.95	0.82	0.83	0.71	0.68	**0.87**
Backcast	0.42	0.35	0.44	0.39	0.35	0.38	0.38	0.43	0.44	0.30	0.28	0.29	**0.37**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.39	1.34	1.11	1.00	1.24	1.08	0.89	0.82	1.12	1.72	0.80	0.74	**1.10**
Nowcast	1.05	1.02	1.03	0.67	0.80	0.68	0.58	0.43	0.98	1.08	0.42	0.43	**0.77**
Backcast	0.43	0.37	0.40	0.15	0.32	0.37	0.23	0.21	0.42	0.33	0.07	0.12	**0.28**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.8: CRPS (Multiplied by 100) for Productivity Growth: When $c^{r,j} = 0$ in (1), $c^{j} = 0$ in (2), and $\sigma^2_{j,r} = 0$ in the MF-VAR Models

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	3.10	3.03	4.35	2.63	2.59	3.38	2.51	3.93	2.39	3.11	2.25	1.91	**2.93**
Nowcast	1.96	2.23	2.46	2.29	1.93	2.12	2.14	2.42	1.91	1.95	1.79	1.33	**2.04**
Backcast	1.66	1.91	1.89	1.85	1.73	1.70	1.69	1.87	1.35	1.71	1.32	1.24	**1.66**
MF-VAR model - (with both aggregation constraints)													
Forecast	3.27	1.96	2.02	2.15	2.67	2.32	1.41	1.86	2.40	2.81	1.60	1.01	**2.12**
Nowcast	1.51	1.43	1.86	0.45	1.17	1.53	0.88	0.96	1.64	1.17	0.38	0.70	**1.14**
Backcast	0.93	1.04	1.16	0.20	0.84	0.88	0.48	0.32	1.07	0.75	0.18	0.21	**0.67**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	3.48	2.05	1.99	2.16	2.70	2.33	1.42	1.88	2.57	2.95	1.58	1.03	**2.18**
Nowcast	1.57	1.45	1.99	0.45	1.17	1.59	0.89	0.98	1.67	1.17	0.40	0.64	**1.17**
Backcast	1.05	1.15	1.30	0.21	0.85	1.04	0.47	0.33	1.23	0.82	0.20	0.22	**0.74**
MF-VAR model - (aggregation constraint only in L)													
Forecast	2.97	2.42	2.62	2.52	2.14	2.57	2.34	2.79	2.56	2.01	2.08	1.98	**2.42**
Nowcast	1.68	1.74	2.05	1.94	1.55	1.82	1.84	2.08	1.78	1.29	1.60	1.20	**1.71**
Backcast	0.91	1.00	1.16	1.18	0.89	1.06	1.08	1.14	1.09	0.78	0.93	0.67	**0.99**
MF-VAR model - (No aggregation constraints)													
Forecast	2.65	2.42	2.63	2.45	2.13	2.57	2.35	2.82	2.35	1.94	2.10	1.75	**2.35**
Nowcast	1.63	1.79	2.12	1.93	1.55	1.84	1.84	2.14	1.77	1.29	1.62	1.17	**1.72**
Backcast	0.93	1.04	1.21	1.17	0.87	1.05	1.07	1.18	1.12	0.79	0.93	0.63	**1.00**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	3.20	2.23	1.93	1.86	2.43	1.96	1.39	1.83	2.41	3.10	1.36	1.02	**2.06**
Nowcast	1.62	1.63	1.94	0.51	1.23	1.64	0.87	0.96	1.78	1.32	0.39	0.71	**1.22**
Backcast	0.98	1.11	1.25	0.21	0.86	0.95	0.46	0.33	1.14	0.79	0.15	0.22	**0.71**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.9: RMSFE (Multiplied by 100) for Output Growth: When $c^{r,j} = 0$ in (1), $c^j = 0$ in (2), and $\sigma^2_{j,r} = 0$ in the MF-VAR Models

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.61	1.41	2.07	1.40	1.67	1.68	1.36	1.83	1.08	1.37	0.95	0.91	**1.45**
Nowcast	0.85	0.89	1.06	0.92	0.88	0.91	0.88	1.01	0.76	0.81	0.68	0.58	**0.85**
Backcast	0.70	0.71	0.80	0.67	0.71	0.70	0.64	0.72	0.49	0.63	0.43	0.49	**0.64**
MF-VAR model - (with both aggregation constraints)													
Forecast	1.23	0.85	0.91	0.89	1.02	1.04	0.66	0.76	0.93	1.14	0.65	0.49	**0.88**
Nowcast	0.65	0.51	0.78	0.27	0.51	0.56	0.41	0.42	0.55	0.52	0.18	0.32	**0.47**
Backcast	0.35	0.31	0.35	0.12	0.28	0.31	0.21	0.17	0.36	0.29	0.08	0.11	**0.24**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	1.29	0.89	0.93	0.89	1.04	1.05	0.67	0.78	0.95	1.17	0.60	0.47	**0.89**
Nowcast	0.78	0.59	0.91	0.33	0.58	0.67	0.47	0.50	0.63	0.61	0.17	0.32	**0.55**
Backcast	0.42	0.36	0.42	0.15	0.31	0.38	0.23	0.21	0.44	0.34	0.08	0.12	**0.29**
MF-VAR model - (aggregation constraint only in L)													
Forecast	1.59	1.06	1.24	1.18	1.22	1.23	1.04	1.18	1.25	1.05	0.86	1.00	**1.16**
Nowcast	1.01	0.77	0.96	0.90	0.88	0.88	0.81	0.91	0.87	0.70	0.65	0.63	**0.83**
Backcast	0.42	0.34	0.43	0.39	0.35	0.38	0.38	0.42	0.41	0.30	0.28	0.29	**0.37**
MF-VAR model - (No aggregation constraints)													
Forecast	1.43	1.06	1.24	1.13	1.20	1.22	1.04	1.18	1.14	0.98	0.87	0.89	**1.11**
Nowcast	0.94	0.79	0.98	0.88	0.87	0.88	0.81	0.94	0.83	0.68	0.65	0.61	**0.82**
Backcast	0.42	0.35	0.44	0.39	0.35	0.38	0.38	0.43	0.44	0.30	0.28	0.29	**0.37**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.30	0.97	0.90	0.82	0.99	0.91	0.66	0.77	0.92	1.16	0.54	0.47	**0.87**
Nowcast	0.81	0.62	0.86	0.35	0.61	0.67	0.47	0.51	0.65	0.64	0.17	0.35	**0.56**
Backcast	0.43	0.37	0.40	0.15	0.32	0.37	0.23	0.21	0.42	0.33	0.07	0.12	**0.28**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.10: CRPS (Multiplied by 100) for Output Growth: When $c^{r,j} = 0$ in (1), $c^j = 0$ in (2), and $\sigma^2_{j,r} = 0$ in the MF-VAR Models

