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Keywords: Consumer Price Index, National Accounts, Telecommunications, Index numbers, Consumption, Productivity, Regulation

JEL classification: C43, E01, E23

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Alternative Approaches to the Treatment of Access Charges in Price Index Construction

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We propose new approaches to the treatment of fixed access charges in price indexes. These charges are payments for accessing goods and services, but are not dependent on the amounts ultimately purchased. This relatively neglected topic is increasingly important given the growth in telecommunications, data and entertainment services that use an access charge model. Basing our models in alternative consumer behaviour frameworks and using UK telecommunications data, we show that the choice of treatment can be very empirically consequential for price indexes. As result, it is also consequential for the measurement of national output, consumption and productivity.

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1. Introduction

A problem faced by national statistical institutes (NSIs) is how to treat fixed access charges. Examples of such charges are annual club memberships, annual fees for the use of a credit card and fixed charges for access to telecommunication services. Such a charge is called "fixed" as it is independent of the actual consumption of the goods and services that it allows consumers to purchase. Alternative approaches to the treatment of access charges exist and the choice can dramatically impact on the resulting price indexes produced by an NSI.

This was recently illustrated by Abdirahman, Coyle, Heys and Stewart (2022) who examined this issue using UK telecommunications data. Whereas the current published price index showed little change over 2010 to 2017, their proposed new options resulted in declines of between 64% and 85%. As price indexes are used as deflators in the national accounts, the chosen treatment of access charges can therefore also make a big difference to the measurement of national output, consumption and productivity.

In this paper, we consider different approaches that could be used by consumer price statisticians to deal with fixed access charges, based in alternative theoretical models of consumer behaviour. Using a subset of the UK telecommunications data of Abdirahman et al. (2022), we similarly find the choice between our approaches to be consequential.

The rest of the paper is organised as follows. The next section describes the three utility maximization models that inform our alternative approaches to price index construction in the presence of access charges. Section 3 sets out the Laspeyres indexes motivated by the models of Section 2, and derives mathematical relationships between them. Section 4 presents empirical results, starting with a range of unweighted indexes before considering the Laspeyres indexes from Section 3. Results from using the corresponding Paasche and Fisher indexes are also reported.[1] Section 5 concludes. An Appendix provides Paasche

[1] These index number formulae were advocated by Laspeyres (1871), Paasche (1874) and Fisher (1922), respectively.

index comparisons, corresponding to those for the Laspeyres indexes presented in Section 3.

2. Utility Maximization Models

In the analysis in the following sections will look at some "practical" price indexes and compare their magnitudes. Before we define these indexes, it is useful to look at three alternative utility maximization models which will help to motivate the alternative practical indexes. The models and their corresponding empirical Laspeyres indexes are summarized in Box 1 in the following section.

It is useful to introduce some notation at this stage. Let $p^t \equiv [p_1^t, \ldots, p_N^t]$ and $q^t \equiv [q_1^t, \ldots, q_N^t]$ the period t price and quantity vectors for the purchases of the goods or services that the payment of the access charge $P^t > 0$ allows the consumer or group of consumers to purchase for periods $t = 0{,}1$.

Define e^t as the *period t expenditure* on the actual goods and services purchased and v^t as the *value of period t total expenditures* on the group of commodities which is equal to e^t plus the period t access fixed charge P^t. It is also useful to define the period t fixed cost *margin* m^t as the ratio of P^t to e^t. Thus we have the following definitions, for $t = 0{,}1$:

$$\begin{aligned} e^t &= p^t \cdot q^t = \sum_{n=1}^{N} p_n^t\, q_n^t; \\ v^t &= p^t \cdot q^t + P^t = e^t + P^t; \\ m^t &= P^t / e^t. \end{aligned} \tag{1}$$

2.1 Model 1

Suppose the consumer has the utility function $f(q)$. The first utility maximization model that we will consider is a "traditional" model which treats the period t fixed charge as a charge on the "income" that the consumer allocates to the N

commodities in the group of commodities under consideration. The *Model 1* period t utility maximization problem for the subgroup of commodities under consideration is then the following one, where 0_N is a vector with all $n = 1, \ldots, N$ elements equal to zero, and $q > 0_N$ implies that at least one element of the q vector is greater than zero:

$$\max_q \{f(q) : p^t \cdot q^t \leq v^t - P^t = e^t; q \geq 0_N\}. \tag{2}$$

If the consumer price index were constructed in only a single stage, then Model 1 is a "practical" model that price statisticians could use to guide the construction of the national CPI. The period t CPI subindex would be appropriate for deflating the actual commodity expenditures e^t but *the subindex would not be appropriate for deflating actual group expenditures* (including the fixed charges), v^t. A typical CPI is constructed by aggregating over both commodity groupings and outlets or households. To implement the Model 1 approach, price statisticians would have to keep track of the various fixed charges that occur for various outlets and commodity groups as well as collecting the basic price and quantity information. The CPI subindexes which would be computed using this approach would also have to include (separately) information on the fixed charges by commodity group. The national accounts division of the national statistical agency would not be able to take a CPI subindex and use it for deflation purposes if that subgroup of commodities included substantial fixed charges.

2.2 Model 2

The second utility maximization problem treats the access charge as a separate commodity that gives utility to consumers even if they do not consume any products or services that the access charge enables.[2] The new utility function is $f^*(q, Q)$ where $Q = 1$ represents the contribution of access to overall utility for the subgroup of commodities under consideration. The *Model 2* period t utility maximization problem for the subgroup of commodities under consideration is then the following:

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[2] This way of thinking about fixed charges in the telecommunications context is used by national regulators. The approach taken to the treatment of access charges is of some importance in measuring the productivity of telecommunications firms

$$\max_q \{f^*(q,1): p^t \cdot q + P^t \le v^t; q \ge 0_N\}. \tag{3}$$

The advantage of this approach is that the CPI index that is constructed using this framework will be suitable for national accounts deflation purposes; i.e., the period t subindex that is a result of using this approach can be used to deflate total period t expenditures v^t on the commodity class.

2.3 Model 3

The third utility maximization problem allocates the period t fixed charge P^t in a proportional-to-expenditure manner across the "usage" prices p^t. Recall that (1) defined the period t margin m^t as P^t/e^t. The margin is treated in much the same way that a general sales tax is treated; i.e., it is added on to the period t usage prices p^t. Thus the *Model 3* period t utility maximization problem for the subgroup of commodities under consideration is the following one:[3]

$$\max_q \{f(q): (1+m^t)p^t \cdot q \le v^t; q \ge 0_N\}. \tag{4}$$

When price statisticians apply the economic approach to index number theory, it is assumed that the observed period t quantity vector q^t solves the corresponding period t utility maximization problem. It is also assumed that the first inequality constraint in problems (2)-(4) holds with equality. Thus if q^t solves problem (2) for period t, then $p^t \cdot q^t = v^t - P^t = e^t$ for $t = 0,1$; if q^t solves problem (3) for period t, then $p^t \cdot q^t = v^t - P^t = e^t$ for $t = 0,1$ and if q^t solves problem (4) for period t, then $(1+m^t)p^t \cdot q^t = v^t$ for $t = 0,1$. Using the definitions for m^t, e^t and v^t in (1), it can be seen that $(1+m^t)p^t \cdot q^t = [1+(P^t/e^t)]p^t \cdot q^t = [1+(P^t/p^t \cdot q^t)]p^t \cdot q^t = p^t \cdot q^t + P^t = v^t$ for $t = 0,1$. Hence for all three utility maximization problems, it is assumed that the various equalities in definitions (1) are satisfied.

[3] Models 1 and 3 will not work if $q^t = 0_N$ for some period t. If this case occurs empirically, then Model 2 or some other model will have to be used.

3. Index Number Comparisons

In this section we use the alternative models of economic behavior in Section 2 to motivate the definitions of alternative Laspeyres indexes. That is, we will define the Laspeyres indexes that correspond to the three models and derive their relationships. Box 1 summarizes the models from the previous section, and introduces the Laspreyres indexes that correspond to each of these models. Note that a fourth model is introduced, which is alternative empirical approximation to Model 2 of Section 2.

The Laspeyres index comparing the prices of period 1 to the corresponding prices of period 0 *using the Model 1 framework,* P_{L1}, is defined as follows:

$$P_{L1} \equiv \frac{p^1 \cdot q^0}{p^0 \cdot q^0}; \tag{5}$$

The Laspeyres index comparing the prices of period 1 to the corresponding prices of period 0 *using the Model 2 framework,* P_{L2}, is defined as follows:

$$P_{L2} \equiv \frac{p^1 \cdot q^0 + P^1}{p^0 \cdot q^0 + P^0} = \frac{P_{L1} + P^1/e^0}{1 + P^0/e^0}, \tag{6}$$

where the second equality in (6) results from dividing the numerator and denominator by $e^0 = p^0 \cdot q^0$.

