Accounting for latent quality in stochastic frontier analysis: Theory and application to hospital efficiency analysis

Bao Hoang Nguyen
(CBEH, University of Queensland)
Robin Sickles
(Rice University)
Valentin Zelenyuk
(University of Queensland)

ESCoE Conference, London, UK
21-23 May 2025

Outline

Motivation

- **Stochastic frontier analysis** (SFA) is one of the most popular approaches to study productivity and efficiency of production units in general and in healthcare sector (especially hospitals) in particular.
  - Aigner, Lovell & Schmidt (1977 JoE,...)
  - Schmidt & Sickles (1984 JBES, ...)
  - Kneip, Sickles and Simar ...
  - Battese and Coelli (1988 JoE, ...)
  - Greene (2004, 2005, etc.)
  - Kumbhakar et al. (1991, ... 2007, 2022)
  - Griffiths & Hajargasht (2016 JoE)
  - Amsler, Prokhorov & Schmidt (2016, 2017, JoE)
  - Tsionas, Parmeter & Zelenyuk (2024, JoE)
  - ...
  - see the book by Sickles & Zelenyuk (2019) for many details and references

Motivation, cont.

- **Key challenge in any performance analysis (including SFA)**: Accounting for the **heterogeneity** in outputs, especially, heterogeneity in **quality**.

Motivation, cont.

- **Key challenge in any performance analysis (including SFA)**: Accounting for the **heterogeneity** in outputs, especially, heterogeneity in **quality**.
- Basically, the efforts to incorporate quality to hospital efficiency analysis were built around the utilization of some **indicators of hospital quality**:

Motivation, cont.

- **Key challenge in any performance analysis (including SFA)**: Accounting for the **heterogeneity** in outputs, especially, heterogeneity in **quality**.
- Basically, the efforts to incorporate quality to hospital efficiency analysis were built around the utilization of some **indicators of hospital quality**:[1]
  - usually **not easy to obtain** in practice;

[1] See a recent systematic review on health care quality in efficiency studies in Sommersguter-Reichmann (2021).

Motivation, cont.

- **Key challenge in any performance analysis (including SFA)**: Accounting for the **heterogeneity** in outputs, especially, heterogeneity in **quality**.
- Basically, the efforts to incorporate quality to hospital efficiency analysis were built around the utilization of some **indicators of hospital quality**:[1]
  - usually **not easy to obtain** in practice;
  - can only capture some **partial aspects** of hospital quality;

[1] See a recent systematic review on health care quality in efficiency studies in Sommersguter-Reichmann (2021).

Motivation, cont.

- **Key challenge in any performance analysis (including SFA)**: Accounting for the **heterogeneity** in outputs, especially, heterogeneity in **quality**.
- Basically, the efforts to incorporate quality to hospital efficiency analysis were built around the utilization of some **indicators of hospital quality**:[1]
  - usually **not easy to obtain** in practice;
  - can only capture some **partial aspects** of hospital quality;
  - can be **subjective** to selection bias.

[1]See a recent systematic review on health care quality in efficiency studies in Sommersguter-Reichmann (2021).

Overview

Goals: In this work, we develop an **alternative approach to account for quality** in hospital efficiency analysis.

- We consider **quality** as a **latent variable**, which is:

Overview

Goals: In this work, we develop an **alternative approach to account for quality** in hospital efficiency analysis.

- We consider **quality** as a **latent variable**, which is:
  - **known** by the relevant decision-makers of hospitals

Overview

Goals: In this work, we develop an **alternative approach to account for quality** in hospital efficiency analysis.

- We consider **quality** as a **latent variable**, which is:
  - **known** by the relevant decision-makers of hospitals
  - **affect** hospitals' choices of **inputs**,

[2] An early attempt was made by Shee & Stefanou (2014), who adapted the approach of Levinsohn & Petrin (2003) to study the efficiency of the Colombian food manufacturing sector.

Overview

Goals: In this work, we develop an **alternative approach to account for quality** in hospital efficiency analysis.

- We consider **quality** as a **latent variable**, which is:
  - **known** by the relevant decision-makers of hospitals
  - **affect** hospitals' choices of **inputs**,
  - **unobserved** by econometricians.

[2] An early attempt was made by Shee & Stefanou (2014), who adapted the approach of Levinsohn & Petrin (2003) to study the efficiency of the Colombian food manufacturing sector.

Overview

Goals: In this work, we develop an **alternative approach to account for quality** in hospital efficiency analysis.

- We consider **quality** as a **latent variable**, which is:
  - **known** by the relevant decision-makers of hospitals
  - **affect** hospitals' choices of **inputs**,
  - **unobserved** by econometricians.
- We adapt the **control function approach**, which was originated by

[2] An early attempt was made by Shee & Stefanou (2014), who adapted the approach of Levinsohn & Petrin (2003) to study the efficiency of the Colombian food manufacturing sector.

Overview

Goals: In this work, we develop an **alternative approach to account for quality** in hospital efficiency analysis.