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	2.10	3.22	2.39	2.91	2.28	2.98	2.88	2.36	3.77	2.68	2.34	2.51	**2.70**
Nowcast	0.97	1.01	1.10	1.20	0.80	0.93	0.94	1.06	1.07	0.96	1.05	1.09	**1.01**
MF-VAR model - (with both aggregation constraints)													
Forecast	1.92	1.84	1.77	1.88	1.57	1.82	1.71	1.89	2.00	1.67	1.82	1.87	**1.81**
Nowcast	0.93	1.05	1.14	1.38	0.74	1.02	0.96	1.10	1.07	0.95	1.14	1.22	**1.06**
MF-VAR model - (aggregation constraint only in Q)													
Forecast	1.93	1.88	1.78	1.88	1.56	1.80	1.72	1.88	1.99	1.70	1.83	1.86	**1.82**
Nowcast	0.95	1.02	1.06	1.16	0.76	0.93	0.91	1.03	1.04	0.92	1.04	1.07	**0.99**
MF-VAR model - (aggregation constraint only in L)													
Forecast	2.05	1.78	1.80	1.91	1.65	1.81	1.71	1.84	1.98	1.69	1.81	1.93	**1.83**
Nowcast	1.14	1.07	1.24	1.39	0.77	1.04	0.99	1.17	1.09	1.00	1.16	1.27	**1.11**
MF-VAR model - (No aggregation constraints)													
Forecast	2.08	1.86	1.85	1.98	1.75	1.91	1.79	1.90	2.06	1.74	1.87	1.95	**1.90**
Nowcast	1.36	1.22	1.43	1.59	0.91	1.21	1.12	1.33	1.24	1.10	1.29	1.35	**1.26**
MF-VAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.92	1.80	1.74	1.87	1.52	1.75	1.67	1.84	1.95	1.71	1.76	1.85	**1.78**
Nowcast	0.92	1.09	1.18	1.45	0.73	1.07	1.00	1.15	1.10	0.99	1.15	1.23	**1.09**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.11: RMSFE (Multiplied by 100) for Growth in Hours Worked: When $c^{r,j} = 0$ in (1), $c^j = 0$ in (2), and $\sigma^2_{j,r} = 0$ in the MF-VAR Models

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
							AR(1) model						
Forecast	1.06	2.10	1.53	1.33	1.19	1.83	1.74	1.29	2.17	1.70	1.49	1.43	**1.57**
Nowcast	0.39	0.41	0.41	0.43	0.33	0.38	0.36	0.38	0.41	0.41	0.40	0.41	**0.39**
							MF-VAR model - (with both aggregation constraints)						
Forecast	0.87	0.89	0.79	0.76	0.65	0.81	0.73	0.79	0.91	0.92	0.82	0.84	**0.82**
Nowcast	0.41	0.46	0.46	0.52	0.27	0.43	0.38	0.43	0.44	0.47	0.48	0.52	**0.44**
							MF-VAR model - (aggregation constraint only in Q)						
Forecast	0.90	0.94	0.79	0.76	0.62	0.82	0.74	0.78	0.92	0.97	0.83	0.83	**0.83**
Nowcast	0.48	0.49	0.44	0.46	0.32	0.41	0.36	0.41	0.45	0.51	0.46	0.48	**0.44**
							MF-VAR model - (aggregation constraint only in L)						
Forecast	0.95	0.91	0.82	0.78	0.70	0.83	0.74	0.79	0.93	0.97	0.84	0.87	**0.84**
Nowcast	0.58	0.55	0.55	0.57	0.34	0.47	0.44	0.52	0.51	0.58	0.55	0.59	**0.52**
							MF-VAR model - (No aggregation constraints)						
Forecast	0.96	0.95	0.82	0.80	0.69	0.87	0.77	0.80	0.96	1.00	0.86	0.87	**0.86**
Nowcast	0.65	0.60	0.60	0.62	0.40	0.54	0.48	0.56	0.57	0.62	0.59	0.61	**0.57**
							MF-VAR model - (with both aggregation constraints but no exogenous predictors)						
Forecast	0.90	0.90	0.79	0.76	0.64	0.79	0.71	0.79	0.90	0.95	0.80	0.83	**0.81**
Nowcast	0.47	0.55	0.52	0.59	0.33	0.48	0.43	0.51	0.50	0.56	0.54	0.56	**0.50**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.12: CRPS (Multiplied by 100) for Growth in Hours Worked: When $c^{r,j} = 0$ in (1), $c^j = 0$ in (2), and $\sigma^2_{j,r} = 0$ in the MF-VAR Models

B.3 Hyperparameter Sensitivity Analysis

So far we have set relatively non-informative priors for the error variances, $\eta^r_{t,j} \sim N(0, \sigma^2_{j,r})$ and $\eta_{t,j} \sim N(0, \sigma^2_j)$. Here we consider an alternative that reflects a belief that σ^2_j and $\sigma^2_{j,r}$ are smaller. We present results comparing against the AR the accuracy of our MF-VAR model (with both aggregation constraints imposed in stochastic form with intercepts, as specified in (1) and (2)) when σ^2_j and $\sigma^2_{j,r} \sim IG(10, 0.01)$.

