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A Nonparametric Approach to the Measurement of Aggregate Markups

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Some References

- De Loecker, J., Eeckhout, J., and Unger, G., 2020. “The Rise of Market Power and the Macroeconomic Implications”, *Quarterly Journal of Economics* 135(2), 561–644.

- Diewert, W.E. and K.J. Fox (2008), “On the Estimation of Returns to Scale, Technical Progress and Monopolistic Markups”, *Journal of Econometrics* 145, 174-193.

- Diewert W.E. and K.J. Fox (2018), “Alternative User Costs, Productivity and Inequality in US Business Sectors”, pp. 21-69 in *Productivity and Inequality*, W.H. Green. L. Khalaf, P. Makdissi, M. Veall and M.-C. Voia (eds.), New York: Springer.

- Schreyer, P. (2014), “Priorities and Directions for Future Productivity Research: An OECD Perspective,” *International Productivity Monitor*, Number 27, Fall, 7-9.

Summary

- We introduce a new and general approach to the estimation of markups that uses the nonparametric approach to production theory
- Past vectors of inputs and outputs for the production unit are used to build up an approximation to the "true" technology
- Our markup growth estimate can be decomposed into contributions from output and input price changes, efficiency change, and technical progress and returns to scale

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A (very) Brief History of Markup Estimates

- Harberger (1954): deadweight loss due to market power "no more than a thirteenth of a percent of the national income". Perhaps a reason for the low priority placed on market power by macroeconomists until recently.

- Hall (1988, 1990): Industry level markups
  - Nordhaus (1990, p. 151): "The results are an embarrassment to the theory, with scale factors of 33 in food products and 138 in chemicals. The results are completely inconsistent with engineering production function,…, and common sense."
  - Baily (1990, p. 147): "[a]nother important innovation of this research program has been the introduction of some rather whacky variables as instruments . . . . In general, it is problematic to insert instrumental variables without a model to guide how to interpret what is found"

A (very) Brief History of Markup Estimates

- A lot of “stress testing” done on Hall’s work
  - Alternative instruments (e.g. Burnside 1996)
  - Estimates a markup parameter then inverts it, which is not the same as estimating it directly (Bartelsman 1995).

- Morrison (1992): 1.286 for U.S. Manufacturing, 1.427 for Japan
- Basu and Fernald (1997): Markups of up to 1.72 in “Manufacturing Durables” and “Private Economy”
- Diewert and Fox (2008, J. Econometrics)
- Generally, markups were found to be quite high but stable.

What's new in the recent resurgence of interest?

- Firm level (CompuStat) data
- Sophisticated econometrics
- And startling results!

The contributions from
Jan De Loecker and co-authors
have reinvigorated the literature
on markups

"Stress testing" has generated
new innovations and insights

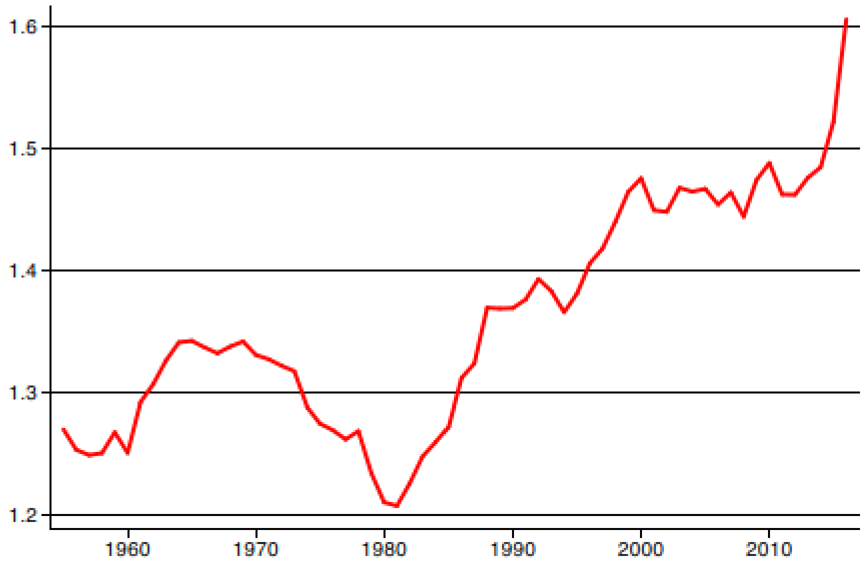

FIGURE I

Average Markups

Output elasticities θ_{st} from the estimated production function are time-varying and sector-specific (two-digit). The average is revenue weighted. The figure illustrates the evolution of the average markup from 1955 to 2016.

De Loecker, Eeckhout and Unger (2020)