- We consider **quality** as a **latent variable**, which is:
  - **known** by the relevant decision-makers of hospitals
  - **affect** hospitals' choices of **inputs**,
  - **unobserved** by econometricians.
- We adapt the **control function approach**, which was originated by
  - Olley & Pakes (1996) (**OP**)

[2] An early attempt was made by Shee & Stefanou (2014), who adapted the approach of Levinsohn & Petrin (2003) to study the efficiency of the Colombian food manufacturing sector.

Overview

Goals: In this work, we develop an **alternative approach to account for quality** in hospital efficiency analysis.

- We consider **quality** as a **latent variable**, which is:
  - **known** by the relevant decision-makers of hospitals
  - **affect** hospitals' choices of **inputs**,
  - **unobserved** by econometricians.
- We adapt the **control function approach**, which was originated by
  - Olley & Pakes (1996) (**OP**)
  - Levinsohn & Petrin (2003) (**LP**)

[2] An early attempt was made by Shee & Stefanou (2014), who adapted the approach of Levinsohn & Petrin (2003) to study the efficiency of the Colombian food manufacturing sector.

Overview

Goals: In this work, we develop an **alternative approach to account for quality** in hospital efficiency analysis.

- We consider **quality** as a **latent variable**, which is:
  - **known** by the relevant decision-makers of hospitals
  - **affect** hospitals' choices of **inputs**,
  - **unobserved** by econometricians.
- We adapt the **control function approach**, which was originated by
  - Olley & Pakes (1996) (**OP**)
  - Levinsohn & Petrin (2003) (**LP**)
  - Ackerberg, Caves and Frazer, 2015 (**ACF**), etc.

[2] An early attempt was made by Shee & Stefanou (2014), who adapted the approach of Levinsohn & Petrin (2003) to study the efficiency of the Colombian food manufacturing sector.

Overview

Goals: In this work, we develop an **alternative approach to account for quality** in hospital efficiency analysis.

- We consider **quality** as a **latent variable**, which is:
  - **known** by the relevant decision-makers of hospitals
  - **affect** hospitals' choices of **inputs**,
  - **unobserved** by econometricians.
- We adapt the **control function approach**, which was originated by
  - Olley & Pakes (1996) (**OP**)
  - Levinsohn & Petrin (2003) (**LP**)
  - Ackerberg, Caves and Frazer, 2015 (**ACF**), etc.
- I.e., we try to **synthesize** this OP-LP-ACF framework with SFA framework, to account for latent quality heterogeneity.[2]

[2] An early attempt was made by Shee & Stefanou (2014), who adapted the approach of Levinsohn & Petrin (2003) to study the efficiency of the Colombian food manufacturing sector.

Overview (Cont.)

Our Monte-Carlo (MC) simulations (analogous to ACF, but with SFA features) confirm good **finite-sample performance** of our approach:

- It handles the latent quality well, confirming that **failing to account for latent quality** may lead to:

Overview (Cont.)

Our Monte-Carlo (MC) simulations (analogous to ACF, but with SFA features) confirm good **finite-sample performance** of our approach:

- It handles the latent quality well, confirming that **failing to account for latent quality** may lead to:
  - an *overestimation* of **labour** coefficient,

Overview (Cont.)

Our Monte-Carlo (MC) simulations (analogous to ACF, but with SFA features) confirm good **finite-sample performance** of our approach:

- It handles the latent quality well, confirming that **failing to account for latent quality** may lead to:
  - an *overestimation* of **labour** coefficient,
  - an *underestimation* of **capital** coefficient, and

Overview (Cont.)

Our Monte-Carlo (MC) simulations (analogous to ACF, but with SFA features) confirm good **finite-sample performance** of our approach:

- It handles the latent quality well, confirming that **failing to account for latent quality** may lead to:
  - an *overestimation* of **labour** coefficient,
  - an *underestimation* of **capital** coefficient, and
  - an *overestimation* of **inefficiency**.

Overview (Cont.)

Our Monte-Carlo (MC) simulations (analogous to ACF, but with SFA features) confirm good **finite-sample performance** of our approach:

- It handles the latent quality well, confirming that **failing to account for latent quality** may lead to:
  - an *overestimation* of **labour** coefficient,
  - an *underestimation* of **capital** coefficient, and
  - an *overestimation* of **inefficiency**.
- The proposed measure of relative quality **shows the ability to capture** various dimensions of hospital quality.

Overview (Cont.)

Our Monte-Carlo (MC) simulations (analogous to ACF, but with SFA features) confirm good **finite-sample performance** of our approach:

- It handles the latent quality well, confirming that **failing to account for latent quality** may lead to:
  - an *overestimation* of **labour** coefficient,
  - an *underestimation* of **capital** coefficient, and
  - an *overestimation* of **inefficiency**.
- The proposed measure of relative quality **shows the ability to capture** various dimensions of hospital quality.
- We also provide an **empirical application** to Queensland hospital data.

A model of quality-adjusted output

- Model of **quality-adjusted output (Y^*)**

$$Y_{it}^* = Y_{it} e^{\omega_{it}} = F\left(K_{it}, L_{it}, M_{it}\right) e^{-u_{it}} e^{v_{it}} \tag{1}$$

Y_{it} is *output*, L_{it} is *labor* input, K_{it} is *capital* input, M_{it} is *material* input,
v_{it} is the idiosyncratic error component,
$u_{it} \geq 0$ is the non-negative inefficiency component,
ω_{it} is the quality component, unobserved (latent).