Box 1: Consumer Theoretical Models and their Corresponding Laspeyres Indexes

Model 1: Consumers regard an access charge as a charge on income. They then decide the expenditure on products using the remaining net income.
- The price index is then constructed only over products purchased, excluding fixed charges:

$$P_{L1} \equiv \frac{p^1 \cdot q^0}{p^0 \cdot q^0}$$

Model 2: Consumers regard the access charge as a separate product that gives utility even if they do not purchase any products that the access charge allows them to purchase.
- The price index is then constructed by the addition of the fixed charge, P^t, $t = 0,1$, where the quantity is equal to one and the charge is the price:

$$P_{L2} \equiv \frac{p^1 \cdot q^0 + P^1}{p^0 \cdot q^0 + P^0}$$

If aggregate data are used rather than individual household consumption data, this is only an approximation to Model 2. Model 2 is a model that applies to a single household. P_{L2} neglects the complications that arise when aggregating over households.

Model 3: Consumers allocate the access charge in a proportional-to-expenditure manner across the usage prices.
- The price index is then constructed using these adjusted prices, where the *margin* m^t is the ratio of access charges P^t to expenditures e^t:

$$P_{L3} \equiv \frac{(1 + m^1)p^1 \cdot q^0}{(1 + m^0)p^0 \cdot \; q^0}$$

Model 4: This is an alternative empirical approximation to the theoretical consumer framework in Model 2. The fixed charge is split into price and quantity components, where unlike in P_{L2}, the aggregate quantity is not taken to be equal to 1 in both periods. It is an approach which is often used by regulators when constructing producer price indexes for the telecom sector.
- The price index is calculated using some appropriate quantity, such as the number of line connections, q_a^t, as the output measure for the access charge, so that $p_a^t = \; P^t/q_a^t$:

$$P_{L4} \equiv \frac{p^1 \cdot q^0 + p_a^1 q_a^0}{p^0 \cdot q^0 + P_a^0 q_a^0}$$

Using definitions (5) and (6), it is possible to compare P_{L1} to P_{L2}:

$$\begin{aligned} P_{L1} - P_{L2} &= P_{L1} - \frac{P_{L1} + P^1/e^0}{1 + P^0/e^0} \\ &= \frac{P_{L1}(1 + P^0/e^0)\; - P_{L1} - P^1/e^0}{1 + P^0/e^0} \\ &= \frac{P_{L1}(P^0/e^0) - P^1/e^0}{1 + m^0} \\ &= \frac{P_{L1}(P^0/e^0) - (P^1/P^0)(P^0/e^0)}{1 + m^0} \\ &= \left[\frac{m^0}{1 + m^0}\right]\left[P_{L1} - \frac{P^1}{P^0}\right]. \end{aligned} \tag{7}$$

From (7) we see that if the Laspeyres price index P_{L1} for the N products that are made available by paying the access charge in each period is equal to one plus the growth rate in the access charges, P^1/P^0, then P_{L1} will be equal to P_{L2} (which is the Laspeyres price index that treats the access charge as a normal commodity). If P_{L1} is greater than P^1/P^0, then P_{L1} will be greater than P_{L2}; if P_{L1} is less than P^1/P^0, then P_{L1} will be less than P_{L2}. If m^0 is large and the difference between P_{L1} and $P^1/P^0$is also large, then the difference between P_{L1} and P_{L2} can be substantial. This can occur in the case of a telecommunications subindex.[4]

The Laspeyres index comparing the prices of period 1 to the corresponding prices of period 0 *using the Model 3 framework,* P_{L3}, is defined as follows:

$$\begin{aligned} P_{L3} &\equiv \frac{(1 + m^1)p^1 \cdot q^0}{(1 + m^0)p^0 \cdot\; q^0} \\ &= \left[\frac{1 + m^1}{1 + m^0}\right] P_{L1}\,, \end{aligned} \tag{8}$$

[4] Our analysis for the case of Laspeyres price indexes also applies to other fixed basket indexes; i.e., simply replace the base period quantity vector q^0 by the fixed basket quantity vector q^* and apply our analysis pertaining to the differences between the various Laspeyres indexes. The definitions for e^0, v^0 and m^0 become $e^0 \equiv p^0 \cdot q^*$, $v^0 \equiv e^0 + P^0$ and $m^0 \equiv P^0/e^0$. P_{L1} becomes $p^1 \cdot q^*/p^0 \cdot q^*$, P_{L2} becomes $[p^1 \cdot q^* + P^1]/[p^0 \cdot q^* + P^0]$ and P_{L3} (which will be defined shortly) becomes $(1 + m^1)p^1 \cdot q^*/(1 + m^0)p^0 \cdot q^*$ where $e^1 \equiv p^1 \cdot q^*$, $v^1 \equiv e^1 + P^1$ and $m^1 \equiv P^1/e^1$.

where the last line in (8) results from dividing the numerator and denominator of the first line by $e^0 = p^0 \cdot q^0$.

It is very easy to compare P_{L3} to P_{L1}. Using definitions (8) and (5), we have:

$$\frac{P_{L3}}{P_{L1}} = \frac{1+m^1}{1+m^0}. \tag{9}$$

From (9), P_{L3} will equal P_{L1} if $m^1 = P^1/e^1$ is equal to $m^0 = P^0/e^0$or if $P^1/P^0 = e^1/e^0$. P_{L3} will be greater than P_{L1} if $m^1 > m^0$ or if $P^1/P^0 > e^1/e^0$. These results are very straightforward and easy to understand.

The more interesting comparisons are between P_{L3} and P_{L2}. Using (6) and (8), we can derive the following equality:

$$P_{L2} - P_{L3} = \frac{m^1}{1+m^0}\left[\frac{e^1}{e^0} - P_{L1}\right]. \tag{10}$$

From (10), if the usage expenditure ratio, e^1/e^0, is equal to the Laspeyres price index for the available products or services, P_{L1}, then P_{L2} will equal P_{L3}. If usage expenditures grow more rapidly than the usage Laspeyres price index so that e^1/e^0 is greater than P_{L1}, this will imply that P_{L2} will be greater than P_{L3}. If m^1 is also large, then P_{L2} will be substantially greater than P_{L3}.[5]

We turn now to Model 4, an alternative empirical approximation to the theoretical consumer framework in Model 2, where the price index is calculated using some appropriate quantity, such as the number of line connections, q_a^t, as the output measure for

[5] Note that $e^1/e^0 = P_{L1}Q_{P1}$ where $Q_{P1} \equiv p^1 \cdot q^1/p^1 \cdot q^0$ is the Paasche quantity index for usage expenditures. Thus (10) can be rewritten as $P_{L2} - P_{L3} = m^1[1+m^0]^{-1}P_{L1}[Q_{P1} - 1]$. Hence if $Q_{p1} > 1$, then $P_{L2} > P_{L3}$.

the access charge, so that $p_a^t = P^t/q_a^t$; see Box 1. The Laspeyres index comparing the prices of period 1 to the corresponding prices of period 0 *using Model 4*, P_{L4}, is then defined as follows:

$$P_{L4} \equiv \frac{p^1 \cdot q^0 + p_a^1 q_a^0}{p^0 \cdot q^0 + P_a^0 q_a^0} = \frac{P_{L1} + (p_a^1 q_a^0)/e^0}{1 + P^0/e^0}, \tag{11}$$

where the last expression in (11) results from dividing the numerator and denominator by $e^0 = p^0 \cdot q^0$. As P_{L4} is an alternative empirical representation of Model 2, in terms of relationships with the other indexes, reference can be made with the relationships of P_{L2} with $P_{L1}$and P_{L3} in (7) and (10), respectively.

Comparing P_{L4} with P_{L1} using (5) and (11), we can derive the following equality:

$$P_{L1} - P_{L4} = \frac{m^0}{1 + m^0}\left[P_{L1} - \frac{p_a^1}{p_a^0}\right]. \tag{12}$$

That is, the term $P^1/P^0 = p_a^1 q_a^1/p_a^0 q_a^0$ in (7) is replaced by $p_a^1 q_a^0/p_a^0 q_a^0 = p_a^1/p_a^0$. From (12) we see that, for example, P_{L1} will be less than P_{L4} if P_{L1} is less than p_a^1/p_a^0.

Directly comparing the two alternative empirical approaches to implementing Model 2 using Laspeyres indexes, P_{L2} and P_{L4}, we can derive the following equality using (6) and (11):

$$P_{L2} - P_{L4} = \frac{(q_a^1 - q_a^0) P_a^1/e^0}{1 + m^0}. \tag{13}$$

If, for example, the access quantity is growing over time, then q_a^1 will be greater than q_a^0 and hence P_{L2} will be greater than P_{L4}.

Comparing P_{L4} with P_{L3} using (8) and (11), we can derive the following equality:

$$P_{L4} - P_{L3} = \frac{m^1}{1+m^0}\left[\frac{q_a^0}{q_a^1}\left(\frac{e^1}{e^0}\right) - P_{L1}\right]. \tag{14}$$

Equation (14) can be compared with the comparison of P_{L2} with P_{L3} in (10). It can be seen that the term (e^1/e^0) in (10) becomes $(q_a^0/q_a^1)(e^1/e^0)$ in (14). If, for example, $P_{L1}$is greater than $(q_a^0/q_a^1)(e^1/e^0)$, then P_{L3} will be greater than P_{L4}.