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	2.35	3.11	4.00	2.16	1.91	3.18	2.51	2.48	2.12	3.80	2.04	1.81	**2.62**
Nowcast	1.63	1.86	1.96	2.10	1.78	1.86	1.85	2.03	1.51	1.66	1.51	1.03	**1.73**
Backcast	1.66	1.91	1.89	1.85	1.73	1.70	1.69	1.87	1.35	1.71	1.32	1.24	**1.66**
MF-VAR model - (with both aggregation constraints)													
Forecast	4.31	2.80	1.90	3.32	4.08	3.72	2.04	1.77	3.39	3.76	2.74	2.05	**2.99**
Nowcast	2.15	1.53	1.68	1.26	2.50	2.30	0.73	0.76	1.71	1.77	0.98	0.84	**1.52**
Backcast	1.02	1.14	1.08	0.20	0.88	0.86	0.45	0.32	1.15	0.82	0.18	0.24	**0.69**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.13: RMSFE (Multiplied by 100) for Productivity Growth: When the Constraints are Imposed in Stochastic Form, as in (1) and (2), with a Tighter Prior: σ^2_j and $\sigma^2_{j,r} \sim IG(10, 0.01)$

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
						AR(1) model							
Forecast	1.44	2.04	2.65	1.31	1.21	2.18	1.66	1.56	1.41	2.36	1.31	1.13	**1.69**
Nowcast	0.76	0.80	0.95	0.78	0.76	0.82	0.79	0.85	0.62	0.77	0.59	0.50	**0.75**
Backcast	0.70	0.71	0.80	0.67	0.71	0.70	0.64	0.72	0.49	0.63	0.43	0.49	**0.64**
						MF-VAR model - (with both aggregation constraints)							
Forecast	1.49	1.32	1.06	1.16	1.40	1.53	0.95	0.82	1.19	1.69	0.97	0.86	**1.20**
Nowcast	0.95	0.83	0.76	0.53	1.00	1.04	0.42	0.40	0.75	0.94	0.44	0.40	**0.71**
Backcast	0.37	0.34	0.34	0.11	0.28	0.29	0.20	0.17	0.38	0.31	0.08	0.13	**0.25**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.14: CRPS (Multiplied by 100) for Productivity Growth: When the Constraints are Imposed in Stochastic Form, as in (1) and (2), with a Tighter Prior: σ_j^2 and $\sigma_{j,r}^2 \sim IG(10, 0.01)$

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
						AR(1) model							
Forecast	3.10	3.03	4.35	2.63	2.59	3.38	2.51	3.93	2.39	3.11	2.25	1.91	**2.93**
Nowcast	1.96	2.23	2.46	2.29	1.93	2.12	2.14	2.42	1.91	1.95	1.79	1.33	**2.04**
Backcast	1.66	1.91	1.89	1.85	1.73	1.70	1.69	1.87	1.35	1.71	1.32	1.24	**1.66**
						MF-VAR model - (with both aggregation constraints)							
Forecast	3.07	1.93	2.05	2.33	2.86	2.47	1.44	1.87	2.19	2.79	1.63	1.14	**2.15**
Nowcast	1.51	1.44	1.64	0.46	1.27	1.46	0.85	0.94	1.58	1.20	0.38	0.77	**1.12**
Backcast	1.02	1.14	1.08	0.20	0.88	0.86	0.45	0.32	1.15	0.82	0.18	0.24	**0.69**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.15: RMSFE (Multiplied by 100) for Output Growth: When the Constraints are Imposed in Stochastic Form, as in (1) and (2), with a Tighter Prior: σ_j^2 and $\sigma_{j,r}^2 \sim IG(10, 0.01)$

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
						AR(1) model							
Forecast	1.61	1.41	2.07	1.40	1.67	1.68	1.36	1.83	1.08	1.37	0.95	0.91	**1.45**
Nowcast	0.85	0.89	1.06	0.92	0.88	0.91	0.88	1.01	0.76	0.81	0.68	0.58	**0.85**
Backcast	0.70	0.71	0.80	0.67	0.71	0.70	0.64	0.72	0.49	0.63	0.43	0.49	**0.64**
						MF-VAR model - (with both aggregation constraints)							
Forecast	1.22	0.85	0.95	0.96	1.11	1.07	0.68	0.77	0.86	1.16	0.63	0.58	**0.90**
Nowcast	0.67	0.53	0.74	0.27	0.57	0.53	0.40	0.43	0.51	0.54	0.18	0.37	**0.48**
Backcast	0.37	0.34	0.34	0.11	0.28	0.29	0.20	0.17	0.38	0.31	0.08	0.13	**0.25**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.16: CRPS (Multiplied by 100) for Output Growth: When the Constraints are Imposed in Stochastic Form, as in (1) and (2), with a Tighter Prior: σ_j^2 and $\sigma_{j,r}^2 \sim IG(10, 0.01)$

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
						AR(1) model							
Forecast	2.10	3.22	2.39	2.91	2.28	2.98	2.88	2.36	3.77	2.68	2.34	2.51	**2.70**
Nowcast	0.97	1.01	1.10	1.20	0.80	0.93	0.94	1.06	1.07	0.96	1.05	1.09	**1.01**
						MF-VAR model - (with both aggregation constraints)							
Forecast	2.05	1.93	1.89	2.06	1.73	2.14	1.89	2.05	2.29	1.79	1.96	2.05	**1.99**
Nowcast	1.11	1.13	1.11	1.22	1.35	1.29	1.05	1.23	1.25	1.04	1.08	1.15	**1.17**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.17: RMSFE (Multiplied by 100) for Growth in Hours Worked: When the Constraints are Imposed in Stochastic Form, as in (1) and (2), with a Tighter Prior: σ_j^2 and $\sigma_{j,r}^2 \sim IG(10, 0.01)$