A model of quality-adjusted output

- Model of **quality-adjusted output (Y^*)**

$$Y_{it}^* = Y_{it} e^{\omega_{it}} = F\left(K_{it}, L_{it}, M_{it}\right) e^{-u_{it}} e^{v_{it}} \tag{1}$$

  Y_{it} is *output*, L_{it} is *labor* input, K_{it} is *capital* input, M_{it} is *material* input,
  v_{it} is the idiosyncratic error component,
  $u_{it} \geq 0$ is the non-negative inefficiency component,
  ω_{it} is the quality component, unobserved (latent).

-

$$Y_{it} = F\left(K_{it}, L_{it}, M_{it}\right) e^{-\omega_{it}} e^{-u_{it}} e^{v_{it}}, \tag{2}$$

A model of quality-adjusted output

- Model of **quality-adjusted output (Y^*)**

$$Y_{it}^* = Y_{it} e^{\omega_{it}} = F(K_{it}, L_{it}, M_{it})\, e^{-u_{it}} e^{v_{it}} \tag{1}$$

  Y_{it} is *output*, L_{it} is *labor* input, K_{it} is *capital* input, M_{it} is *material* input,
  v_{it} is the idiosyncratic error component,
  $u_{it} \geq 0$ is the non-negative inefficiency component,
  ω_{it} is the quality component, unobserved (latent).

-

$$Y_{it} = F(K_{it}, L_{it}, M_{it})\, e^{-\omega_{it}} e^{-u_{it}} e^{v_{it}}, \tag{2}$$

- A Cobb-Douglas value-added production function (as in ACF)

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} - \omega_{it} - u_{it} + v_{it}. \tag{3}$$

A model of quality-adjusted output

- Model of **quality-adjusted output (Y^*)**

$$Y^*_{it} = Y_{it} e^{\omega_{it}} = F\left(K_{it}, L_{it}, M_{it}\right) e^{-u_{it}} e^{v_{it}} \quad (1)$$

Y_{it} is *output*, L_{it} is *labor* input, K_{it} is *capital* input, M_{it} is *material* input,
v_{it} is the idiosyncratic error component,
$u_{it} \geq 0$ is the non-negative inefficiency component,
ω_{it} is the quality component, unobserved (latent).

- $$Y_{it} = F\left(K_{it}, L_{it}, M_{it}\right) e^{-\omega_{it}} e^{-u_{it}} e^{v_{it}}, \quad (2)$$

- A Cobb-Douglas value-added production function (as in ACF)

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} - \omega_{it} - u_{it} + v_{it}. \quad (3)$$

- As usual, the idiosyncratic error is assumed to be zero-mean and mean independent of all covariates

Timing of input choices

- **Capital** for period t is chosen at **period** $t-1$ with the full information of latent quality at period $t-1$.

Timing of input choices

- **Capital** for period t is chosen at **period** $t-1$ with the full information of latent quality at period $t-1$.
- **Labor input** for period t is chosen at some time **between** $t-1$ **and** t, say $t-b$ for $b \in [0,1]$, taking into account the latent quality at $t-b$.

Timing of input choices

- **Capital** for period t is chosen at **period** $t-1$ with the full information of latent quality at period $t-1$.
- **Labor input** for period t is chosen at some time **between** $t-1$ **and** t, say $t-b$ for $b \in [0,1]$, taking into account the latent quality at $t-b$.
- **Material input** is chosen with the full information about latent quality **at period** t as well as about the choices of other inputs.

Timing of input choices

- **Capital** for period t is chosen at **period** $t-1$ with the full information of latent quality at period $t-1$.
- **Labor input** for period t is chosen at some time **between** $t-1$ **and** t, say $t-b$ for $b \in [0,1]$, taking into account the latent quality at $t-b$.
- **Material input** is chosen with the full information about latent quality **at period** t as well as about the choices of other inputs.
- The choice of **material input** is assumed to be a **strictly increasing function** in the latent quality

$$m_{it} = f\left(\omega_{it}, k_{it}, l_{it}\right), \tag{4}$$

m_{it} is the logarithm of the material input,
$f(.)$ is an unknown but increasing function in ω_{it}.

Timing of input choices

- **Capital** for period t is chosen at **period** $t-1$ with the full information of latent quality at period $t-1$.
- **Labor input** for period t is chosen at some time **between** $t-1$ **and** t, say $t-b$ for $b \in [0,1]$, taking into account the latent quality at $t-b$.
- **Material input** is chosen with the full information about latent quality **at period** t as well as about the choices of other inputs.
- The choice of **material input** is assumed to be a **strictly increasing function** in the latent quality

$$m_{it} = f\left(\omega_{it}, k_{it}, l_{it}\right), \tag{4}$$

  m_{it} is the logarithm of the material input,
  $f(.)$ is an unknown but increasing function in ω_{it}.
- We can **invert out** ω_{it} as

$$\omega_{it} = f^{-1}\left(m_{it}, k_{it}, l_{it}\right). \tag{5}$$

Data generating process of latent quality

$$\omega_{it} = g\left(\omega_{i,t-1}, \mathbf{Q}_{it}\right) + \xi_{it}, \tag{6}$$

$$\mathbb{E}\left(\xi_{it} \mid \mathcal{I}_{i,t-1}, \omega_{i,t-1}, \mathbf{Q}_{it}\right) = \mathbb{E}\left(\xi_{it} \mid \omega_{i,t-1}, \mathbf{Q}_{it}\right) = 0. \tag{7}$$

$g(\cdot)$ is an unknown function,
$\mathbf{Q}_{it}$ is a vector of exogenous variables in period t,
$\mathcal{I}_{i,t-1}$ is the information set for period $t-1$.