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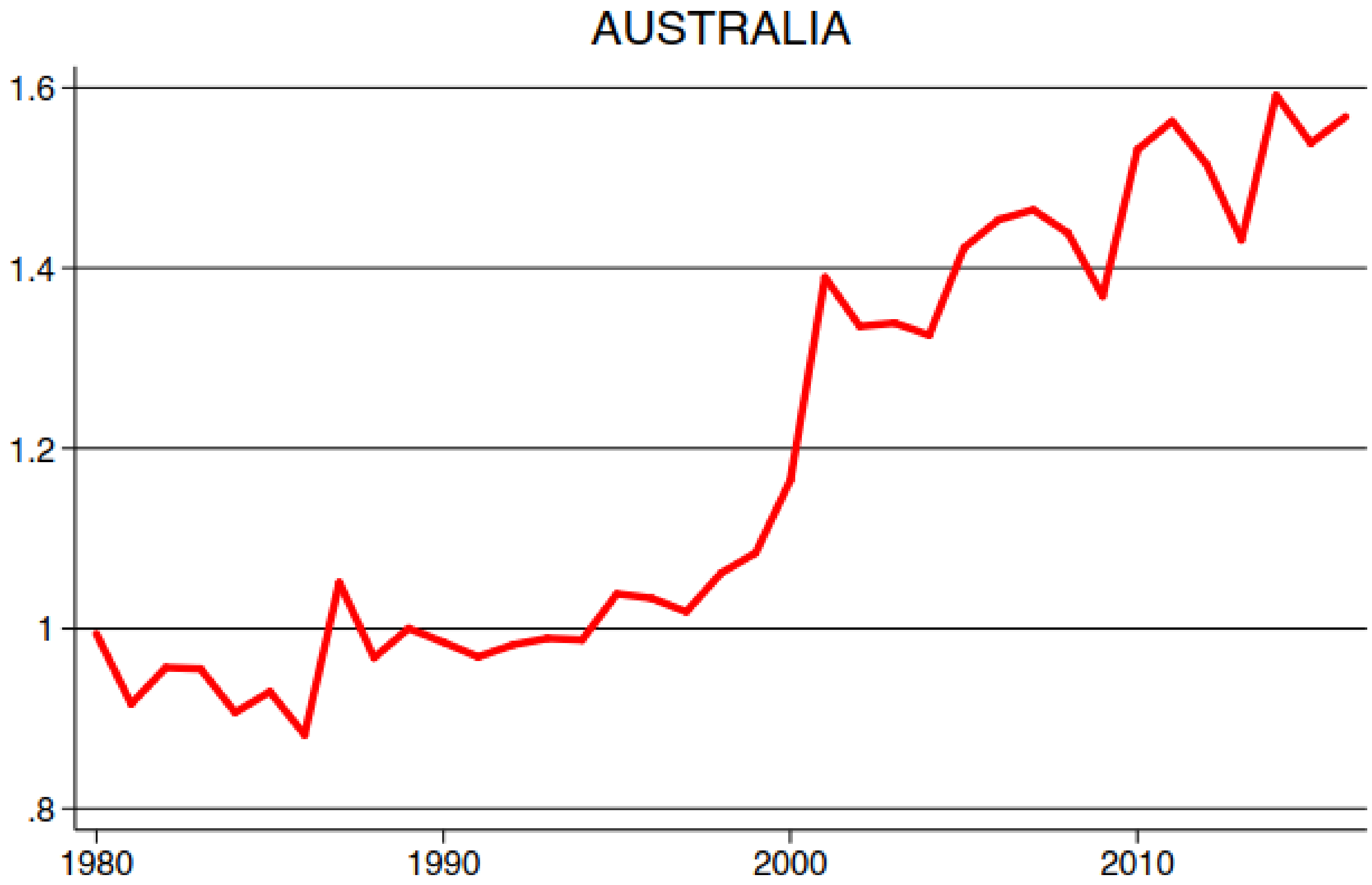
What's new in the recent resurgence of interest?

Figure A.2, De Loecker and Eeckhout (2018a)



Our take on De Loecker, Eeckhout and Unger (2020)

Suppose we have a production unit that produces a single output q using N variable inputs x and one capital input k which we hold fixed in the short run.

Production function: $q = f(x,k)$ where $x \equiv [x_1,x_2,\ldots,x_N]$ is a vector of variable inputs.

The variable cost minimization problem for the production unit is the following minimization problem:

$$\min_x \{w \cdot x : f(x,k) = q\} \equiv C(q,k,w)$$

- $w \equiv [w_1,w_2,\ldots,w_N]$ is the vector of positive input prices that the unit faces
- $C(q,k,w)$ is the variable cost function
- $w \cdot x \equiv \sum_{n=1}^{N} w_n x_n$

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Our take on De Loecker, Eeckhout and Unger (2020)

Lagrangian: $L(x,\lambda) \equiv w \cdot x - \lambda\{f(x,k) - q\}$.

Assume f is differentiable with respect to the components of x at the solution to the variable cost minimization problem, $x^* \equiv [x_1^*, x_2^*, \ldots, x_N^*]$

First order necessary conditions that should be satisfied by the solution x^* to the variable cost minimization problem:

$$w_n = \lambda^* f_n(x^*,k); \quad n = 1,\ldots,N;$$

$$q = f(x^*,k)$$

where $f_n(x^*,k) \equiv \partial f(x^*,k)/\partial x_n$ for n = 1,…,N.

The producer's *optimal variable cost* is then: $C(q,k,w) \equiv w \cdot x^*$

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Our take on De Loecker, Eeckhout and Unger (2020)

Optimal marginal (variable) cost is equal to the Lagrange multiplier solution λ^* to the system of equations defined by the FOC:

$$\partial C(q,k,w)/\partial y = \lambda^*$$

Define the marginal (variable) *cost ratio* μ as the price of a unit of output, p, divided by marginal (variable) cost, λ^*:

$$\mu \equiv p/\lambda^* = 1 + m$$

The (variable cost) *markup* m is defined as $\mu - 1$.