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.06	2.10	1.53	1.33	1.19	1.83	1.74	1.29	2.17	1.70	1.49	1.43	**1.57**
Nowcast	0.39	0.41	0.41	0.43	0.33	0.38	0.36	0.38	0.41	0.41	0.40	0.41	**0.39**
MF-VAR model - (with both aggregation constraints)													
Forecast	0.92	1.03	0.91	0.94	0.73	1.00	0.85	0.89	1.14	1.07	0.93	0.94	**0.95**
Nowcast	0.54	0.59	0.49	0.47	0.53	0.58	0.44	0.52	0.65	0.63	0.47	0.47	**0.53**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.18: CRPS (Multiplied by 100) for Growth in Hours Worked: When the Constraints are Imposed in Stochastic Form, as in (1) and (2), with a Tighter Prior: σ_j^2 and $\sigma_{j,r}^2 \sim IG(10, 0.01)$

B.4 Horseshoe: Prior Sensitivity Analysis

The results in the body of the paper used the adaptive Lasso prior which has desirable theoretical properties and requires minimal subjective input from the researcher. To convince the reader that our results are robust to prior choice, in this appendix we present forecasting, nowcasting, and backcasting results in the same format as in the paper, but using the Horseshoe prior of Carvalho et al. (2010). This does not require the selection of any prior hyperparameters. It can be seen that the adaptive Lasso and Horseshoe prior results are pretty similar to one another.

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	2.35	3.11	4.00	2.16	1.91	3.18	2.51	2.48	2.12	3.80	2.04	1.81	**2.62**
Nowcast	1.63	1.86	1.96	2.10	1.78	1.86	1.85	2.03	1.51	1.66	1.51	1.03	**1.73**
Backcast	1.66	1.91	1.89	1.85	1.73	1.70	1.69	1.87	1.35	1.71	1.32	1.24	**1.66**
VB MFVAR model - (with both aggregation constraints)													
Forecast	2.64	1.83	2.36	3.15	4.47	3.74	1.70	2.30	2.60	2.04	2.32	1.94	**2.59**
Nowcast	1.12	1.51	1.88	1.65	1.65	1.74	1.39	0.95	1.17	1.04	1.23	1.22	**1.38**
Backcast	0.38	0.75	0.96	0.17	0.55	0.82	0.65	0.25	0.55	0.42	0.19	0.45	**0.51**
VB MFVAR model - (aggregation constraint only in Q)													
Forecast	2.78	1.74	2.34	3.36	4.71	3.98	1.73	2.35	2.80	2.39	2.50	2.03	**2.73**
Nowcast	1.13	1.21	1.90	1.65	1.81	1.41	0.99	1.02	1.31	1.10	1.12	0.97	**1.30**
Backcast	0.37	0.70	0.98	0.19	0.69	0.50	0.59	0.31	0.53	0.30	0.17	0.40	**0.48**
VB MFVAR model - (aggregation constraint only in L)													
Forecast	1.38	1.60	2.77	1.58	1.59	1.86	1.42	1.82	1.38	1.87	1.42	1.37	**1.67**
Nowcast	1.42	1.53	2.16	2.47	1.72	2.06	1.79	2.19	1.72	1.47	1.94	1.31	**1.81**
Backcast	0.88	0.94	1.09	1.23	0.96	1.11	1.09	1.15	1.06	0.78	1.02	0.69	**1.00**
VB MFVAR model - (No aggregation constraints)													
Forecast	1.32	1.51	2.64	1.39	1.39	1.83	1.35	1.51	1.17	1.92	1.42	1.41	**1.57**
Nowcast	1.39	1.50	2.09	2.38	1.67	2.01	1.73	2.12	1.65	1.47	1.90	1.28	**1.77**
Backcast	0.89	0.93	1.09	1.23	0.95	1.12	1.10	1.14	1.07	0.79	1.02	0.68	**1.00**
VB MFVAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	2.93	2.33	2.16	3.40	4.42	3.73	1.82	2.57	2.79	2.58	2.96	2.04	**2.81**
Nowcast	1.22	1.53	1.80	1.81	1.56	1.35	1.27	0.78	1.20	1.28	1.26	1.10	**1.35**
Backcast	0.38	0.74	0.85	0.18	0.52	0.74	0.63	0.25	0.62	0.36	0.11	0.39	**0.48**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.19: RMSFE (Multiplied by 100) for Productivity Growth: Horseshoe Prior

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.44	2.04	2.65	1.31	1.21	2.18	1.66	1.56	1.41	2.36	1.31	1.13	**1.69**
Nowcast	0.76	0.80	0.95	0.78	0.76	0.82	0.79	0.85	0.62	0.77	0.59	0.50	**0.75**
Backcast	0.70	0.71	0.80	0.67	0.71	0.70	0.64	0.72	0.49	0.63	0.43	0.49	**0.64**
VB MFVAR model - (with both aggregation constraints)													
Forecast	1.12	1.13	1.46	1.11	1.48	1.38	0.90	0.93	1.04	1.18	0.88	0.87	**1.12**
Nowcast	0.66	0.77	1.13	0.77	0.73	0.86	0.62	0.49	0.61	0.63	0.57	0.58	**0.70**
Backcast	0.20	0.32	0.51	0.12	0.24	0.30	0.27	0.16	0.27	0.21	0.10	0.19	**0.24**
VB MFVAR model - (aggregation constraint only in Q)													
Forecast	1.24	1.13	1.54	1.16	1.53	1.47	0.91	0.96	1.10	1.33	0.94	0.95	**1.19**
Nowcast	0.76	0.80	1.33	0.82	0.87	0.76	0.59	0.60	0.79	0.78	0.64	0.62	**0.78**
Backcast	0.23	0.36	0.59	0.14	0.30	0.25	0.28	0.23	0.33	0.17	0.12	0.22	**0.27**
VB MFVAR model - (aggregation constraint only in L)													
Forecast	1.44	1.30	1.00	1.07	1.27	1.30	0.92	0.86	1.14	1.67	0.91	0.73	**1.13**
Nowcast	0.97	0.85	0.88	0.58	0.84	0.86	0.50	0.42	0.84	0.98	0.39	0.40	**0.71**
Backcast	0.42	0.36	0.42	0.15	0.31	0.38	0.23	0.21	0.43	0.34	0.08	0.12	**0.29**
VB MFVAR model - (No aggregation constraints)													
Forecast	0.79	0.92	1.83	0.79	0.85	1.13	0.82	0.88	0.67	1.27	0.82	0.78	**0.96**
Nowcast	0.88	0.80	1.49	0.98	0.93	1.07	0.87	0.97	0.87	1.01	0.94	0.75	**0.96**
Backcast	0.40	0.36	0.63	0.46	0.42	0.46	0.40	0.45	0.44	0.35	0.41	0.34	**0.43**
VB MFVAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.19	1.50	1.33	1.21	1.50	1.35	0.97	0.99	1.07	1.33	0.98	0.88	**1.19**
Nowcast	0.75	1.05	1.14	0.89	0.79	0.78	0.66	0.47	0.75	0.82	0.61	0.59	**0.78**
Backcast	0.23	0.43	0.47	0.14	0.26	0.34	0.28	0.19	0.37	0.19	0.08	0.20	**0.27**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.20: Average CRPS (Multiplied by 100) for Productivity Growth: Horseshoe Prior