Scaling property of inefficiency component

- We assume the **scaling property** for the inefficiency component.[3]

$$u_{it} = h\left(\mathbf{Z}_{it}; \boldsymbol{\lambda}\right) u_{it}^{*} \tag{8}$$

u_{it}^{*} is a **positive random variable** following the same distribution across all i and t (called the **basic distribution**),
$h\left(.;.\right)$ is a **known positive function** (called the **scale function**).

[3] See Caudill, Ford & Gropper (1995), Wang & Schmidt (2002), Wang & Ho (2010), Amsler, Prokhorov & Schmidt (2017), Simar, Van Keilegom & Zelenyuk (2017), Kutlu, Tran & Tsionas (2019) and others.

Scaling property of inefficiency component

- We assume the **scaling property** for the inefficiency component.[3]

$$u_{it} = h\left(\mathbf{Z}_{it}; \boldsymbol{\lambda}\right) u_{it}^{*} \tag{8}$$

u_{it}^{*} is a **positive random variable** following the same distribution across all i and t (called the **basic distribution**),
$h\left(.;.\right)$ is a **known positive function** (called the **scale function**).

- For simplicity, we assume

$$h\left(\mathbf{Z}_{it}; \boldsymbol{\lambda}\right) = \exp(\mathbf{Z}_{it}\boldsymbol{\lambda}), \tag{9}$$

and

$$E\left(u_{it}^{*} \mid k_{it}, l_{it}, m_{it}, \mathbf{Z}_{it}\right) = E\left(u_{it}^{*}\right) = \mu, \tag{10}$$

$$Var\left(u_{it}^{*} \mid k_{it}, l_{it}, m_{it}, \mathbf{Z}_{it}\right) = Var\left(u_{it}^{*}\right) = \sigma_u^2. \tag{11}$$

[3] See Caudill, Ford & Gropper (1995), Wang & Schmidt (2002), Wang & Ho (2010), Amsler, Prokhorov & Schmidt (2017), Simar, Van Keilegom & Zelenyuk (2017), Kutlu, Tran & Tsionas (2019) and others.

Final estimable model

- Substituting (5), (8), and (9) into (3)

$$y_{it} = \Theta\left(m_{it}, k_{it}, l_{it}\right) - \mu \exp(\mathbf{Z}_{it}\boldsymbol{\lambda}) + \varepsilon_{it}, \tag{12}$$

where

$$\Theta\left(m_{it}, k_{it}, l_{it}\right) = \beta_0 + \beta_l l_{it} + \beta_k k_{it} - \omega_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} - f^{-1}\left(m_{it}, k_{it}, l_{it}\right), \tag{13}$$

and

$$\varepsilon_{it} = v_{it} - \exp(\mathbf{Z}_{it}\boldsymbol{\lambda})\left[u^*_{it} - \mu\right]. \tag{14}$$

Estimation procedure

- Adapt the two-stage estimation routine in ACF to estimate β_l, β_k, $\boldsymbol{\lambda}$, and μ

Estimation Routine

Estimation procedure

- Adapt the two-stage estimation routine in ACF to estimate β_l, β_k, $\boldsymbol{\lambda}$, and μ
  Estimation Routine
- Use the two moments conditions

$$E\left(\varepsilon_{it} \mid k_{it}, l_{it}, m_{it}, \mathbf{Z}_{it}\right) = 0. \tag{15}$$

$$E\left(\xi_{it} \mid \omega_{it-1}, k_{it}, l_{it-1}, m_{it-1}, \mathbf{Z}_{it-1}, \mathbf{Q}_{it}\right) = 0. \tag{16}$$

Estimation procedure

- Adapt the two-stage estimation routine in ACF to estimate β_l, β_k, $\boldsymbol{\lambda}$, and μ
  Estimation Routine
- Use the two moments conditions

$$E\left(\varepsilon_{it} \mid k_{it}, l_{it}, m_{it}, \mathbf{Z}_{it}\right) = 0. \tag{15}$$

$$E\left(\xi_{it} \mid \omega_{it-1}, k_{it}, l_{it-1}, m_{it-1}, \mathbf{Z}_{it-1}, \mathbf{Q}_{it}\right) = 0. \tag{16}$$