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Our take on De Loecker, Eeckhout and Unger (2020)

The *output elasticity of input n*, θ_n, is defined as follows:

$$\theta_n(x^*,k) \equiv f_n(x^*,k)x_n^*/f(x^*,k) \qquad \text{for } n = 1,\ldots,N$$

$$= f_n(x^*,k)x_n^*/q \qquad \text{where } q \equiv f(x^*,k)$$

$$= w_n x_n^*/\lambda^* q \qquad \text{using the first FOC}$$

Thus marginal variable cost λ^* satisfies the following N equations:

$$\lambda^* = w_n x_n^*/\theta_n(x^*,k)q \;; \quad n = 1,\ldots,N$$

Therefore, using $\mu \equiv p/\lambda^*$:

$$\mu = \theta_n(x^*,k)pq/w_n x_n^* \;; \quad n = 1,\ldots,N.$$

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Our take on De Loecker, Eeckhout and Unger (2020)

Choose some input n, and choose some output elasticity of demand for that input, $\theta_n(x^*,k)$, that is based on exogenous knowledge.

Using an observed input cost $w_n x_n^*$ for the chosen input n along with an observed value of output produced, pq, they have their estimate for the marginal (variable) cost ratio and hence the markup over marginal variable cost.

Our take on De Loecker, Eeckhout and Unger (2020)

Our concerns with this methodology:

- **There is not much policy interest in determining markups over marginal variable cost**. Fixed costs have to be amortized over the lifetime of the “fixed” input. If they are not paid for, then no one will supply financial capital to the production unit to purchase the durable inputs that are need to produce the output.

- **The first order conditions need to be checked**. Are they satisfied for the production unit under consideration? A suitable functional form for the production function f(x,k,t) needs to be chosen, and derivatives of f with respect to each variable input calculated to check if $f_n(x^*,k,t)/w_n$ is a constant across all variable inputs n.
  - If these conditions are not satisfied, then variable cost has not been minimized.

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Our take on De Loecker, Eeckhout and Unger (2020)

Our concerns with this methodology (cont.):

- **The Cobb-Douglas functional form is not a “suitable” functional form** since it implies that variable input cost shares are constant over time (which is not the case in the real world).

- Time t is added here as an additional explanatory variable that must appear in the production function because the authors are looking at time series of outputs and inputs and technical progress or **Total Factor Productivity improvements cannot be ignored**.

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Our take on De Loecker, Eeckhout and Unger (2020)

- The authors recognize that markups over marginal variable cost are not of great interest and work out results for markups over "full cost" including the (user) costs of durable inputs.

- They find that their big markups become small markups that have increased over time.

- However, their results have a ***missing input problem***: they do not include land or monetary holdings of firms.

Our take on De Loecker, Eeckhout and Unger (2020)

- Firm holdings of money have greatly increased and the price of land has also greatly increased over the last two decades

- Including these missing assets will increase total costs of production substantially and hence lessen the growing markups over full costs that were found.

- Schreyer (2014) flagged the importance of missing assets for the measurement of TFP and Diewert and Fox (2018) document the large increases in the value of land and monetary assets for US producers.

Finally, our approach

Consider the problem of measuring the total factor productivity (TFPG) of a one output, one input firm, between periods t = 0, 1

$$\text{Revenue: } R^t = p^t y^t$$

$$\text{Cost: } C^t = w^t x^t$$

$$TFPG(1) \equiv [y^1/y^0]/[x^1/x^0]$$

Or, generalizing a method originally suggested by Jorgenson and Griliches (1967; 252):

$$1+m^t \equiv R^t/C^t$$

$$TFPG(2) \equiv \{(1+m^1)/(1+m^0)\}\{[w^1/w^0]/[p^1/p^0]\}$$

i.e. the markup growth rate $(1+m^1)/(1+m^0)$ times the rate of increase in input prices w^1/w^0 divided by the rate of increase in output prices p^1/p^0.

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Our approach

TFPG(1) = TFPG(2) but this equivalence does not generally extend to the multiple output, multiple input case.

TFPG(2) indicates the relationship between total factor productivity and increased profitability:

$$(1+m^1)/(1+m^0) = [TFPG(2)][p^1/p^0]/[w^1/w^0]$$

Our approach is to generalize this decomposition of markup growth to the multiple output, multiple input case

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Our approach

In period t, a production unit:

- produces combinations of M net outputs, $y^t \equiv [y_1^t,...,y_M^t]$,
- using N primary inputs $x^t \equiv [x_1^t,...,x_N^t] > 0_N$,
- while facing the strictly positive vector of net output prices $p^t \equiv [p_1^t,...,p_M^t] >> 0_M$
- and the strictly positive vector of primary input prices $w^t \equiv [w_1^t,...,w_N^t] >> 0_N$.

The production unit's *observed period t markup* m^t is defined as follows, t =1,…,T:

$$1 + m^t \equiv p^t \cdot y^t / w^t \cdot x^t$$

Our approach

Denote the *period t production possibilities set* for the sector by S^t: $(y,x) \in S^t$

We assume that S^t satisfies the following regularity conditions:

(i) S^t is a closed set;

(ii) for every $x \geq 0_N$, $(0_M,x) \in S^t$;

(iii) if $(y,x) \in S^t$ and $y^* \leq y$, then $(y^*,x) \in S^t$ (free disposability of net outputs);

(iv) if $(y,x) \in S^t$ and $x^* \geq x$, then $(y,x^*) \in S^t$ (free disposability of primary inputs);

(v) if $x \geq 0_N$ and $(y,x) \in S^t$, then $y \leq b(x)$ where the upper bounding vector b can depend on x (bounded primary inputs implies bounded from above net outputs).