In the telecommunications context, the choice of index number method will matter as will be shown in Section 4, with the relativities between the different indexes summarized in Box 3.

4. Empirical Results

For empirical evidence on the huge differences in actual national indexes that the alternative treatment of access charges can make in the telecommunications context, we use a subset of the UK data that are listed in Abdirahman et al. (2022).[6] Specifically, we use only fixed line retail data for ease of interpretation of results. The UK retail telecom revenues for fixed lines $v^t \equiv p_n^t q_n^t$ and the corresponding quantities q_n^t for the years 2010-2017 are listed in Table 1. These data are not pure CPI data in that they do not refer to the purchases by households but instead refer to all retail purchases. However, these data will serve as an example that will show that the above three alternative treatment of access charges can lead to significantly different price (and quantity) indexes.

[6] The data come from Ofcom's Telecommunications Market Data Tables and the Communications Market Reports for 2016, 2017 and 2018. Some data for 2010 is missing so is estimated; see Abdirahman et al. (2022), p. 55.

Table 1: Fixed Line UK Retail Telecommunications Revenues and Quantities

Year t	v_1^t	v_2^t	v_3^t	v_4^t	v_5^t	q_1^t	q_2^t	q_3^t	q_4^t	q_5^t	e^t	v^t
2010	935	293	849	824	3259	65134	4850	5642	14736	23752	2901	6160
2011	787	237	675	742	3375	56083	4570	4471	13066	23872	2441	5816
2012	723	198	566	659	3706	51985	4111	3902	11506	24462	2146	5852
2013	673	155	488	620	3964	46191	3455	3351	10681	24970	1936	5900
2014	577	132	430	620	4148	40766	3015	2940	9028	25549	1759	5907
2015	498	123	369	604	4462	35586	2749	2735	8855	26075	1594	6056
2016	428	111	270	596	4776	30471	2169	2811	7826	26482	1405	6181
2017	362	89	228	543	4969	24705	1550	2587	6126	26661	1222	6191

The revenues in Table 1 are expressed in millions of pounds sterling. The five "products" and their units of measurement for the corresponding quantities are as follows:

- 1 = UK geographic calls in millions of minutes;
- 2 = International calls in millions of minutes;
- 3 = Calls to mobile phones in millions of minutes;
- 4 = Other calls in millions of minutes;
- 5 = Fixed line access charges; units are the number of lines in thousands.

Note that $e^t \equiv v_1^t + v_2^t + v_3^t + v_4^t$ is the total revenue or expenditure for year t on the various types of calls made from fixed lines in the UK and $v^t \equiv e^t + v_5^t$ is total expenditure including access charges v_5^t, where $v_5^t = P^t$ from our prior notation. The ratio of access charges in year t to the corresponding total call revenues is the *margin* $m^t = v_5^t/e^t$ which is listed in Table 2. From Table 2, it can be seen that m^t increases steadily from 1.12 in 2010 to 4.07 in 2017. Thus, from the relationships derived in Section 3, the treatment of access charges is likely to make a substantial difference to any telecom price index based on the above data.

The *unit value prices* for each product can be constructed using the information in Table 1; i.e., we have $p_n^t \equiv v_n^t/q_n^{t*}$ for $n = 1, \ldots, 5$ and $t = 2010, \ldots, 2017$. To see more clearly how the prices of the various telecom products have changed over the sample period, normalize the unit value prices to equal 1 in the base year, 2010; i.e., define the *normalized prices and quantities,* p_n^{t*} and q_n^{t*}, as follows:[7]

$$p_n^{t*} \equiv p_n^t/p_n^{2010};\ q_n^{t*} \equiv q_n^t p_n^{2010}; \quad n = 1, \ldots, 5; t = 2010, \ldots, 2017. \qquad (15)$$

Table 2 lists the normalized prices and quantities for the five products along with the margin series, $m^t \equiv P^t/e^t = v_5^t/e^t$.

Table 2: Normalized Prices and Quantities for the UK Fixed Line Retail Sector

Year t	p_1^{t*}	p_2^{t*}	p_3^{t*}	p_4^{t*}	p_5^{t*}	q_1^{t*}	q_2^{t*}	q_3^{t*}	q_4^{t*}	q_5^{t*}	m^t
2010	1.0000	1.0000	1.0000	1.0000	1.0000	935.00	293.00	849.00	824.00	3259.00	1.1234
2011	0.9776	0.8584	1.0033	1.0156	1.0304	805.07	276.08	672.79	730.62	3275.47	1.3826
2012	0.9689	0.7973	0.9640	1.0243	1.1042	746.25	248.36	587.17	643.39	3356.42	1.7269
2013	1.0150	0.7426	0.9678	1.0381	1.1570	663.07	208.72	504.25	597.25	3426.12	2.0475
2014	0.9860	0.7247	0.9720	1.2282	1.1833	585.20	182.14	442.41	504.82	3505.57	2.3582
2015	0.9749	0.7406	0.8966	1.2198	1.2472	510.84	166.07	411.56	495.15	3577.74	2.7993
2016	0.9785	0.8471	0.6383	1.3619	1.3144	437.41	131.03	423.00	437.61	3633.58	3.3993
2017	1.0208	0.9505	0.5857	1.5852	1.3583	354.64	93.64	389.29	342.55	3658.14	4.0663

It can be seen that relative prices and relative quantities vary considerably over the sample period. This will lead to dispersion among alternative index number formulae.

Section 4.1 Unweighted Price Indexes

Before turning to the (weighted) Laspeyres and Paasche index numbers introduced in Section 3, Table 3 presents various "unweighted" price indexes that could be used by NSIs in the case where appropriate weights for use in price indexes are unavailable. See Box 2 for definitions of each unweighted index considered.

[7] If we change the units of measurement of prices, then we have to change the corresponding units of measurement for quantities in the opposite direction in order to preserve values.

Box 2: Unweighted Indexes

The term “unweighted” really means “equally weighted”. These indexes do not make any use of quantity or value information, so they do not take into account the economic importance of each product. This is not a problem if expenditure shares are roughly equal but typically this is not the case.

The following definitions are for indexes between periods 0 and 1, with prices for goods $n = 1, \ldots, N$.

Definition 1: Dutot (1738) Index

$$P_D(p^0, p^1) \equiv \frac{\frac{1}{N}\sum_{n=1}^{N} p_n^1}{\frac{1}{N}\sum_{n=1}^{N} p_n^0} = \frac{\sum_{n=1}^{N} p_n^1}{\sum_{n=1}^{N} p_n^0}$$

Definition 2: Carli (1764) Index

$$P_C(p^0, p^1) \equiv \frac{1}{N}\sum_{n=1}^{N} \frac{p_n^1}{p_n^0}$$

Definition 3: Jevons (1863) Index

$$P_J(p^0, p^1) \equiv \prod_{n=1}^{N} \left(\frac{p_n^1}{p_n^0}\right)^{\frac{1}{N}}$$

Definition 4: Harmonic Mean Index (Jevons, 1865: Coggeshall, 1887)

$$P_H(p^0, p^1) \equiv \left[\frac{1}{N}\sum_{n=1}^{N} \left(\frac{p_n^1}{p_n^0}\right)^{-1}\right]^{-1}$$

Definition 5: Carruthers, Sellwood and Ward (1980) and Dalén (1992) Index

$$P_{CSWD}(p^0, p^1) \equiv [P_C(p^0, p^1) P_H(p^0, p^1)]^{\frac{1}{2}}$$

For more on these equally weighted indexes, see the Chapter 20 in ILO, IMF, OECD, UNECE, Eurostat and World Bank (2004).

Later we will consider including fixed access charges as a separate product, but for now we consider products 1 to 4. That is, we consider treating access charges as per Model 1 of

Section 2, so they do not appear in our index calculations. The fixed base Harmonic, Caruthers-Sellwood-Ward-Dalén, and Carli indexes, P_H^t, P_{CWSD}^t, P_C^t and their chained counterparts, P_{HCH}^t, P_{CWSDCH}^t, P_{CCH}^t, are listed in this table. The fixed base and chained Dutot indexes coincide as do the fixed base and chained Jevons indexes so are simply listed as P_D^t and P_J^t in Table 3. These indexes were calculated using the prices $p_n^t = v_n^t/q_n^t$ where the v_n^t and q_n^t for $n = 1, 2, 3, 4$ are listed in Table 1. All these indexes except for the Dutot index are independent of the units of measurement. This means that for all the index number formulae that appear in Table 3, except for the Dutot index P_D^t, it does not matter whether we use the prices and quantities listed in Table 1 or their normalized counterparts listed in Table 2.

Instead of using the original units of measurement to calculate the Dutot index, we could use the normalized prices $p_n^{t*} \equiv p^t/p_n^{2010}$ listed in Table 2 and calculate a new Dutot index.[8] It turns out that this new Dutot index, P_{DN}^t using normalized prices in place of the original prices, is equal to the fixed base Carli index P_C^t so we did not list in P_{DN}^t Table 3.