- Obtain standard errors via bootstrapping

Estimate inefficiency and quality

- We suggest to use $E\left(u_{it} \mid \mathbf{Z}_{it}\right)$ as the point estimate of the inefficiency of hospital i at time t, which can be estimated by

$$\widehat{u}_{it} = \widehat{E}\left(u_{it} \mid \mathbf{Z}_{it}\right) = \widehat{\mu} \exp\left(\widehat{\boldsymbol{\lambda}} \mathbf{Z}_{it}\right). \tag{17}$$

Estimate inefficiency and quality

- We suggest to use $E\left(u_{it} \mid \mathbf{Z}_{it}\right)$ as the point estimate of the inefficiency of hospital i at time t, which can be estimated by

$$\widehat{u}_{it} = \widehat{E}\left(u_{it} \mid \mathbf{Z}_{it}\right) = \widehat{\mu}\exp\left(\widehat{\boldsymbol{\lambda}}\mathbf{Z}_{it}\right). \tag{17}$$

- We suggest to use the following normalized measure of hospital quality

$$\tilde{\omega}_{it} = \frac{\omega_{it} - \min\limits_{\substack{j\in\{1,\ldots,N\}\\ s\in\{1,\ldots,T\}}} \omega_{js}}{\max\limits_{\substack{j\in\{1,\ldots,N\}\\ s\in\{1,\ldots,T\}}} \omega_{js} - \min\limits_{\substack{j\in\{1,\ldots,N\}\\ s\in\{1,\ldots,T\}}} \omega_{js}}, \tag{18}$$

which can be estimated by

$$\widehat{\tilde{\omega}}_{it} = \frac{\widehat{\omega_{it} - \beta_0} - \min\limits_{\substack{j\in\{1,\ldots,N\}\\ s\in\{1,\ldots,T\}}} \widehat{\omega_{js} - \beta_0}}{\max\limits_{\substack{j\in\{1,\ldots,N\}\\ s\in\{1,\ldots,T\}}} \widehat{\omega_{js} - \beta_0} - \min\limits_{\substack{j\in\{1,\ldots,N\}\\ s\in\{1,\ldots,T\}}} \widehat{\omega_{js} - \beta_0}}. \tag{19}$$

Monte Carlo Simulation - DGP

The Data Generating Process is analogous to ACF, except for DGP

- SFA features are added: $u_{it} = \exp(\lambda Z_{it}) u_{it}^*, \quad u_{it}^* \sim \mathbb{N}^+ (0, \sigma_u^2)$.
- Exogenous variables are added to the DGP of ω_{it}:
  $\omega_{it} = g(\omega_{i,t-1}, Q_{it}, Z_{it}) + \xi_{it}$
- The correlations between ω and Z and between ω and Q are governed by coefficients α_z and α_q, respectively.

Monte Carlo Simulation - Results

Table 1: Simulation Results

α_z	α_q	Proposed approach				Traditional NLS			
		$\hat{\mu}$	$\hat{\lambda}$	$\hat{\beta}_l$	$\hat{\beta}_k$	$\hat{\mu}$	$\hat{\lambda}$	$\hat{\beta}_l$	$\hat{\beta}_k$
0.00	0.00	0.241	5.036	0.600	0.400	0.242	5.027	0.257	0.731
		(0.034)	(0.590)	(0.011)	(0.017)	(0.036)	(0.614)	(0.005)	(0.008)
0.00	2.00	0.244	4.982	0.600	0.400	0.244	4.980	0.226	0.763
		(0.034)	(0.581)	(0.010)	(0.016)	(0.036)	(0.602)	(0.003)	(0.007)
2.00	0.00	0.242	5.005	0.600	0.401	0.321	4.447	0.238	0.748
		(0.036)	(0.598)	(0.010)	(0.017)	(0.047)	(0.546)	(0.004)	(0.008)
2.00	2.00	0.239	5.062	0.600	0.401	0.282	4.727	0.222	0.768
		(0.035)	(0.624)	(0.010)	(0.015)	(0.042)	(0.597)	(0.003)	(0.007)

(i) The true values of μ, λ, β_l, β_k are 0.24, 5, 0.6, and 0.4, respectively.
(ii) **Proposed approach**: *our proposed approach.* **Traditional NLS**: *A non-linear least squares estimator (NLS) that simply treats the latent quality as a component of the error term.*

An Empirical Study of Queensland Public Hospitals

- We use the data on public hospitals in Queensland, Australia, including 57 hospitals in 12 year periods from FY 2005/06 to FY 2016/17. Data
- Estimates of Hospital Production Frontier Coefficients

Table 2: Estimated coefficients

	Output factor (log)	
	Proposed approach	Traditional NLS
Labor factor (log)	0.644***	0.561***
	(0.086)	(0.065)
Total beds (log)	0.288***	0.336***
	(0.097)	(0.083)

Notes: (i) p-value: $^{*} < 0.1$, $^{**} < 0.05$, $^{***} < 0.01$.
(ii) Bootstrapped standard errors are reported in parentheses.