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Our approach

The production unit's period t *(Maximum) Markup Function*:

$$M^{t*}(p,w) \equiv \max_{y,x} \{p \cdot y / w \cdot x : (y,x) \in S^t\}$$

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Our approach

Since the observed period t output and input vectors (y^t,x^t) belong to the period t production possibilities set, the observed period t *markup* m^t will satisfy the following inequality:

$$1 + m^t \equiv p^t \cdot y^t / w^t \cdot x^t \leq M^{t*}(p^t,w^t)$$

Assume that there is *no technological regress* in the technology of the production unit over time:

$$S^1 \subset S^2 \; ; \; S^2 \subset S^3 \; ; \; \ldots \; ; \; S^{T-1} \subset S^T \; .$$

Thus the production possibilities sets do not become smaller over time.

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Our approach

We want to utilize the Maximum Markup Function in the time series context without having to undertake econometric estimation of the production functions that could represent the various production possibilities sets S^t.

Thus we will *approximate* the period t production possibilities set S^t by the union of the observed output and input vectors for periods 1,2,…,t: (y^1,x^1), (y^2,x^2), …, (y^t,x^t).

Using this very rough approximation to reality, the period t *Approximate Maximum Markup Function* $M^t(p,w)$ becomes the following function:

$$M^t(p,w) \equiv \max_j \{p \cdot y^j / w \cdot x^j : j = 1,2,\ldots,t\}$$

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Our approach

Thus:

$$M^1(p,w) = p \cdot y^1 / w \cdot x^1;$$

$$M^2(p,w) = \max\{p \cdot y^1 / w \cdot x^1;\ p \cdot y^2 / w \cdot x^2\};$$

$$M^3(p,w) = \max\{p \cdot y^1 / w \cdot x^1;\ p \cdot y^2 / w \cdot x^2;\ p \cdot y^3 / w \cdot x^3\}$$

…and so on.

We want to decompose maximum markup growth into the product of a term that represents

- changes in prices going from period t–1 to period t
- and a term that represents the combined effects of technical progress and changes in returns to scale.

Our approach

There are two decompositions of maximum markup growth that are equally valid. The first decomposition is:

$$M^t(p^t,w^t)/M^{t-1}(p^{t-1},w^{t-1}) = [M^t(p^t,w^t)/M^{t-1}(p^t,w^t)][M^{t-1}(p^t,w^t)/M^{t-1}(p^{t-1},w^{t-1})]$$

$$= \tau_P{}^t \alpha_L{}^t$$

where the *period t Paasche type index of the combined effects of technical progress and returns to scale on markup growth holding prices constant* $\tau_P{}^t$ is defined as:

$$\tau_P{}^t \equiv M^t(p^t,w^t)/M^{t-1}(p^t,w^t) \geq 1$$

The $\tau_P{}^t$ measure of markup change holds prices constant at their period t levels.

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Our approach

$$M^t(p^t,w^t)/M^{t-1}(p^{t-1},w^{t-1}) = [M^t(p^t,w^t)/M^{t-1}(p^t,w^t)][M^{t-1}(p^t,w^t)/M^{t-1}(p^{t-1},w^{t-1})]$$

$$= \tau_P{}^t \alpha_L{}^t$$

$$\tau_P{}^t \equiv M^t(p^t,w^t)/M^{t-1}(p^t,w^t) \geq 1$$

The numerator and denominator of $\tau_P{}^t$ both calculate the maximum markup that is attainable by the production unit if it faces period t output and input prices but:

- the numerator markup uses the period t technology
- while the denominator markup uses the technology of the previous period

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Our approach

$$M^t(p^t,w^t)/M^{t-1}(p^{t-1},w^{t-1}) = [M^t(p^t,w^t)/M^{t-1}(p^t,w^t)][M^{t-1}(p^t,w^t)/M^{t-1}(p^{t-1},w^{t-1})]$$

$$= \tau_P{}^t \alpha_L{}^t$$

The *period t Laspeyres type measure of the effect of output and input price change on markup growth* is:

$$\alpha_L{}^t \equiv M^{t-1}(p^t,w^t)/M^{t-1}(p^{t-1},w^{t-1})$$

$\alpha_L{}^t$ compares the maximum markup in period t to the maximum markup in period t–1 holding the technology constant at the period t–1 levels.

Our approach

The second decomposition is:

$$M^t(p^t,w^t)/M^{t-1}(p^{t-1},w^{t-1}) = [M^t(p^t,w^t)/M^t(p^{t-1},w^{t-1})][M^t(p^{t-1},w^{t-1})/M^{t-1}(p^{t-1},w^{t-1})]$$

$$= \alpha_P{}^t \tau_L{}^t$$

Where:

$$\tau_L{}^t \equiv M^t(p^{t-1},w^{t-1})/M^{t-1}(p^{t-1},w^{t-1}) \geq 1$$

$$\alpha_P{}^t \equiv M^t(p^t,w^t)/M^t(p^{t-1},w^{t-1})$$

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Our approach

Since τ_L^t and τ_P^t are equally relevant measures of generalized technical progress between periods t–1 and t, we use their geometric mean τ^t as our final estimate of ***generalized technical progress***:

$$\tau^t \equiv [\tau_L^t \tau_P^t]^{1/2}$$

Similarly for our final measure of ***price change***:

$$\alpha^t \equiv [\alpha_L^t \alpha_P^t]^{1/2}$$

Decomposition for the rate of growth of the Maximum Markup Function:

$$M^t(p^t,w^t)/M^{t-1}(p^{t-1},w^{t-1}) = \alpha^t \tau^t$$

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Our approach

Following the example of Balk (1998; 143) and Diewert and Fox (2018; 629), we define the *markup efficiency* of the production unit during period t as follows:

$$e^t \equiv [p^t \cdot y^t / w^t \cdot x^t] / M^t(p^t, w^t) \leq 1$$

If $e^t = 1$, then production is allocatively efficient.