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
						AR(1) model							
Forecast	3.10	3.03	4.35	2.63	2.59	3.38	2.51	3.93	2.39	3.11	2.25	1.91	**2.93**
Nowcast	1.96	2.23	2.46	2.29	1.93	2.12	2.14	2.42	1.91	1.95	1.79	1.33	**2.04**
Backcast	1.66	1.91	1.89	1.85	1.73	1.70	1.69	1.87	1.35	1.71	1.32	1.24	**1.66**
						VB MFVAR model - (with both aggregation constraints)							
Forecast	1.83	1.94	2.69	2.19	3.48	2.90	1.58	1.91	1.70	1.71	1.48	1.12	**2.04**
Nowcast	0.74	1.36	1.97	0.39	1.11	1.53	1.14	0.64	0.96	1.09	0.45	0.62	**1.00**
Backcast	0.38	0.75	0.96	0.17	0.55	0.82	0.65	0.25	0.55	0.42	0.19	0.45	**0.51**
						VB MFVAR model - (aggregation constraint only in Q)							
Forecast	1.90	2.10	3.04	2.27	3.54	2.95	1.54	1.91	1.88	1.73	1.55	1.17	**2.13**
Nowcast	0.71	1.41	2.21	0.43	1.26	0.86	1.00	0.63	0.91	0.75	0.43	0.68	**0.94**
Backcast	0.37	0.70	0.98	0.19	0.69	0.50	0.59	0.31	0.53	0.30	0.17	0.40	**0.48**
						VB MFVAR model - (aggregation constraint only in L)							
Forecast	2.56	2.58	3.48	2.97	2.52	2.94	2.59	3.11	2.60	2.27	2.17	2.02	**2.65**
Nowcast	1.75	1.74	2.36	2.31	1.90	2.15	1.94	2.26	1.99	1.45	1.80	1.40	**1.92**
Backcast	0.88	0.94	1.09	1.23	0.96	1.11	1.09	1.15	1.06	0.78	1.02	0.69	**1.00**
						VB MFVAR model - (No aggregation constraints)							
Forecast	2.42	2.53	3.45	2.86	2.44	2.91	2.57	2.93	2.34	2.20	2.56	1.95	**2.60**
Nowcast	1.67	1.71	2.36	2.23	1.84	2.11	1.95	2.18	1.86	1.42	1.96	1.34	**1.89**
Backcast	0.89	0.93	1.09	1.23	0.95	1.12	1.10	1.14	1.07	0.79	1.02	0.68	**1.00**
						VB MFVAR model - (with both aggregation constraints but no exogenous predictors)							
Forecast	1.94	2.48	2.57	2.31	3.44	2.91	1.57	2.04	1.87	1.61	1.90	1.17	**2.15**
Nowcast	0.70	1.53	1.83	0.45	1.03	1.23	1.03	0.69	1.03	0.77	0.22	0.63	**0.93**
Backcast	0.38	0.74	0.85	0.18	0.52	0.74	0.63	0.25	0.62	0.36	0.11	0.39	**0.48**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.21: RMSFE (Multiplied by 100) for Output Growth: Horseshoe Prior

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.61	1.41	2.07	1.40	1.67	1.68	1.36	1.83	1.08	1.37	0.95	0.91	**1.45**
Nowcast	0.85	0.89	1.06	0.92	0.88	0.91	0.88	1.01	0.76	0.81	0.68	0.58	**0.85**
Backcast	0.70	0.71	0.80	0.67	0.71	0.70	0.64	0.72	0.49	0.63	0.43	0.49	**0.64**
VB MFVAR model - (with both aggregation constraints)													
Forecast	0.89	1.01	1.67	0.89	1.24	1.20	0.73	0.85	0.83	0.81	0.68	0.53	**0.94**
Nowcast	0.45	0.72	1.17	0.26	0.56	0.65	0.56	0.39	0.49	0.56	0.27	0.31	**0.53**
Backcast	0.20	0.32	0.51	0.12	0.24	0.30	0.27	0.16	0.27	0.21	0.10	0.19	**0.24**
VB MFVAR model - (aggregation constraint only in Q)													
Forecast	1.03	1.16	1.92	0.91	1.27	1.20	0.71	0.87	1.01	0.88	0.72	0.58	**1.02**
Nowcast	0.56	0.85	1.33	0.32	0.68	0.56	0.56	0.46	0.62	0.50	0.33	0.41	**0.60**
Backcast	0.23	0.36	0.59	0.14	0.30	0.25	0.28	0.23	0.33	0.17	0.12	0.22	**0.27**
VB MFVAR model - (aggregation constraint only in L)													
Forecast	1.31	1.23	2.22	1.38	1.35	1.33	1.17	1.42	1.28	1.21	1.05	1.06	**1.33**
Nowcast	0.97	0.86	1.49	1.08	1.04	1.02	0.90	1.09	1.01	0.83	0.89	0.76	**0.99**
Backcast	0.41	0.38	0.64	0.45	0.42	0.43	0.41	0.47	0.47	0.34	0.43	0.35	**0.43**
VB MFVAR model - (No aggregation constraints)													
Forecast	1.26	1.16	2.21	1.40	1.36	1.39	1.13	1.33	1.18	1.22	1.14	1.03	**1.32**
Nowcast	0.95	0.81	1.49	1.09	1.06	1.06	0.89	1.04	0.93	0.86	0.91	0.73	**0.98**
Backcast	0.40	0.36	0.63	0.46	0.42	0.46	0.40	0.45	0.44	0.35	0.41	0.34	**0.43**
VB MFVAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	1.00	1.54	1.60	0.96	1.23	1.22	0.75	0.93	0.99	0.79	0.72	0.59	**1.03**
Nowcast	0.50	1.04	1.13	0.33	0.61	0.68	0.60	0.49	0.69	0.49	0.17	0.36	**0.59**
Backcast	0.23	0.43	0.47	0.14	0.26	0.34	0.28	0.19	0.37	0.19	0.08	0.20	**0.27**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.22: Average CRPS (Multiplied by 100) for Output Growth: Horseshoe Prior