Empirical Illustration - Estimated coefficients

Table 3: Estimated coefficients

	Output factor (log)	
	Proposed approach	Traditional NLS
Labor factor (log)	0.644***	0.561***
	(0.086)	(0.065)
Total beds (log)	0.288***	0.336***
	(0.097)	(0.083)
$\hat{\mu}$	4.085	2.323
	(9.727)	(10.126)
Major city hospitals	−0.874	−0.796
	(4.651)	(3.964)
Teaching hospitals	0.239	0.284
	(0.227)	(0.246)
Large hospitals	−1.299	−2.000
	(1.447)	(2.685)
The proportion of unit producing personnel	−0.044***	−0.037***
	(0.013)	(0.014)
The outpatient-inpatient ratio	−0.065***	−0.065***
	(0.022)	(0.024)
Case-mix index	1.546**	1.832**
	(0.656)	(0.841)

Notes: (i) Bootstrapped p-value: $^{*} < 0.1$, $^{**} < 0.05$, $^{***} < 0.01$.
(ii) Bootstrapped standard errors are reported in parentheses.

An Empirical Study of Queensland Public Hospitals (cont.)

- Estimates of Hospital Inefficiency

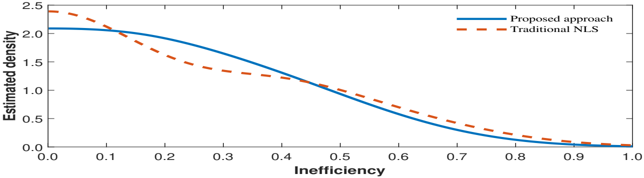

Figure 1: Estimated density of inefficiency estimates.

- Estimates of Hospital Relative Quality

Table 4: Correlation coefficients between the proposed relative quality measure and partial indicators of quality

Variable	Spearman correlation
Nurse to beds ratio	0.42
Hospital standardized mortality ratio	-0.21

Note: Due to data limitation, the correlations are only calculated for a subset of 49 hospitals in our sample in FY 2012/13.

Concluding Remarks

- Our study proposes an econometric approach to account for quality in hospital efficiency analysis.

Concluding Remarks

- Our study proposes an econometric approach to account for quality in hospital efficiency analysis.
- The simulation results and empirical illustration demonstrate that our proposed approach handles the endogeneity caused by the latent quality well.

Concluding Remarks

- Our study proposes an econometric approach to account for quality in hospital efficiency analysis.
- The simulation results and empirical illustration demonstrate that our proposed approach handles the endogeneity caused by the latent quality well.
- Our proposed measure of relative quality shows ability to capture various dimensions of hospital quality.

THANK YOU!!!

References

Amsler, C., Prokhorov, A. & Schmidt, P. (2017), 'Endogenous environmental variables in stochastic frontier models', *Journal of Econometrics* **199**(2), 131–140.

Caudill, S. B., Ford, J. M. & Gropper, D. M. (1995), 'Frontier estimation and firm-specific inefficiency measures in the presence of heteroscedasticity', *Journal of Business & Economic Statistics* **13**(1), 105–111.

Daraio, C. & Simar, L. (2007), Economies of scale, scope and experience in the Italian motor-vehicle sector, *in* C. Daraio & L. Simar, eds, 'Advanced Robust and Nonparametric Methods in Efficiency Analysis: Methodology and Applications', Springer Science & Business Media, pp. 135–165.

Kutlu, L., Tran, K. C. & Tsionas, M. G. (2019), 'A time-varying true individual effects model with endogenous regressors', *Journal of Econometrics* .

Levinsohn, J. & Petrin, A. (2003), 'Estimating production functions using inputs to control for unobservables', *The Review of Economic Studies* **70**(2), 317–341.

Olley, G. S. & Pakes, A. (1996), 'The dynamics of productivity in the telecommunications equipment industry', *Econometrica* **6**(1), 21–37.

Shee, A. & Stefanou, S. E. (2014), 'Endogeneity corrected stochastic production frontier and technical efficiency', *American Journal of Agricultural Economics* **97**(3), 939–952.

Simar, L., Van Keilegom, I. & Zelenyuk, V. (2017), 'Nonparametric least squares methods for stochastic frontier models', *Journal of Productivity Analysis* **47**(3), 189–204.

Sommersguter-Reichmann, M. (2021), 'Health care quality in nonparametric efficiency studies: a review', *Central European Journal of Operations Research* **30**, 1–65.

Wang, H. J. & Ho, C. W. (2010), 'Estimating fixed-effect panel stochastic frontier models by model transformation', *Journal of Econometrics* **18**(2), 129–144.

Wang, H. J. & Schmidt, P. (2002), 'One-step and two-step estimation of the effects of exogenous variables on technical efficiency levels', *Journal of Productivity Analysis* **157**, 286–296.