Index of the ***change in markup efficiency*** ε^t for the production unit over the two periods as follows:

$$\varepsilon^t \equiv e^t / e^{t-1}$$

Thus if $\varepsilon^t > 1$, then markup efficiency has *improved* going from period t–1 to t whereas it has *fallen* if $\varepsilon^t < 1$.

Our approach

Using the above definitions:

$$[p^t \cdot y^t / w^t \cdot x^t]/[p^{t-1} \cdot y^{t-1} / w^{t-1} \cdot x^{t-1}] = (1+m^t)/(1+m^{t-1})$$

$$= \varepsilon^t \alpha^t \tau^t$$

That is, observed markup growth is decomposed into:

- efficiency change, ε^t;
- price change, α^t;
- and generalized technical progress (including returns to scale)

Our approach

Reorganise to give us a ***growth accounting decomposition*** of nominal net output growth, $p^t \cdot y^t / p^{t-1} \cdot y^{t-1}$:

$$p^t \cdot y^t / p^{t-1} \cdot y^{t-1} = [w^t \cdot x^t / w^{t-1} \cdot x^{t-1}] \varepsilon^t \alpha^t \tau^t$$

Define ***Multifactor Productivity Growth*** going from period t–1 to period t, π^t, as the product of efficiency growth ε^t times generalized technical progress τ^t:

$$\pi^t \equiv \varepsilon^t \tau^t$$

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Our approach

We can also decompose the overall price change index α^t into separate indexes of output price change β^t and input price change γ^t

$$p^t \cdot y^t / p^{t-1} \cdot y^{t-1} = \varepsilon^t \beta^t \tau^t [w^t \cdot x^t / w^{t-1} \cdot x^{t-1}] / \gamma^t$$

$[w^t \cdot x^t / w^{t-1} \cdot x^{t-1}] / \gamma^t$ is nominal input growth divided by an input price index, γ^t.

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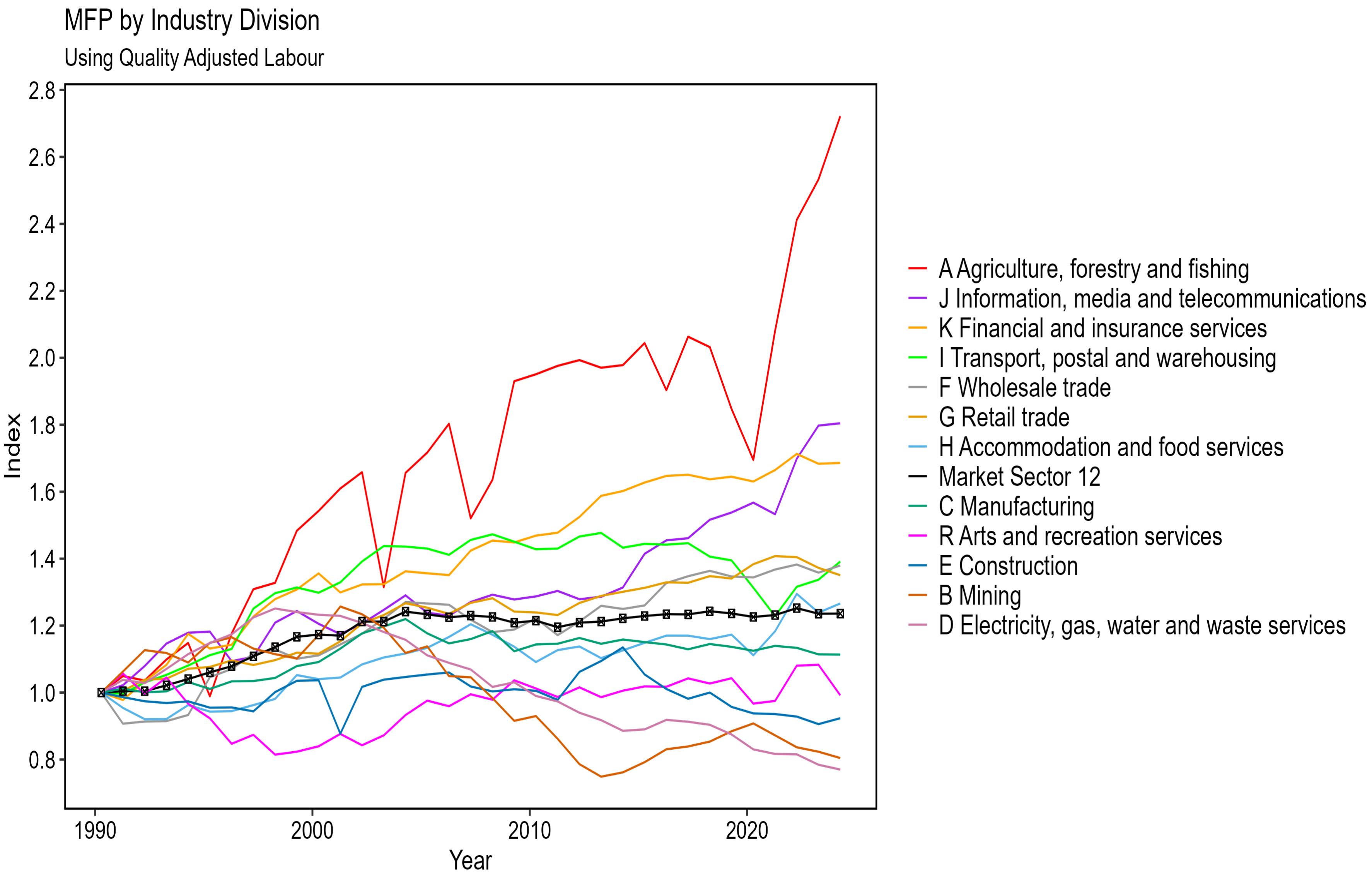
Application to Australian Retail Sector