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	2.10	3.22	2.39	2.91	2.28	2.98	2.88	2.36	3.77	2.68	2.34	2.51	**2.70**
Nowcast	0.97	1.01	1.10	1.20	0.80	0.93	0.94	1.06	1.07	0.96	1.05	1.09	**1.01**
VB MFVAR model - (with both aggregation constraints)													
Forecast	2.01	1.78	1.84	1.94	1.56	1.84	1.75	1.82	1.95	1.70	1.85	1.94	**1.83**
Nowcast	0.97	1.11	1.17	1.49	0.83	0.92	1.05	1.25	1.00	0.92	1.26	1.35	**1.11**
VB MFVAR model - (aggregation constraint only in Q)													
Forecast	2.04	1.84	1.90	2.02	1.68	1.95	1.82	1.89	2.05	1.74	1.90	1.96	**1.90**
Nowcast	1.00	1.11	1.25	1.40	0.87	1.09	1.00	1.17	1.12	1.01	1.16	1.23	**1.12**
VB MFVAR model - (aggregation constraint only in L)													
Forecast	2.02	1.77	1.84	1.95	1.57	1.85	1.75	1.82	1.95	1.70	1.84	1.94	**1.83**
Nowcast	0.99	1.08	1.25	1.41	0.76	1.05	0.98	1.14	1.08	0.99	1.16	1.26	**1.10**
VB MFVAR model - (No aggregation constraints)													
Forecast	2.06	1.84	1.90	2.01	1.68	1.94	1.83	1.88	2.04	1.75	1.89	1.96	**1.90**
Nowcast	1.01	1.11	1.26	1.42	0.85	1.10	1.00	1.17	1.13	1.02	1.17	1.24	**1.12**
VB MFVAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	2.01	1.78	1.84	2.01	1.56	1.84	1.74	1.87	1.96	1.79	1.81	1.92	**1.85**
Nowcast	0.98	1.07	1.35	1.60	0.78	0.99	1.08	1.02	1.00	1.04	1.26	1.34	**1.13**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.23: RMSFE (Multiplied by 100) for Growth in Hours Worked: Horseshoe Prior

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI	Average
AR(1) model													
Forecast	1.06	2.10	1.53	1.33	1.19	1.83	1.74	1.29	2.17	1.70	1.49	1.43	**1.57**
Nowcast	0.39	0.41	0.41	0.43	0.33	0.38	0.36	0.38	0.41	0.41	0.40	0.41	**0.39**
VB MFVAR model - (with both aggregation constraints)													
Forecast	0.93	0.90	0.83	0.79	0.67	0.84	0.75	0.78	0.91	0.94	0.83	0.87	**0.84**
Nowcast	0.47	0.55	0.51	0.59	0.36	0.40	0.45	0.53	0.41	0.48	0.58	0.60	**0.49**
VB MFVAR model - (aggregation constraint only in Q)													
Forecast	0.92	0.89	0.85	0.86	0.74	0.87	0.81	0.81	0.92	0.89	0.84	0.88	**0.86**
Nowcast	0.42	0.48	0.49	0.53	0.36	0.44	0.39	0.45	0.45	0.45	0.46	0.50	**0.45**
VB MFVAR model - (aggregation constraint only in L)													
Forecast	0.91	0.86	0.84	0.83	0.70	0.83	0.78	0.80	0.90	0.87	0.83	0.88	**0.84**
Nowcast	0.42	0.47	0.50	0.53	0.32	0.43	0.40	0.44	0.44	0.45	0.47	0.50	**0.45**
VB MFVAR model - (No aggregation constraints)													
Forecast	0.94	0.94	0.85	0.83	0.69	0.88	0.78	0.79	0.95	0.97	0.85	0.87	**0.86**
Nowcast	0.52	0.58	0.57	0.59	0.40	0.51	0.45	0.51	0.53	0.59	0.56	0.58	**0.53**
VB MFVAR model - (with both aggregation constraints but no exogenous predictors)													
Forecast	0.93	0.90	0.84	0.83	0.67	0.84	0.75	0.80	0.91	0.97	0.82	0.86	**0.84**
Nowcast	0.51	0.56	0.61	0.65	0.36	0.46	0.49	0.45	0.44	0.59	0.60	0.62	**0.53**

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.24: Average CRPS (Multiplied by 100) for Growth in Hours Worked: Horseshoe Prior

B.5 P-Values of Equal Forecast Accuracy Between the MF-VAR Models, as seen in Tables 1-6, and the AR(1)