Methodology - Adapted estimation routine

1. Obtain estimates of $\boldsymbol{\lambda}$ and μ using moment conditions in (15).
   1.1 Approximate $\Theta(\cdot)$ by a full third-order polynomial in $l_{it}, k_{it},$ and m_{it}.
   1.2 Use non-linear least square (NLS) estimator to estimate (12) to obtain $\widehat{\Theta(\cdot)}$, $\hat{\boldsymbol{\lambda}}$, and $\hat{\mu}$.

$$\widehat{\Theta(\cdot)}, \hat{\boldsymbol{\lambda}}, \hat{\mu} = \underset{\Theta(\cdot),\boldsymbol{\lambda},\mu}{\operatorname{argmin}} \frac{1}{N \times T} \sum_{t=1}^{T} \sum_{i=1}^{n} \left(y_{it} - \Theta(\cdot) + \exp\left(\mathbf{Z}_{it}\boldsymbol{\lambda} + \mu\right)\right)^2, \qquad (20)$$

2. Obtain estimates of β_l and β_k using moment conditions in (16).
   2.1 For a hypothesis guess of the parameters β_l and β_k, from (13) we can construct

$$\omega_{it}\left(\widehat{\beta_l, \beta_k}\right) - \beta_0 = -\widehat{\Theta}\left(l_{it}, k_{it}, m_{it}\right) + \beta_l l_{it} + \beta_k k_{it} \qquad (21)$$

   2.2 Regress $\omega_{it}\left(\widehat{\beta_l, \beta_k}\right) - \beta_0$ on a full third order polynomial in $\omega_{it-1}\left(\widehat{\beta_l, \beta_k}\right) - \beta_0$ and $\mathbf{Q}_{it}$ to obtain the residual, $\widehat{\xi_{it}}\left(\beta_l, \beta_k\right)$, from (6).
   2.3 Search over β_l and β_k space to find $\widehat{\beta_l}$ and $\widehat{\beta_k}$ that minimize the following moment condition (i.e., using method of moments estimator)

$$\mathbf{E}\left[\widehat{\xi_{it}}\left(\beta_l, \beta_k\right) \otimes \begin{pmatrix} k_{it} \\ l_{it-1} \end{pmatrix}\right] = 0 \qquad (22)$$

Simulation evidence - Data Generating Process

- We adapt the first data generating process (DGP) discussed in ACF to obtain a production model that is analytically solvable for all input choices.
- The production function for the data generating process is assumed to be Leontief in material input as follows

$$Y_{it} = \min\left\{\beta_0 L_{it}^{\beta_l} K_{it}^{\beta_k} \exp(-\omega_{it}), \beta_m M_{it}\right\} \exp(v_{it}) \exp(-u_{it}). \tag{23}$$

- The choice of capital for period t (i.e., K_{it}) is made at period $t-1$, determined by the level of investment at $t-1$ with the investment adjustment cost varying across firms.
- The choice of labour input (i.e., L_{it}) is made at $t-b$ and $b \in [0,1]$. Moreover, when making decision about the labour input, firms are assumed to face different wages.
- Wages (in logs) and latent quality are generated from AR(1) processes.
- The optimal choices of capital and labour inputs then can be solved analytically.

Simulation evidence - Data Generating Process

- The AR(1) process of the latent quality from $t-1$ to t is decomposed into two sub-processes: (1) from $t-1$ to $t-b$, and (2) from $t-b$ to t.

$$\begin{aligned} \omega_{it-b} &= \left(\rho^{\omega}\right)^{(1-b)} \omega_{i,t-1} + \alpha_q Q_{it} + \alpha_z Z_{it} + \xi_{it}^{\omega 1}, \\ \xi_{it}^{\omega 1} &\overset{iid}{\sim} \mathbb{N}\left(0, \sigma^2_{\xi^{\omega 1}}\right), \end{aligned} \tag{24}$$

and

$$\begin{aligned} \omega_{it} &= \left(\rho^{\omega}\right)^{b} \omega_{i,t-b} + \xi_{it}^{\omega 2}, \\ \xi_{it}^{\omega 2} &\overset{iid}{\sim} \mathbb{N}\left(0, \sigma^2_{\xi^{\omega 2}}\right). \end{aligned} \tag{25}$$

- Z_{it} and Q_{it} are generated from AR(1) processes

Simulation evidence - Data Generating Process (Cont.)

- The optimal choice of material input is given by the Leontief first order condition

$$\beta_m M_{it} = \beta_0 L_{it}^{\beta_l} K_{it}^{\beta_k} \exp(-\omega_{it}). \tag{26}$$

- The inefficiency term is generated as follows

$$\begin{aligned} u_{it} &= \exp\left(\lambda Z_{it}\right) u_{it}^{*}, \\ u_{it}^{*} &\sim \mathbb{N}^{+}\left(0, \sigma_u^2\right), \end{aligned} \tag{27}$$

- The error is generated as follows

$$v_{it} \sim \mathbb{N}\left(0, \sigma_v^2\right). \tag{28}$$

- The output is then generated from the following value-added production function (in logs)

$$y_{it} = \ln \beta_0 + \beta_l l_{it} + \beta_k k_{it} - \omega_{it} - u_{it} + v_{it}. \tag{29}$$

MC Simulation

Empirical Illustration - Data and Variables

- We use the data on public hospitals in Queensland, Australia, including 57 hospitals in 12 year periods from FY 2005/06 to FY 2016/17.

Empirical Illustration - Data and Variables

- We use the data on public hospitals in Queensland, Australia, including 57 hospitals in 12 year periods from FY 2005/06 to FY 2016/17.
- Inputs

Empirical Illustration - Data and Variables

- We use the data on public hospitals in Queensland, Australia, including 57 hospitals in 12 year periods from FY 2005/06 to FY 2016/17.
- Inputs
  - Full-time equivalent (FTE) staff is used as a measure of labour input. The aggregation approach proposed by Daraio & Simar (2007) is utilized to aggregate FTE in six different labour categories into a single measure of labour input (called labour factor).