Table 3: Model 1 Unweighted Price Indexes for Products 1-4

Year t	P_D^t	P_J^t	P_H^t	P_{CSWD}^t	P_C^t	P_{HCH}^t	P_{CSWDCH}^t	P_{CCH}^t
2010	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2011	**0.9733**	**0.9616**	**0.9594**	**0.9616**	**0.9637**	**0.9594**	**0.9616**	**0.9637**
2012	**0.9404**	**0.9345**	**0.9302**	**0.9344**	**0.9386**	**0.9319**	**0.9345**	**0.9370**
2013	**0.9358**	**0.9328**	**0.9241**	**0.9324**	**0.9409**	**0.9294**	**0.9328**	**0.9362**
2014	**0.9705**	**0.9610**	**0.9440**	**0.9607**	**0.9777**	**0.9544**	**0.9611**	**0.9677**
2015	**0.9314**	**0.9427**	**0.9278**	**0.9428**	**0.9580**	**0.9355**	**0.9427**	**0.9499**
2016	**0.8445**	**0.9213**	**0.8882**	**0.9217**	**0.9565**	**0.8972**	**0.9204**	**0.9442**
2017	**0.8851**	**0.9742**	**0.9153**	**0.9736**	**1.0355**	**0.9447**	**0.9731**	**1.0024**

[8] Define $P_{DN}^t \equiv [p_1^{t*} + p_2^{t*} + p_3^{t*} + p_4^{t*}]/[p_1^{2010*} + p_2^{2010*} + p_3^{2010*} + p_4^{2010*}] = [p_1^{t*} + p_2^{t*} + p_3^{t*} + p_4^{t*}]/4 = (1/4)\sum_{n=1}^{4}(p_n^t/p_n^{2010}) \equiv P_C^t$] where the second equality follows using $p_n^{2010*} = 1$ for $n = 1, 2, 3, 4$. The Dutot index using normalized prices in place of the initial prices is equal to the fixed base Carli index, P_C^t, for $t = 2010, \ldots, 2017$.

The Dutot index using normalized prices, $P_{DN}^t = P_C^t$, ends up well above its Jevons index counterpart, P_J^t, when $t = 2017$. Normally, the Jevons and Dutot indexes should be approximately equal; i.e., the following approximate equality can be derived:[9]

$$P_J^t \approx P_{DN}^t[1 + (1/2)\mathrm{var}(\epsilon^{2010}) - (1/2)\mathrm{var}(\epsilon^t)\,; \;\; t = 2011, \ldots, 2017. \qquad (16)$$

where $\epsilon_n^t \equiv (p_n^{t*}/\,p_A^t) - 1$ and $p_A^t \equiv (1/4)(\,p_1^{t*} + p_2^{t*} + p_3^{t*} + p_4^{t*})$, for $n = 1, 2, 3, 4$, $\epsilon^t \equiv [\epsilon_1^t, \epsilon_2^t, \epsilon_3^t, \epsilon_4^t]$ and $\mathrm{var}(\epsilon^t) \equiv (1/4)\sum_{n=1}^{4}(\epsilon_n^t)^2$ for $t = 2011, \ldots, 2017$. Since the normalized prices p_n^{t*} all equal 1 when $t = 2010$, we see that $\mathrm{var}(\epsilon^{2010}) = 0$. Moreover, because p_3^{t*} trends down and p_4^{t*} trends up as t increases, $\mathrm{var}(\epsilon^t)$ is increasing over time, using the above approximate equality it can be seen that P_J^t will tend to be less than P_{DN}^t with the gap growing over time as the variance $\mathrm{var}(\epsilon^t)$ increases. We therefore have an explanation for why the gap between P_J^t and $P_{DN}^t = P_C^t$ increases over time.[10]

The large differences between the Dutot index using the original units of measurement, P_D^t, and the version of the Dutot index that uses normalized prices, P_{DN}^t, indicates that *the Dutot formula should be used with extreme caution* even if there are common units of measurement for the individual commodities in scope for the index.

From Table 3, it can be seen that the Jevons index is approximately equal to both the fixed base and chained Carruthers, Ward, Sellwood and Dalén indexes; i.e., we have the following approximate equalities:[11]

$$P_J^t \approx P_{CSWD}^t; \;\; t = 2011, \ldots, 2017. \qquad (17)$$

Looking at Table 3, the following inequalities hold:

[9] See ILO, IMF, OECD, UNECE, Eurostat and World Bank (2004; 362).
[10] As we have seen above, using normalized prices in the Dutot formula converts the fixed base Dutot index into a fixed base Carli index. Hence the divergence is explained by the fact that a geometric mean of numbers that are not all equal (the Jevons index) will always be less than the corresponding arithmetic mean (the Dutot index using normalized prices which is the fixed base Carli index). Recall that the indexes other than the Dutot index are invariant to the units of measurement.
[11] See ILO, IMF, OECD, UNECE, Eurostat and World Bank (2004; 363).

$$P_H^t < P_J^t < P_C^t; P_{HCH}^t < P_J^t < P_{CCH}^t;\ \ t = 2011, \ldots, 2017. \tag{18}$$

Note that in 2017, the Dutot index P_D^{2017} was equal to 0.8851 while the fixed base Carli index P_c^{2017} was equal to 1.0355, so that $P_D^{2017}/P_D^{2017} = 1.0355\,/0.8851 = \ 1.17$. That is, there is a 17.0 % spread between these indexes listed in Table 3, which is substantial. *The choice of an unweighted index number formula matters*.

To conclude this section, in Table 4 we list the same unweighted indexes as were listed in Table 3 but using the prices of all products 1 to 5. That is, we consider treating access charges as per Model 4 of Section 2, so they appear in our index calculations.

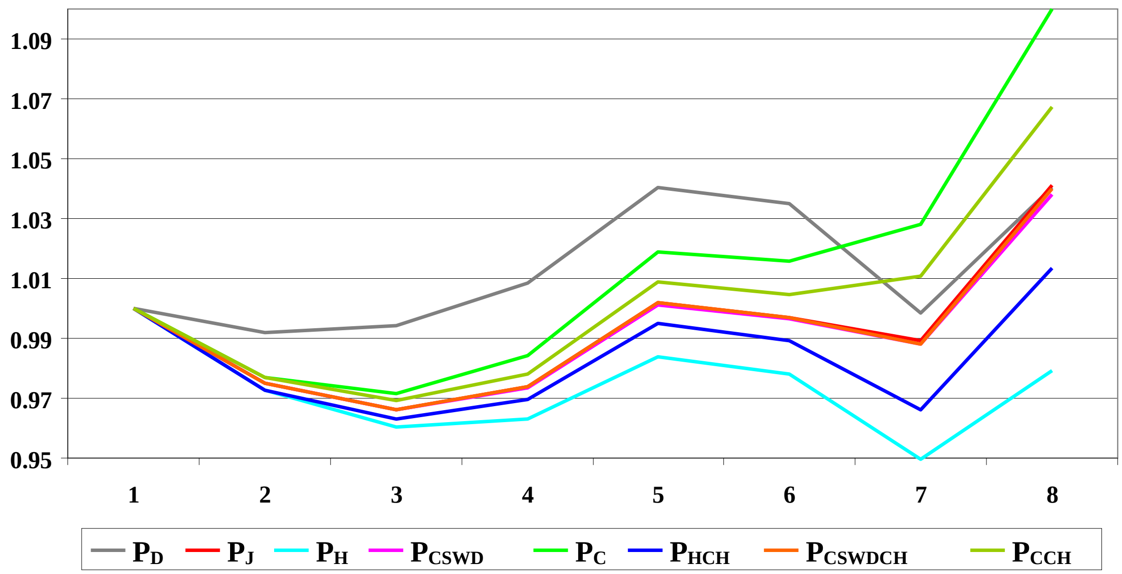
Table 4: Model 4 Unweighted Price Indexes

Year t	P_D^t	P_J^t	P_H^t	P_{CSWD}^t	P_C^t	P_{HCH}^t	P_{CSWDCH}^t	P_{CCH}^t
2010	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2011	**0.9920**	**0.9750**	**0.9728**	**0.9749**	**0.9770**	**0.9728**	**0.9749**	**0.9770**
2012	**0.9941**	**0.9662**	**0.9605**	**0.9661**	**0.9717**	**0.9629**	**0.9662**	**0.9694**
2013	**1.0083**	**0.9739**	**0.9629**	**0.9734**	**0.9841**	**0.9697**	**0.9738**	**0.9780**
2014	**1.0403**	**1.0019**	**0.9838**	**1.0012**	**1.0188**	**0.9950**	**1.0019**	**1.0088**
2015	**1.0349**	**0.9970**	**0.9779**	**0.9967**	**1.0158**	**0.9891**	**0.9970**	**1.0048**
2016	**0.9986**	**0.9892**	**0.9498**	**0.9882**	**1.0280**	**0.9660**	**0.9882**	**1.0108**
2017	**1.0403**	**1.0412**	**0.9792**	**1.0379**	**1.1001**	**1.0133**	**1.0400**	**1.0674**

The 2017 spread in the above unweighted indexes is 1.1001/0.9792 = 1.123 or 12.3%. Recall that the corresponding index spread for the Model 1 unweighted price indexes was 17.0% so the addition of product 5 has lowered the spread significantly. The above indexes used the prices that correspond to the values and quantities listed in Table 1. Recall that the Dutot index using normalized prices, P_{DN}^t, was equal to the chained Carli index, P_{CCH}^t, listed in Table 3. A similar result holds here: P_{DN}^t is equal to P_{CCH}^t listed in Table 9. The indexes listed in Table 9 are plotted on Figure 2. It can be seen that P_J^t, P_{CSWD}^t and P_{CSWDCH}^t

cannot be distinguished on Figure 2. These series are in the middle of the listed indexes, with the chained Carli and Carli indexes, P_{CCH}^t and P_C^t, well above the middle series and the chained Harmonic and Harmonic indexes, P_{HCH}^t and P_H^t, well below the middle series. The Dutot series P_D^t is initially well above the other series but it joins up with the middle series at the end of the sample period. The Dutot index P_{DN}^t using the normalized prices listed in Table 2 coincides with the fixed base Carli index P_C^t. Thus there is a substantial difference in the Dutot indexes as the units of measurement change. The remaining indexes are invariant to changes in the units of measurement.