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI
MF-VAR model - (with both aggregation constraints)												
Forecast	0.21	0.20	0.08	0.27	0.20	0.18	0.25	0.24	0.31	0.24	0.30	0.25
Nowcast	0.24	0.33	0.75	0.45	0.78	0.20	0.24	0.22	0.19	0.36	0.44	0.49
Backcast	0.47	0.70	0.01	0.61	0.43	0.88	0.36	0.02	0.51	1.00	0.71	0.97
MF-VAR model - (aggregation constraint only in Q)												
Forecast	0.22	0.21	0.07	0.27	0.20	0.16	0.25	0.24	0.32	0.24	0.30	0.25
Nowcast	0.60	0.37	0.99	0.45	0.85	0.23	0.25	0.22	0.28	0.79	0.44	0.49
Backcast	0.47	0.75	0.01	0.58	0.43	1.00	0.42	0.05	0.50	0.99	0.69	0.92
MF-VAR model - (aggregation constraint only in L)												
Forecast	0.19	0.22	0.07	0.24	0.21	0.15	0.22	0.18	0.27	0.22	0.29	0.25
Nowcast	0.85	0.14	0.75	0.82	0.27	0.13	0.20	0.11	0.75	0.16	1.00	0.27
Backcast	0.59	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.03	0.01	0.00	0.88
MF-VAR model - (No aggregation constraints)												
Forecast	0.22	0.22	0.07	0.25	0.21	0.16	0.22	0.18	0.27	0.22	0.29	0.27
Nowcast	0.31	0.76	0.52	0.39	0.22	0.77	0.77	0.48	0.32	0.39	0.39	0.23
Backcast	0.52	0.02	0.01	0.01	0.01	0.00	0.00	0.01	0.00	0.01	0.00	0.59
MF-VAR model - (with both aggregation constraints but no exogenous predictors)												
Forecast	0.22	0.21	0.07	0.27	0.20	0.17	0.25	0.24	0.30	0.24	0.30	0.25
Nowcast	0.40	0.30	0.57	0.46	0.98	0.21	0.24	0.23	0.18	0.53	0.42	0.52
Backcast	0.47	0.72	0.01	0.59	0.43	0.93	0.43	0.23	0.52	0.96	0.75	0.89

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.25: P-Values of Equal Forecast Accuracy Between each of the MF-VAR Models, as seen in Table 1, and the AR(1): RMSFE for Productivity Growth

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI
MF-VAR model - (with both aggregation constraints)												
Forecast	0.00	0.00	0.01	0.07	0.00	0.00	0.04	0.07	0.08	0.03	0.10	0.06
Nowcast	0.43	0.62	0.47	0.53	0.80	0.28	0.10	0.08	0.36	0.40	0.33	0.46
Backcast	0.94	0.09	0.00	0.68	0.90	0.14	0.07	0.00	0.59	0.17	0.39	0.25
MF-VAR model - (aggregation constraint only in Q)												
Forecast	0.01	0.00	0.01	0.07	0.00	0.00	0.05	0.07	0.12	0.03	0.10	0.06
Nowcast	0.78	0.39	0.30	0.50	1.00	0.44	0.13	0.11	0.52	0.72	0.33	0.43
Backcast	0.95	0.10	0.00	0.72	0.90	0.19	0.08	0.00	0.65	0.17	0.41	0.27
MF-VAR model - (aggregation constraint only in L)												
Forecast	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.08	0.02	0.00	0.03
Nowcast	0.94	0.06	0.53	0.03	0.15	0.12	0.05	0.09	0.82	0.24	0.07	0.62
Backcast	0.31	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.01	0.01	0.00	0.27
MF-VAR model - (No aggregation constraints)												
Forecast	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.12	0.02	0.00	0.06
Nowcast	0.40	0.57	0.91	0.94	0.20	0.60	0.21	0.86	0.46	0.56	0.84	0.45
Backcast	0.33	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.16
MF-VAR model - (with both aggregation constraints but no exogenous predictors)												
Forecast	0.03	0.00	0.03	0.09	0.00	0.00	0.06	0.09	0.21	0.05	0.10	0.08
Nowcast	0.32	0.17	0.89	0.76	0.56	0.79	0.34	0.17	0.19	0.20	0.42	0.85
Backcast	0.93	0.13	0.00	0.74	0.90	0.18	0.10	0.01	0.56	0.24	0.35	0.32

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.26: P-Values of Equal Forecast Accuracy Between each of the MF-VAR Models, as seen in Table 2, and the AR(1): CRPS for Productivity Growth

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI
MF-VAR model - (with both aggregation constraints)												
Forecast	0.21	0.20	0.08	0.27	0.20	0.18	0.25	0.24	0.31	0.24	0.30	0.25
Nowcast	0.77	0.43	0.37	0.15	0.13	0.50	0.14	0.12	0.93	0.20	0.14	0.16
Backcast	0.90	0.20	0.11	0.09	0.90	0.07	0.20	0.12	0.79	0.35	0.10	0.12
MF-VAR model - (aggregation constraint only in Q)												
Forecast	0.22	0.21	0.07	0.27	0.20	0.16	0.25	0.24	0.32	0.24	0.30	0.25
Nowcast	0.15	0.24	0.50	0.15	0.11	0.42	0.14	0.12	0.58	0.15	0.14	0.16
Backcast	0.92	0.20	0.11	0.09	0.86	0.08	0.21	0.12	0.82	0.36	0.10	0.12
MF-VAR model - (aggregation constraint only in L)												
Forecast	0.19	0.22	0.07	0.24	0.21	0.15	0.22	0.18	0.27	0.22	0.29	0.25
Nowcast	0.32	0.15	0.31	0.22	0.18	0.09	0.22	0.07	0.31	0.14	0.20	0.67
Backcast	0.62	0.19	0.12	0.57	0.11	0.06	0.35	0.06	0.65	0.19	0.06	0.77
MF-VAR model - (No aggregation constraints)												
Forecast	0.22	0.22	0.07	0.25	0.21	0.16	0.22	0.18	0.27	0.22	0.29	0.27
Nowcast	0.53	0.16	0.31	0.26	0.21	0.10	0.23	0.06	0.37	0.14	0.22	0.65
Backcast	0.58	0.20	0.12	0.60	0.13	0.05	0.33	0.06	0.51	0.18	0.05	0.97
MF-VAR model - (with both aggregation constraints but no exogenous predictors)												
Forecast	0.22	0.21	0.07	0.27	0.20	0.17	0.25	0.24	0.30	0.24	0.30	0.25
Nowcast	0.25	0.39	0.63	0.15	0.11	0.50	0.14	0.12	0.84	0.16	0.14	0.19
Backcast	0.95	0.23	0.12	0.10	0.95	0.06	0.20	0.12	0.30	0.28	0.12	0.13