Empirical Illustration - Data and Variables

- We use the data on public hospitals in Queensland, Australia, including 57 hospitals in 12 year periods from FY 2005/06 to FY 2016/17.
- Inputs
  - Full-time equivalent (FTE) staff is used as a measure of labour input. The aggregation approach proposed by Daraio & Simar (2007) is utilized to aggregate FTE in six different labour categories into a single measure of labour input (called labour factor).
  - Expenditure on drug and medical supplies is used as a measure of material input (in FY 2012/13 constant price).

Empirical Illustration - Data and Variables

- We use the data on public hospitals in Queensland, Australia, including 57 hospitals in 12 year periods from FY 2005/06 to FY 2016/17.
- Inputs
  - Full-time equivalent (FTE) staff is used as a measure of labour input. The aggregation approach proposed by Daraio & Simar (2007) is utilized to aggregate FTE in six different labour categories into a single measure of labour input (called labour factor).
  - Expenditure on drug and medical supplies is used as a measure of material input (in FY 2012/13 constant price).
  - The number of beds is utilized as a proxy for capital input.

Empirical Illustration - Data and Variables (Cont.)

- Outputs

Empirical Illustration - Data and Variables (Cont.)

- Outputs
  - Outpatient output is measured by the number of non-admitted occasions of service.

Empirical Illustration - Data and Variables (Cont.)

- Outputs
  - Outpatient output is measured by the number of non-admitted occasions of service.
  - Inpatient output is measured by the number of casemix weighted inpatient episodes.

Empirical Illustration - Data and Variables (Cont.)

- Outputs
  - Outpatient output is measured by the number of non-admitted occasions of service.
  - Inpatient output is measured by the number of casemix weighted inpatient episodes.
  - To deal with multiple outputs in SFA framework, as for the labor input, we aggregate them into a single measure of outputs.

Empirical Illustration - Data and Variables (Cont.)

- Determinations of hospital inefficiency

Empirical Study

Empirical Illustration - Data and Variables (Cont.)

- Determinations of hospital inefficiency
  - Three characteristics of hospitals (e.g., location, size, teaching status).

Empirical Study

Empirical Illustration - Data and Variables (Cont.)

- Determinations of hospital inefficiency
  - Three characteristics of hospitals (e.g., location, size, teaching status).
  - Labor mix: the proportion of unit producing personnel.

Empirical Study

Empirical Illustration - Data and Variables (Cont.)

- Determinations of hospital inefficiency
  - Three characteristics of hospitals (e.g., location, size, teaching status).
  - Labor mix: the proportion of unit producing personnel.
  - Outpatient-inpatient ratio: the ratio of outpatient volume to inpatient volume.

Empirical Study

Empirical Illustration - Data and Variables (Cont.)

- Determinations of hospital inefficiency
  - Three characteristics of hospitals (e.g., location, size, teaching status).
  - Labor mix: the proportion of unit producing personnel.
  - Outpatient-inpatient ratio: the ratio of outpatient volume to inpatient volume.
  - Case-mix index

Empirical Study

Empirical Illustration - Data and Variables (Cont.)

- Determinations of hospital inefficiency
  - Three characteristics of hospitals (e.g., location, size, teaching status).
  - Labor mix: the proportion of unit producing personnel.
  - Outpatient-inpatient ratio: the ratio of outpatient volume to inpatient volume.
  - Case-mix index
- Determinations of latent quality

Empirical Study

Empirical Illustration - Data and Variables (Cont.)

- Determinations of hospital inefficiency
  - Three characteristics of hospitals (e.g., location, size, teaching status).
  - Labor mix: the proportion of unit producing personnel.
  - Outpatient-inpatient ratio: the ratio of outpatient volume to inpatient volume.
  - Case-mix index
- Determinations of latent quality
  - Case-mix index

Empirical Study

Empirical Illustration - Data and Variables (Cont.)

Table 5: Descriptive statistics

	Mean	Std. Dev.	Min	Max
Output factor (*FOUT*)	0.92	1.40	0.03	7.82
Labour factor (*FLABOR*)	1.33	2.40	0.03	14.07
Total beds (*BEDS*)	160.85	244.03	9.00	1218.00
Drug and medical supply expenditure (*DMSUP*) ($100,000s)$^{(a)}$	181.36	372.80	0.90	3454.36
Major city hospitals (*CITY*)	0.19	0.39	0.00	1.00
Teaching hospitals (*TEACH*)	0.32	0.47	0.00	1.00
Large hospitals *LARGE*	0.40	0.49	0.00	1.00
The proportion of unit producing personnel (*UPP*) (%)	65.94	7.07	38.83	88.13
The outpatient-inpatient ratio (*OIR*)	13.97	6.60	4.28	49.34
Case-mix index (*CMI*)	0.92	0.22	0.56	2.49

(*a*): *in FY 2012/13 constant price*