Figure 1: Model 4 Unweighted Indexes



Section 4.2 Weighted Indexes

In this section, we utilize the data in the tables 1 and 2 to compute alternative indexes for each of the approaches to the treatment of access charges, as described in sections 2 and 3.[12]

We start with the *Model 1 weighted indexes* for our UK telecom data set. For the Model 1 indexes, we ignore the access charges and simply compute the alternative indexes using

[12] We will also consider a fourth approach which is relevant for producer price indexes.

only the prices and quantities for the first four products. Denote the year t fixed base Laspeyres, Paasche and Fisher indexes by P_{L1}^t, P_{P1}^t and P_{F1}^t, and their chained counterparts by P_{LCH1}^t, P_{PCH1}^t and P_{FCH1}^t, where the Fisher index is the symmetric geometric mean of the Laspeyres and Paasche indexes, so that $P_{F1}^t = [P_{L1}^t P_{P1}^t]^{1/2}$ and $P_{FCH1}^t = [P_{LCH1}^t P_{PCH1}^t]^{1/2}$ These indexes are listed in Table 5.[13]

Note that the weighted indexes listed in Table 5 are generally higher than their unweighted counterparts listed in Table 3. The chained Laspeyres indexes are always above their chained Paasche counterparts but this is not always the case for the fixed base Laspeyres and Paasche indexes. Note also that the spread between the six weighted indexes listed in Table 5 for 2017 is much smaller than the corresponding spread between the unweighted indexes in Table 3: the highest index value is 1.0690 for the chained Laspeyres index and the lowest index value is 1.0355 for the fixed base Paasche index. Thus the index spread in 2017 is 1.0690/1.0355 = 1.032 or a 3.2% spread which is far smaller than the unweighted index spread in 2017 which was 17.0%.

Table 5: Model 1 Laspeyres, Paasche and Fisher Indexes

Year t	P_{L1}^t	P_{P1}^t	P_{LCH1}^t	P_{PCH1}^t	P_{F1}^t	P_{FCH1}^t
2010	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2011	**0.9839**	**0.9825**	**0.9839**	**0.9825**	**0.9832**	**0.9832**
2012	**0.9658**	**0.9644**	**0.9661**	**0.9648**	**0.9651**	**0.9655**
2013	**0.9802**	**0.9811**	**0.9805**	**0.9797**	**0.9807**	**0.9801**
2014	**1.0243**	**1.0259**	**1.0274**	**1.0249**	**1.0251**	**1.0262**
2015	**0.9979**	**1.0066**	**1.0034**	**1.0009**	**1.0022**	**1.0022**
2016	**0.9746**	**0.9832**	**0.9931**	**0.9790**	**0.9789**	**0.9860**
2017	**1.0466**	**1.0355**	**1.0690**	**1.0481**	**1.0411**	**1.0585**

[13] If we wish to calculate price change between, say, periods 1 and 3, we can use a fixed base index, $P^{1,3}$, which compares the period 3 price level directly with the level in period 1, or by using a chained index. A chained index in this example can be written as $P^{1,2}P^{2,3}$, so that the period 2 price level is compared with that of period 1, and the result is multiplied by the comparison of the period 3 price level with that of period 2. Direct and chained bilateral indexes, such as we are using here, typically do not give the same answers for the price change from period 1 to period 3, except for very restrictive bilateral index number formulae which we do not consider.

Since the Paasche and Laspeyres indexes have equal justifications, we prefer the Fisher index as it is a symmetric geometric mean of these two indexes. It also has the attractive property of satisfying the time reversal test; see e.g. ILO *et al.* (2004; 295). To choose between the fixed base Fisher and its chained counterpart, we look at the spread between the Laspeyres and Paasche indexes in 2017. For the fixed base versions of these indexes, the spread is equal to $P_{LFB}^{2017}/P_{PFB}^{2017} = 1.0466/1.0355 = 1.011$ or 1.1%. For the chained versions of these indexes the spread is equal to $P_{LCH}^{2017}/P_{PCH}^{2017} = 1.0690/1.0481 = 1.020$ or 2.0%. Since the spread is smaller for the fixed base indexes, we prefer the fixed base indexes over the chained indexes and hence our preferred index for the present data set is the Fisher fixed base index, P_{FFB}^{t}.

For the *Model 2 weighted indexes*, we treat the total access charges $v_5^t \equiv P^t$ as the aggregate price of access in year t and we set the corresponding year t quantity, Q^t, equal to 1.[14] The prices and quantities for products 1-4 are the p_n^t and q_n^t that are listed in Table 1. The price of access, v_5^t, is listed in Table 1. Denote the resulting year t fixed base Laspeyres, Paasche and Fisher indexes by P_{L2}^t, P_{P2}^t and P_{F2}^t and their chained counterparts by P_{LCH2}^t, P_{PCH2}^t and P_{FCH2}^t. These indexes are listed in Table 6. We also list (one plus) the rate of growth in the access charges, P^t/P^{2010}, and (one plus) the rate of growth in expenditures on products 1 to 4, e^t/e^{2010}. Note that P^t/P^{2010} increases rapidly over time while e^t/e^{2010} decreases rapidly.

Bringing access charges into the scope of the index has led to a general increase in the weighted index numbers. The fixed base Fisher index for Model 1 ended up at 1.0481 in 2017 whereas the fixed base Fisher index for Model 2 ended up at 1.3442. *This is a very large difference*. The fixed base Laspeyres index ended up at 1.2996 while the counterpart fixed base Paasche index ended up at 1.3946. The corresponding chained indexes ended up at 1.3419 and 1.3465. Therefore, for Model 2, we prefer the chained Fisher index over

[14] As noted in Box 1, this is only an approximation to Model 2 defined by (3) since the UK data is aggregate retail sales data rather than individual household consumption data. Also Model 2 defined by (3) is a model that applies to a single household; we have neglected the complications that arise when aggregating over households.

its fixed base counterpart since the spread between the Laspeyres and Paasche indexes is much smaller for the chained indexes. However, the two Fisher indexes are very close to each other, ending up at 1.3463 and 1.3442, so in this case it does not matter which Fisher index is chosen.

Table 6: Model 2 Laspeyres, Paasche, Fisher Indexes, P^t/P^{2010} and e^t/e^{2010}

Year t	P_{L2}^t	P_{P2}^t	P_{LCH2}^t	P_{PCH2}^t	P_{F2}^t	P_{FCH2}^t	P^t/P^{2010}	e^t/e^{2010}
2010	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2011	1.0112	1.0126	1.0112	1.0126	1.0119	1.0119	1.0356	0.8414
2012	1.0565	1.0671	1.0611	1.0658	1.0618	1.0634	1.1372	0.7397
2013	1.1051	1.1276	1.1137	1.1203	1.1163	1.1170	1.2163	0.6674
2014	1.1558	1.1877	1.1659	1.1722	1.1716	1.1691	1.2728	0.6063
2015	1.1943	1.2506	1.2198	1.2282	1.2221	1.2240	1.3691	0.5495
2016	1.2343	1.3185	1.2797	1.2870	1.2757	1.2834	1.4655	0.4843
2017	1.2996	1.3946	1.3419	1.3465	1.3463	1.3442	1.5247	0.4212

Recall (7) which established the following relationship between the year t Model 1 Laspeyres index, P_{L1}^t, and the Model 2 Laspeyres index, P_{L2}^t: $P_{L1}^t - P_{L2}^t = [m^{2010}/(1 + m^{2010})][P_{L1}^t - (P^t/P^{2010})]$. From Tables 5 and 6, it can be seen that $P_{L1}^t < P^t/P^{2010}$ for all $t > 2010$ and thus $P_{L1}^t < P_{L2}^t$ for $t = 2011, \ldots, 2017$. Similarly, (A.5) in the Appendix establishes the following relationship between the year t Model 1 Paasche index, P_{P1}^t, and the Model 2 Paasche index, P_{P2}^t: $P_{P1}^t - P_{P2}^t = [m^t/(1 + m^t)]P_{P2}^t\,[P^t/P^{2010}]^{-1}\,[P_{P1}^t - (P^t/P^{2010})]$. From Tables 5 and 6, it can be seen that $P_{P1}^t < P^t/P^{2010}$ for all $t > 2010$ and thus $P_{P1}^t < P_{P2}^t$ for $t = 2011, \ldots, 2017$. These inequalities also imply that $P_{F1}^t < P_{F2}^t$ for $t = 2011, \ldots, 2017$. Hence, due to the very rapid growth in access charges over the sample period, the Model 2 Laspeyres, Paasche and Fisher indexes will be much larger than their Model 1 counterparts.