Application to Australian Retail Sector

Endogenous rate of return used in the user cost to ensure that value added = payments to factors of production. That is $i_{i,t}$ in the following is the internal rate of return (Australian System of National Accounts):

19.80 When tax considerations are given to the measurement of capital rental prices (both capital income taxes and indirect business taxes), the tax-adjusted rental price equation becomes:

$$r_{i,j,t} = T_{i,j,t}(i_{i,t}P_{i,j,t-1} + \delta_{j,t}P_{i,j,t} - \pi_{i,j,t}) + x_{i,t}P_{i,j,t-1} \quad \text{-------} (19.3)$$

where i indexes industries, $T_{i,j,t}$ is the income tax parameter and $x_{i,t}$ is the effective net indirect tax rate on production. The description of data sources for constructing the tax parameter is provided in Annex C.

That is:

$$p^t \cdot y^t = w^t \cdot x^t \ \text{ for all periods}$$

Hence:

$$p^t \cdot y^t / w^t \cdot x^t = (1 + m^t) = 1$$

Need to use an exogenous rate of return in the user cost formula to have non-zero markups, m^t.

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Application to Australian Retail Sector

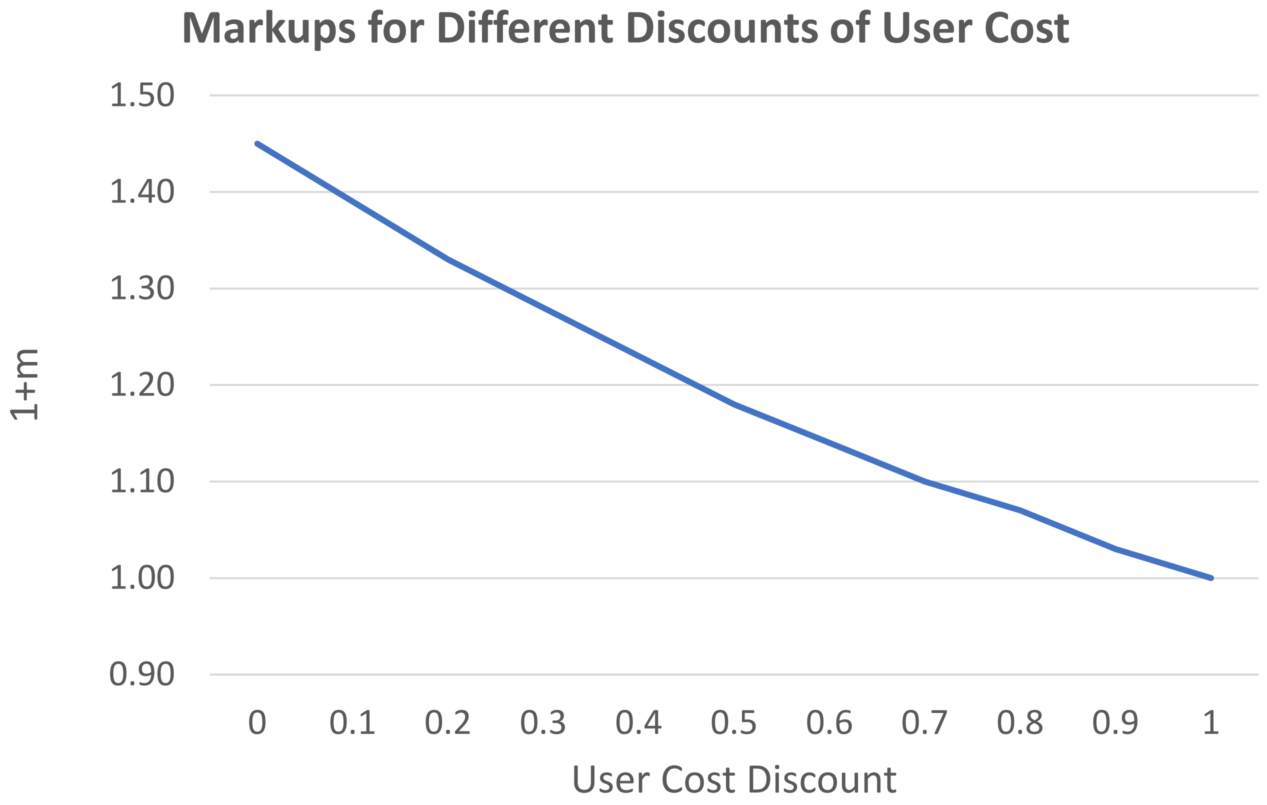
Need to choose an exogenous rate of return, recalculate user cost value for each asset type and create revised capital input aggregate.

Have the (unpublished) user cost data to do this, but have not done it yet!

But as the choice of the exogenous rate of return is somewhat arbitrary, we can carry out experiments using different user costs, considering what is needed in order to get the kinds of markups estimated in the literature.