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.27: P-Values of Equal Forecast Accuracy Between each of the MF-VAR Models, as seen in Table 3, and the AR(1): RMSFE for Output Growth

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI
MF-VAR model - (with both aggregation constraints)												
Forecast	0.00	0.00	0.01	0.07	0.00	0.00	0.04	0.07	0.08	0.03	0.10	0.06
Nowcast	0.37	0.28	0.21	0.09	0.04	0.25	0.08	0.07	0.51	0.13	0.06	0.06
Backcast	0.09	0.07	0.04	0.00	0.01	0.02	0.04	0.05	0.19	0.15	0.03	0.12
MF-VAR model - (aggregation constraint only in Q)												
Forecast	0.01	0.00	0.01	0.07	0.00	0.00	0.05	0.07	0.12	0.03	0.10	0.06
Nowcast	0.06	0.19	0.30	0.09	0.03	0.23	0.08	0.07	0.30	0.09	0.06	0.06
Backcast	0.08	0.07	0.04	0.00	0.01	0.02	0.05	0.05	0.29	0.15	0.03	0.11
MF-VAR model - (aggregation constraint only in L)												
Forecast	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.08	0.02	0.00	0.03
Nowcast	0.78	0.09	0.16	0.11	0.13	0.05	0.05	0.02	0.75	0.13	0.01	0.80
Backcast	0.62	0.06	0.02	0.17	0.01	0.03	0.05	0.01	0.85	0.21	0.03	0.73
MF-VAR model - (No aggregation constraints)												
Forecast	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.12	0.02	0.00	0.06
Nowcast	0.89	0.10	0.17	0.25	0.20	0.06	0.06	0.03	0.96	0.13	0.02	0.79
Backcast	0.64	0.07	0.02	0.19	0.02	0.02	0.05	0.01	0.98	0.18	0.03	0.89
MF-VAR model - (with both aggregation constraints but no exogenous predictors)												
Forecast	0.03	0.00	0.03	0.09	0.00	0.00	0.06	0.09	0.21	0.05	0.10	0.08
Nowcast	0.78	0.39	0.61	0.12	0.13	0.36	0.14	0.09	0.57	0.18	0.04	0.20
Backcast	0.20	0.09	0.05	0.00	0.01	0.01	0.04	0.04	0.15	0.09	0.02	0.11

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.28: P-Values of Equal Forecast Accuracy Between each of the MF-VAR Models, as seen in Table 4, and the AR(1): CRPS for Output Growth

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI
MF-VAR model - (with both aggregation constraints)												
Forecast	0.25	0.24	0.27	0.28	0.24	0.21	0.27	0.28	0.26	0.20	0.26	0.27
Nowcast	0.11	0.00	0.00	0.10	0.07	0.03	0.02	0.05	0.05	0.01	0.00	0.03
MF-VAR model - (aggregation constraint only in Q)												
Forecast	0.25	0.24	0.27	0.28	0.24	0.21	0.27	0.28	0.26	0.20	0.26	0.27
Nowcast	0.11	0.00	0.00	0.10	0.07	0.03	0.02	0.05	0.05	0.01	0.00	0.03
MF-VAR model - (aggregation constraint only in L)												
Forecast	0.25	0.24	0.27	0.28	0.24	0.21	0.27	0.28	0.26	0.20	0.26	0.27
Nowcast	0.11	0.00	0.00	0.10	0.07	0.03	0.02	0.05	0.05	0.01	0.00	0.03
MF-VAR model - (No aggregation constraints)												
Forecast	0.25	0.24	0.27	0.28	0.24	0.21	0.27	0.28	0.26	0.20	0.26	0.27
Nowcast	0.10	0.00	0.00	0.10	0.06	0.03	0.02	0.05	0.05	0.01	0.00	0.02
MF-VAR model - (with both aggregation constraints but no exogenous predictors)												
Forecast	0.25	0.24	0.27	0.28	0.24	0.21	0.27	0.28	0.26	0.20	0.26	0.27
Nowcast	0.11	0.00	0.00	0.10	0.07	0.03	0.02	0.05	0.05	0.01	0.00	0.03

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.29: P-Values of Equal Forecast Accuracy Between each of the MF-VAR Models, as seen in Table 5, and the AR(1): RMSFE for Growth in Hours Worked

	NE	YH	EM	EE	LON	SE	SW	WM	NW	WA	SCOT	NI
MF-VAR model - (with both aggregation constraints)												
Forecast	0.02	0.02	0.03	0.04	0.01	0.01	0.03	0.04	0.03	0.01	0.03	0.03
Nowcast	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MF-VAR model - (aggregation constraint only in Q)												
Forecast	0.02	0.02	0.03	0.04	0.01	0.01	0.03	0.04	0.03	0.01	0.03	0.03
Nowcast	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MF-VAR model - (aggregation constraint only in L)												
Forecast	0.02	0.02	0.03	0.04	0.01	0.01	0.03	0.04	0.03	0.01	0.03	0.03
Nowcast	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MF-VAR model - (No aggregation constraints)												
Forecast	0.02	0.02	0.03	0.04	0.01	0.01	0.03	0.04	0.03	0.01	0.03	0.03
Nowcast	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MF-VAR model - (with both aggregation constraints but no exogenous predictors)												
Forecast	0.02	0.02	0.03	0.04	0.01	0.01	0.03	0.04	0.03	0.01	0.03	0.03
Nowcast	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Regional abbreviations: NE - North East England, YM - Yorkshire and Humber, EM - East Midlands, EE - East of England, LON - London, SE - South East England, SW - South West England, WM - West Midlands, NW - North West England, WA - Wales, SCOT - Scotland, NI - Northern Ireland.

Table B.30: P-Values of Equal Forecast Accuracy Between each of the MF-VAR Models, as seen in Table 6, and the AR(1): CRPS for Growth in Hours Worked