For the *Model 3 weighted indexes*, the access charges are spread across products 1 to 4 in a proportional manner. Thus define $1 + m^t \equiv v^t/e^t$ and $p_n^{t**} \equiv (1 + m^t)p_n^{t*}$ for $n = 1, 2, 3, 4$ and $t = 2010, \ldots, 2017$. The corresponding quantities are the q_n^{t*} listed in Table

2.[15] Denote the Model 3 year t fixed base Laspeyres, Paasche and Fisher indexes by P_{L3}^t, P_{P3}^t and P_{F3}^t and their chained counterparts by P_{LCH3}^t, P_{PCH3}^t and P_{FCH3}^t. These indexes are listed in Table 7.

Table 7: Model 3 Laspeyres, Paasche and Fisher Indexes

Year t	P_{L3}^t	P_{P3}^t	P_{LCH3}^t	P_{PCH3}^t	P_{F3}^t	P_{FCH3}^t
2010	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2011	1.1040	1.1024	1.1040	1.1024	1.1032	1.1032
2012	1.2403	1.2385	1.2407	1.2391	1.2394	1.2399
2013	1.4068	1.4081	1.4072	1.4061	1.4074	1.4066
2014	1.6199	1.6225	1.6249	1.6209	1.6212	1.6229
2015	1.7854	1.8010	1.7953	1.7909	1.7932	1.7931
2016	2.0191	2.0369	2.0575	2.0282	2.0280	2.0428
2017	2.4972	2.4706	2.5506	2.5008	2.4839	2.5256

Allocating the access charges across the first four types of call products leads to a very large increase in the weighted index numbers. The fixed base Fisher indexes for Model 1 and 2 ends up at 1.0481 and 1.3442 respectively in 2017 whereas the fixed base Fisher index for Model 3 ends up at 2.4839. These differences are very large. The Model 3 fixed base Laspeyres and Paasche spread in 2017 was smaller than the corresponding spread in their chained counterparts so the fixed base Fisher index P_{F3}^t is our preferred weighted index for this approach.

Using our current notation, the equalities in (9) and (A.7) translate into the following equalities:

$$\frac{P_{L3}^t}{P_{L1}^t} = \frac{P_{P3}^t}{P_{P1}^t} = \frac{1+m^t}{1+m^{2010}}\ ; \quad t = 2010, \ldots, 2017. \qquad (19)$$

[15] Instead of using the $p_n^{t**} \equiv (1+m^t)p_n^{t*}$ and q_n^{t*} for $n = 1, 2, 3, 4$ from Table 2 as the primary data that is used in the various Laspeyres, Paasche and Fisher indexes, we could use $(1+m^t)p_n^t$ and q_n^t for $n = 1,2,3,4$ from Table 1 as the primary data. The indexes remain the same since the Laspeyres, Paasche and Fisher indexes are invariant to changes in the units of measurement.

From Table 2, we see that m^t is monotonically increasing. Thus using (19), the inequalities $P_{L3}^t > P_{L1}^t$ and $P_{P3}^t > P_{P1}^t$ for $t > 2010$ must hold, as is confirmed by a comparison of the results in tables 4 and 6.

Using our current notation, (10) can be rewritten as follows:

$$\frac{P_{L3}^t}{P_{L2}^t} = \left[\frac{1+m^t}{1+m^{2010}}\right]\left[P_{L1}^t - \frac{e^t}{e^{2010}}\right]; \quad t = 2010, \dots, 2017. \qquad (20)$$

Tables 6 and 5 list the usage expenditure ratios e^t/e^{2010} and the Model 1 Laspeyres indexes P_{L1}^t, respectively. Using these series, it can be seen that $P_{L1}^t > e^t/e^{2010}$ for $t > 2010$. Thus using (20), we must have $P_{L3}^t > P_{L2}^t$ for $t > 2010$, as is confirmed by a comparison of the results in tables 5 and 6.

Using our current notation, (A.10) can be rewritten as follows:

$$P_{P3}^t - P_{P2}^t = \left[\frac{m^{2010}}{1+m^{2010}}\right] P_{P2}^t \left[P_{P1}^t - \frac{e^t}{e^{2010}}\right]; \quad t = 2010, \dots, 2017. \qquad (21)$$

Tables 6 and 5 list the usage expenditure ratios e^t/e^{2010} and the Model 1 Paasche indexes P_{P1}^t, respectively. It can be seen that $P_{P1}^t > e^t/e^{2010}$ for $t > 2010$. Thus using (21), we must have $P_{P3}^t > P_{P2}^t$ for $t > 2010$. It follows that it is also the case that $P_{F3}^t > P_{F2}^t$ for $t > 2010$, as is confirmed by a comparison of the results in tables 6 and 7.

Finally, consider *Model 4*. As noted in Box 1 of Section 3, this is an alternative empirical implementation of the Model 2 consumer theoretic framework of Section 2. This approach is used when constructing producer price indexes for the telecom sector in the regulation literature that attempts to measure the Total Factor Productivity of the sector. Here the number of line connections is used as the output measure for access charges.[16] Thus this approach simply uses all the v_n^t and q_n^t, $n = 1,\ldots,5$, that are listed in Table 1 (and the implied prices $p_n^t \equiv v_n^t/q_n^t$) in the usual index number formulae. Denote the Model 4 year t fixed base Laspeyres, Paasche and Fisher indexes by P_{L4}^t, P_{P4}^t and P_{F4}^t and their chained counterparts by P_{LCH4}^t, P_{PCH4}^t and P_{FCH4}^t. These indexes are listed in Table 8.

Table 8: Model 4 Laspeyres, Paasche and Fisher Indexes

Year t	P_{L4}^t	P_{P4}^t	P_{LCH4}^t	P_{PCH4}^t	P_{F4}^t	P_{FCH4}^t
2010	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2011	**1.0085**	**1.0097**	**1.0085**	**1.0097**	**1.0091**	**1.0091**
2012	**1.0390**	**1.0484**	**1.0427**	**1.0470**	**1.0437**	**1.0449**
2013	**1.0737**	**1.0927**	**1.0800**	**1.0857**	**1.0832**	**1.0829**
2014	**1.1084**	**1.1316**	**1.1135**	**1.1178**	**1.1199**	**1.1156**
2015	**1.1298**	**1.1733**	**1.1479**	**1.1541**	**1.1513**	**1.1510**
2016	**1.1544**	**1.2209**	**1.1904**	**1.1953**	**1.1872**	**1.1929**
2017	**1.2115**	**1.2796**	**1.2419**	**1.2438**	**1.2451**	**1.2428**

Using the Model 4 methodology, the fixed base Paasche index grows more rapidly than the corresponding fixed base Laspeyres index. The addition of product 5 to the first four products has caused this somewhat unusual phenomenon. The price of product 5 increases 1.36 fold over the sample period which is much higher than a weighted average of the prices of the first 4 products; i.e., P_{L1}^t and P_{P1}^t increased 1.047 fold and 1.036 fold respectively over the sample period. At the same time, q_5^t increased while q_1^t to q_4^t all decreased substantially over the sample period. Under these conditions, P_{P4}^t will increase more rapidly than P_{L4}^t. Table 8 also indicates that the spread between P_{L4}^{2017} and P_{P4}^{2017} is larger than the spread between the chained indexes, P_{L4CH}^{2017} and P_{P4CH}^{2017}. Under these conditions, we prefer the chained Fisher index P_{FCH4}^t over its fixed base counterpart P_{F4}^t. However, Table 8 indicates that the difference between the fixed base and chained Fisher

[16] See for example Lawrence and Diewert (2006; 218) where the distributor's number of line connections is regarded as an output of the firm. Their paper is concerned with electricity distribution but the same methodology is used for telecommunication firms.

indexes is negligible using Model 4. Comparing the results across tables 5-8 confirm the relationships with corresponding indexes as derived in (12)-(14) and (A.12)-(A.14).

Table 9 lists the fixed base and chained Fisher indexes for all four approaches.