Application to Australian Retail Sector

Define "discounted" user cost (u) as $u \times d$, where $0 \leq d \leq 1$. Then using data for 2024:



Note that even with $u \times d \approx 0$ markups can't get to levels estimated by de Loecker and colleagues

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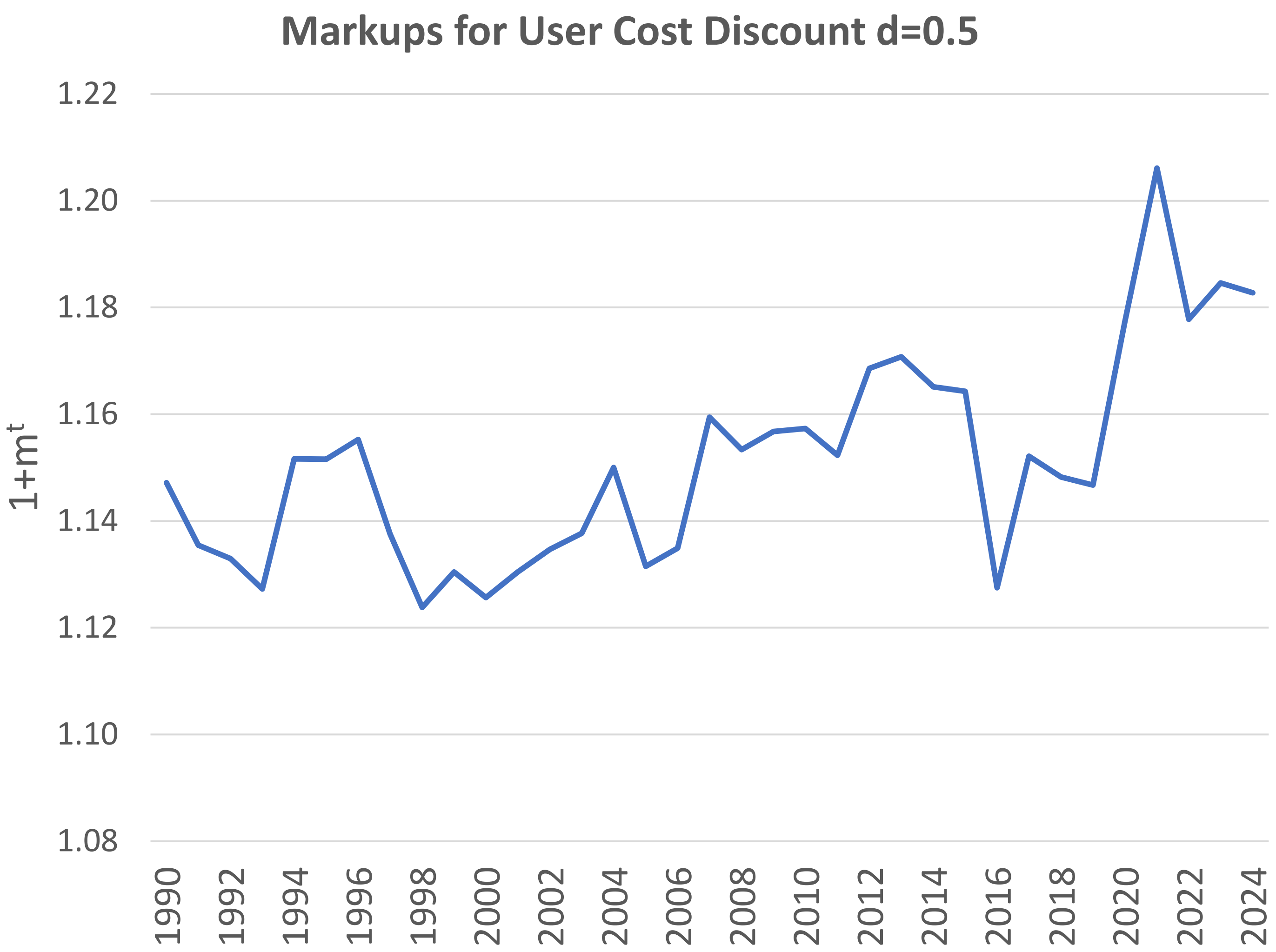
Application to Australian Retail Sector: $m^t > 0$



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Application to Australian Retail Sector

To plot decompositions of interest, define the cumulated variables as follows:

$$MFP^1 \equiv 1; T^1 \equiv 1;\ E^1 \equiv 1; B^1 \equiv 1; G^1 \equiv 1$$

For $t = 2, 3, , \dots, T$, define the preceding variables recursively as follows:

$$MFP^t \equiv \pi^t MFP^{t-1}; T^t \equiv \tau^t T^{t-1}; E^t \equiv \epsilon^t E^{t-1}; B^t \equiv \beta^t B^{t-1}; G^t \equiv \gamma^t G^{t-1}$$

MFP^t = productivity level

T^t = technology and returns to scale level

E^t = efficiency level

B^t = output price level

G^t = input price level

Similarly for the markup – normalise the level to 1 in the first period and cumulate the growth