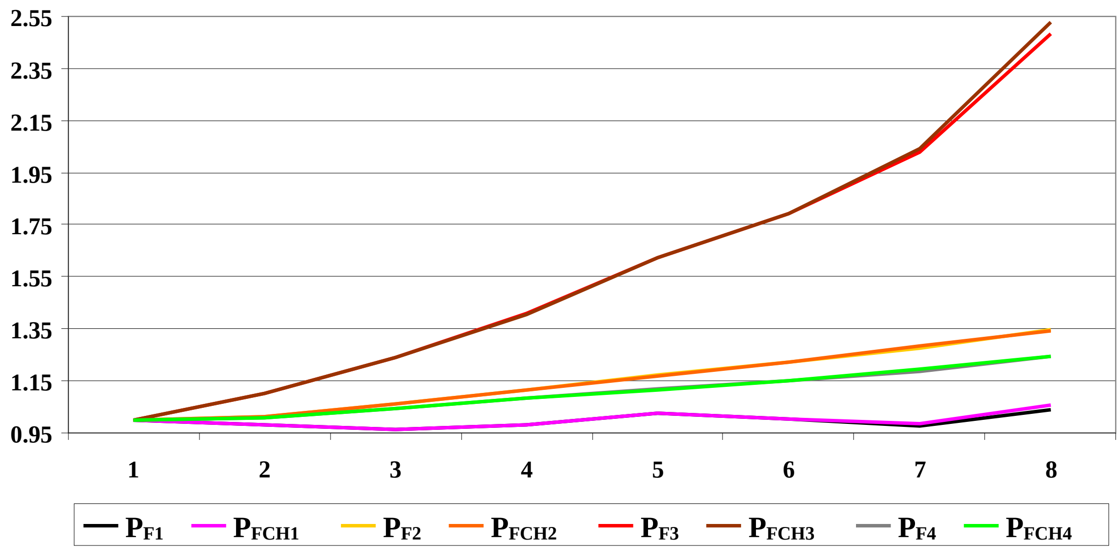
Table 9: Fixed Base and Chained Fisher Indexes for All Four Approaches

Year t	P_{F1}^{t}	P_{FCH1}^{t}	P_{F2}^{t}	P_{FCH2}^{t}	P_{F3}^{t}	P_{FCH3}^{t}	P_{F4}^{t}	P_{FCH4}^{t}
2010	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2011	0.9832	0.9832	1.0119	1.0119	1.1032	1.1032	1.0091	1.0091
2012	0.9651	0.9655	1.0618	1.0634	1.2394	1.2399	1.0437	1.0449
2013	0.9807	0.9801	1.1163	1.1170	1.4074	1.4066	1.0832	1.0829
2014	1.0251	1.0262	1.1716	1.1691	1.6212	1.6229	1.1199	1.1156
2015	1.0022	1.0022	1.2221	1.2240	1.7932	1.7931	1.1513	1.1510
2016	0.9789	0.9860	1.2757	1.2834	2.0280	2.0428	1.1872	1.1929
2017	1.0411	1.0585	1.3463	1.3442	2.4839	2.5256	1.2451	1.2428

From Table 9, it can be seen that the Model 1 Fisher indexes (which ignored the access charges) generate the lowest increase in prices, followed by the Model 4 indexes (include access charges as a regular commodity with the quantity set equal to the number of lines), followed by the Model 2 indexes (include access charges but hold the corresponding quantity fixed at unity) and finally followed by the Model 3 Fisher indexes (which spread the access charges across the other products). These alternative approach Fisher indexes are plotted in Figure 2.

It can be seen that the differences between fixed base and chained Fisher indexes for each approach are small but the differences between the four approaches are very large indeed. It is clear that in the case of fixed line telecommunications services, the choice of approach to the treatment of access charges is consequential.

Figure 2: Alternative Approach Fisher Indexes



Box 3: Summary of Empirical Results for Weighted Indexes

The following table summarizes the empirical findings for the weighted indexes, with the inequalities being the same for Laspeyres, Paasche and (hence) Fisher indexes, for both fixed base and chained indexes.

In the table, P_{ji} denotes the price index for $j = L, P, F$ (for indexes Laspeyres, Paasche and Fisher, respectively) and $i = 1, \ldots, 4$ (for models 1 to 4):

	P_{j1}	P_{j2}	P_{j3}	P_{j4}
P_{j1}	$=$			
P_{j2}	$>$	$=$		
P_{j3}	$>$	$>$	$=$	
P_{j4}	$>$	$<$	$<$	$=$

The way to read the table is as follows. Going down the column with the heading P_{j1}, $P_{j1} = P_{j1}$, $P_{j2} > P_{j1}$, $P_{j3} > P_{j1}$ and $P_{j4} > P_{j1}$. Similarly for the other columns. As the table is of course symmetric, only the bottom triangle of the table is shown.

Hence, while the magnitudes differ depending on the index formula and the choice of fixed base or chaining, the relationships across the models do not change. For example, a Laspeyres, index based on Model 1 is greater than the corresponding Laspeyres index based on Model 3, and the same is true if the Paasche or Fisher index formulae are used.

5. Conclusion

This paper has presented alternative frameworks of consumer behaviour to derive alternative price indexes in the presence of fixed access charges. We have demonstrated relationships between these indexes, both theoretically and empirically.

Using UK fixed line retail telecommunications data, before considering weighted price indexes in their different forms that arise from the alternative frameworks, we presented a range of unweighted prices indexes. Such indexes are particularly relevant in contexts where quantity or expenditure weights are unavailable.

Considering standard unweighted indexes that an NSI might use, we found that results can differ substantially depending on which formula is used, and whether fixed access charges are excluded or included as a separate product. The Carli fixed base and chained indexes are not recommended due to their failure of the time reversal test with a built-in upward bias. The Dutot index is also not recommended due to its lack of invariance to changes in the units of measurement; our empirical example shows that changing the units of measurement can make a huge difference. We recommend the use of the Jevons index as (when there are no missing prices) this has the best properties among the class of unweighted indexes.[17]

Often both price and quantity data will be available to the price statistician, especially in the case of regulated industries.[18] Weighted indexes are preferred over unweighted indexes because they take into account the economic importance of the various outputs of the industry.

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[17] See Diewert (1995) and ILO *et al.* (2004, Chapter 20).

[18] Data submitted to regulators are usually quarterly data which presents challenges in the context of producing a monthly CPI. However, national income accountants must produce quarterly consumer price indexes and perhaps more importantly, national accounts price indexes can be revised. Hence as better information becomes available to the price statistician, better (revised) indexes can be produced.

For our implementation of the alternative weighted indexes that arise out of our models, we find that the approach used to allocate fixed access charges can lead to a substantial difference in the resulting price index. The example in this paper also shows that there can be significant differences between weighted and unweighted indexes.

Model 1 excludes the fixed access charges and hence is not recommended. Model 2 assumes that the quantity corresponding to the fixed charge is always equal to one. This is a reasonable model at the household level, but can only be considered as an approximation to an appropriate index when aggregated data are used. Hence, this model is also not recommended in contexts such as our telecommunications example. To implement Model 4, a quantity variable is be chosen for the fixed access charge. In our empirical example, this is taken to be the number of line rentals. The corresponding unit access price is then equal to the aggregate charge over households divided by the number of line rentals. This is a reasonable approach if the choice of quantity is considered appropriate. Indeed, it is the approach typically used in regulatory contexts. However, the index results will be sensitive to the choice of quantity variable.

Model 3 allocates the access charge across the prices of the different products purchased in proportion to the access charge divided by the total expenditure on the products to which the charge provides access. This treats the access charge in the same manner as a sales tax by adding it to the prices of the purchased products. This is a reasonable approach, as long as households purchase at least one product after paying the access charge. This implies that there may be problems with the use of this model at the household level, but not at more aggregated levels, such as when using national data. Hence, we recommend the use of Model 3, implemented with the Fisher index if the required data are available.[19]

We believe that this paper has contributed to the understanding of choices involved in price index construction in the presence of fixed access charges, particularly in terms of both the underlying assumptions on consumer behaviour and the empirical implications.

[19] The Fisher index is favoured because of it attractive properties under both the axiomatic and economic approaches to index number choices; see Diewert (1992) and ILO et al. (2004, chapters 16 and 17).

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Appendix: Paasche Index Comparisons

In Section 3 we presented alternative Laspeyres indexes which corresponded to our three consumer theoretic models of Section 2. In this appendix, we present the corresponding Paasche indexes and derive their relationships.

The Paasche index comparing the prices of period 1 to the corresponding prices of period 0 *using the Model 1 framework*, P_{P1}, is defined as follows:

$$P_{P1} \equiv \frac{p^1 \cdot q^1}{p^0 \cdot q^1}. \qquad \text{(A.1)}$$

The Paasche index comparing the prices of period 1 to the corresponding prices of period 0 *using the Model 2 framework*, P_{P2}, defined as follows:

$$P_{P2} \equiv \frac{p^1 \cdot q^1 + P^1}{p^0 \cdot q^1 + P^0} = \frac{1 + P^1/e^1}{P_{\mathrm{P1}}^{-1} + P^0/e^1}, \qquad \text{(A.2)}$$

where the second equality in results from dividing the numerator and denominator by $e^1 = p^1 \cdot q^1$, We also used definition (A.1) in deriving the equality in (A.2).

Using definitions (A.1) and (A.2), it is possible to compare P_{P1} to P_{P2} but the resulting formula is a bit more complicated than the corresponding Laspeyres comparison in Section 3:

$$
\begin{aligned}
P_{P2}^{-1} - P_{P1}^{-1} &= P_{P1}^{-1} - \frac{P_{\mathrm{P}1}^{-1} + P^0/e^1}{1 + P^1/e^1} \\
&= \frac{P_{\mathrm{P}1}^{-1}(1 + P^1\,/e^1\,) \, - P_{P1}^{-1} - P^0\,/e^1}{1 + P^1\,/e^1} \\
&= \frac{P_{P1}^{-1}(P^1/e^1) - P^0/e^1}{1 + m^1} \\
&= \frac{P_{P1}^{-1}(P^1/e^1) - (P^0/P^1)(\mathrm{P}^1/e^1)}{1 + m^1} \\
&= \left[\frac{m^1}{1 + m^1}\right]\left[P_{P1}^{-1} - \left(\frac{P^1}{P^0}\right)^{-1}\right].
\end{aligned}
\tag{A.3}
$$

Multiply both sides of (A.3) by $P_{P1}P_{P2}$ and the following expression is obtained:

$$
\begin{aligned}
P_{P2} - P_{P1} &= \left[\frac{m^1}{1 + m^1}\right] P_{P2}\left[1 - P_{P1}\left(\frac{P^1}{P^0}\right)^{-1}\right] \\
&= \left[\frac{m^1}{1 + m^1}\right] P_{P2}\left(\frac{P^1}{P^0}\right)^{-1}\left[\frac{P^1}{P^0} - P_{P1}\right].
\end{aligned}
\tag{A.4}
$$

Finally, multiply both sides of (A.4) through by -1 to obtain the following counterpart to the Laspreyres case in (7):

$$
P_{P1} - P_{P2} = \left[\frac{m^1}{1 + m^1}\right] P_{P2}\left(\frac{P^1}{P^0}\right)^{-1}\left[P_{P1} - \frac{P^1}{P^0}\right]. \tag{A.5}
$$

From (A.5) we see that if the if the Paasche price index P_{P1} for the N products that are made available by paying the access charge in each period is equal to one plus the growth rate in the access charges, P^1/P^0, then P_{P1} will be equal to P_{P2} (which is the Paasche price index that treats the access charge as a normal commodity). If P_{P1} is greater than P^1/P^0, then P_{P1} will be greater than P_{P2}; if P_{P1} is less than P^1/P^0, then P_{P1} will be less than P_{P2}.

If m[1] is large and the difference between P_{P1} and P^1/P^0 is also large, then the difference between P_{P1} and P_{P2} can be substantial.[20]

We turn now to the Model 3 framework. The Paasche index comparing the prices of period 1 to the corresponding prices of period 0 *using the Model 3 framework*, P_{P3}, is defined as follows:

$$\begin{aligned} P_{P3} &\equiv \frac{(1+m^1)p^1 \cdot q^1}{(1+m^0)p^0 \cdot\ q^1} \\ &= \frac{1+m^1}{(1+m^0)P_{P1}^{-1}} \\ &= \left[\frac{1+m^1}{1+m^0}\right] P_{P1}\,, \end{aligned} \tag{A.6}$$

where the second equality in (A.6) results from dividing the numerator and denominator by $e^1 = p^1 \cdot q^1$.

It is very easy to compare to compare P_{P3} to P_{P1} and P_{L3} to P_{L1}. Using definitions (8) and (A.6), we have:

$$\frac{P_{L3}}{P_{L1}} = \frac{P_{P3}}{P_{P1}} = \frac{1+m^1}{1+m^0}\,. \tag{A.7}$$

From (A.7), P_{L3} will equal P_{L1} and P_{P3} will equal P_{P1} if $m^1 = P^1/e^1$ is equal to $m^0 = P^0/e^0$or if $P^1/P^0 = e^1/e^0$. P_{L3} will be greater than P_{L1} and P_{P3} will be greater than P_{P1} if $m^1 > m^0$ or if $P^1/P^0 > e^1/e^0$. These results are very straightforward and easy to understand.

The more interesting comparisons are between between P_{P3} and P_{P2}. Using definitions (A.2) and (A.6), we can derive the following equality:

---

[20] Note that the conditions for "bias" between P_{L1} and P_{L2} and for "bias" between P_{P1} and P_{P2} are very similar in structure.

$$P_{P2}^{-1} - P_{P3}^{-1} = \left[\frac{m^0}{1+m^1}\right]\left[\left(\frac{e^1}{e^0}\right)^{-1} - P_{P1}^{-1}\right]. \tag{A.8}$$

Divide both sides of (A.8) by P_{P3}^{-1} to obtain the following equality, using (A.6):

$$\frac{P_{P3}}{P_{P2}} = \left[\frac{m^0}{1+m^1}\right]\left(\frac{e^1}{e^0}\right)^{-1}\left[P_{P1} - \frac{e^1}{e^0}\right]. \tag{A.9}$$

Multiply both sides of (A.9) by $-P_{P2}$ to obtain the following equality:

$$P_{P2} - P_{P3} = \left[\frac{m^0}{1+m^0}\right]\left(\frac{e^1}{e^0}\right)^{-1} P_{P2}\left[\frac{e^1}{e^0} - P_{P1}\right]. \tag{A.10}$$

Thus if the usage expenditure ratio, e^1/e^0, is equal to the Paasche price index for the available products or services, P_{P1}, then P_{P2} will equal P_{P3}. As noted above, in the telecommunications context, typically usage expenditures will grow more rapidly than the usage Paasche price index so that e^1/e^0 will be much greater than P_{P1} which will imply that P_{P2} will be greater than P_{P3} using (A.10). If m^0 is also large, then P_{P2} will be substantially greater than P_{P3}.[21]

We turn now to Model 4, an alternative empirical approximation to the theoretical consumer framework in Model 2. The Paasche index comparing the prices of period 1 to the corresponding prices of period 0 *using Model 4*, P_{P4}, is defined as follows:

$$P_{P4} \equiv \frac{p^1 \cdot q^1 + p_a^1 q_a^1}{p^0 \cdot q^1 + P_a^0 q_a^1} = \frac{1 + P^1/e^1}{P_{P1}^{-1} + (p_a^0 q_a^1)/e^1}, \tag{A.11}$$

[21] Note that $e^1/e^0 = P_{P1}Q_{L1}$ where $Q_{L1} \equiv p^0 \cdot q^1/p^0 \cdot q^0$ is the Laspeyres quantity index for usage expenditures. Then (A.10)**Error! Reference source not found.** can be rewritten as $P_{P2} - P_{P3} = m^0(1+m^0)^{-1}(e^1/e^0)^{-1}P_{P2}P_{P1}[Q_{L1} - 1]$. Hence if $Q_{L1} > 1$, so that the quantity of services consumed is measured to be growing, then $P_{P2} > P_{P3}$.

where the last expression in (A.11) results from dividing the numerator and denominator by $e^1 = p^1 \cdot q^1$. As P_{P4} is an alternative empirical representation of Model 2, in terms of relationships with the other indexes, reference can be made with the relationships of P_{P2} with $P_{P1}$and P_{P3} in (A.5) and (A.10), respectively.

Comparing P_{P4} with P_{P1} using (A.1) and (A.11), we can derive the following equality:

$$P_{P1} - P_{P4} = \left[\frac{m^1}{1+m^1}\right] P_{P4} \left(\frac{p_a^1}{p_a^0}\right)^{-1} \left[P_{P1} - \frac{p_a^1}{p_a^0}\right]. \qquad \text{(A.12)}$$

That is, the term $P^1/P^0 = p_a^1 q_a^1 / p_a^0 q_a^0$ in (A.5) is replaced by $p_a^1 q_a^1 / p_a^0 q_a^1 = p_a^1/p_a^0$. From (A.12) we see that, for example, P_{P1} will be less than P_{P4} if P_{P1} is less than p_a^1/p_a^0.

Directly comparing the two alternative empirical approaches to implementing Model 2 using Paasche indexes, P_{P2} and P_{P4}, we can derive the following equality using (A.2) and (A.11):

$$P_{P2} - P_{P4} = \frac{(q_a^1 - q_a^0) P_a^0 / e^1}{1+m^1}. \qquad \text{(A.13)}$$

If, for example, the access quantity is growing over time, then q_a^1 will be greater than q_a^0 and hence P_{P2} will be greater than P_{P4}.

Comparing P_{L4} with P_{L3} using (A.6) and (A.11), we can derive the following equality:

$$P_{P4} - P_{P3} \quad = \left[\frac{m^0}{1+m^0}\right] \left(\frac{e^1}{e^0}\right)^{-1} P_{P4} \left[\frac{e^1}{e^0} - P_{P1} \left(\frac{q_a^1}{q_a^0}\right)\right]. \qquad \text{(A.14)}$$

Equation (A.14) can be compared with the comparison of P_{L2} with P_{L3} in (A.10). It can be seen that the term P_{P1} in (A.10) becomes $P_{P1}(q_a^0/q_a^1)$ in (A.14). If, for example, $P_{P1}(q_a^0/q_a^1)$ is greater than $(e^1/e^0\ )$, then P_{P3} will be greater than P_{P4}.