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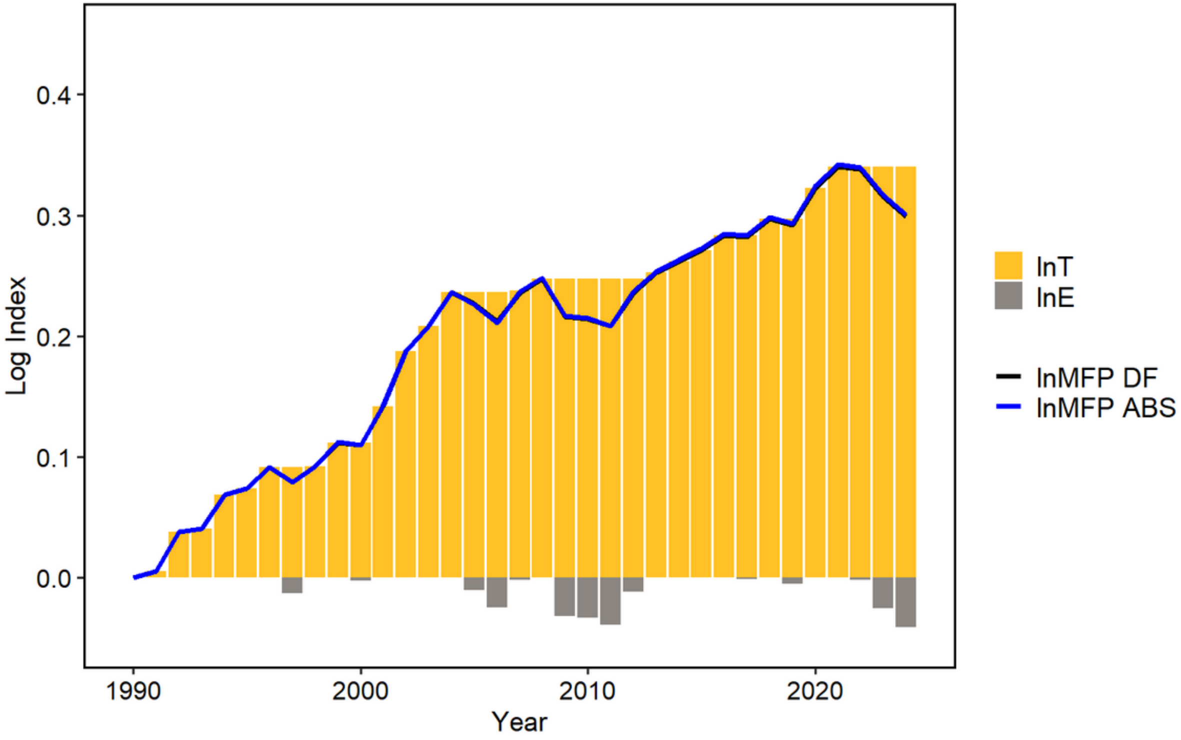
Application to Australian Retail Sector: $m^t = 0$

Productivity decomposition into Technology (T) and Efficiency (E) components:



lnMFP DF is (the log of) multifactor productivity estimated from our markup function method

lnMFP ABS is (the log of) Australian Bureau of Statistics index number based estimate.

Essentially identical!

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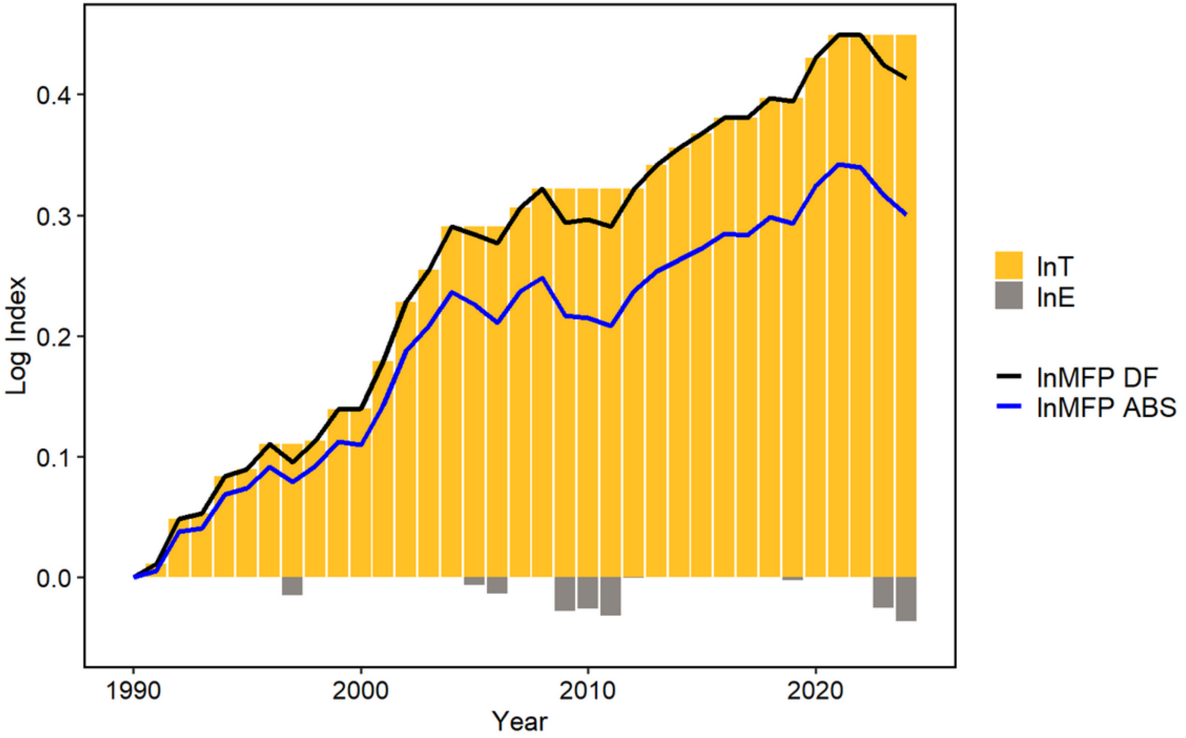
Application to Australian Retail Sector: $m^t > 0$

User cost discount of 0.5. "MFP DF" decomposition into Technology (T) and Efficiency (E) components:

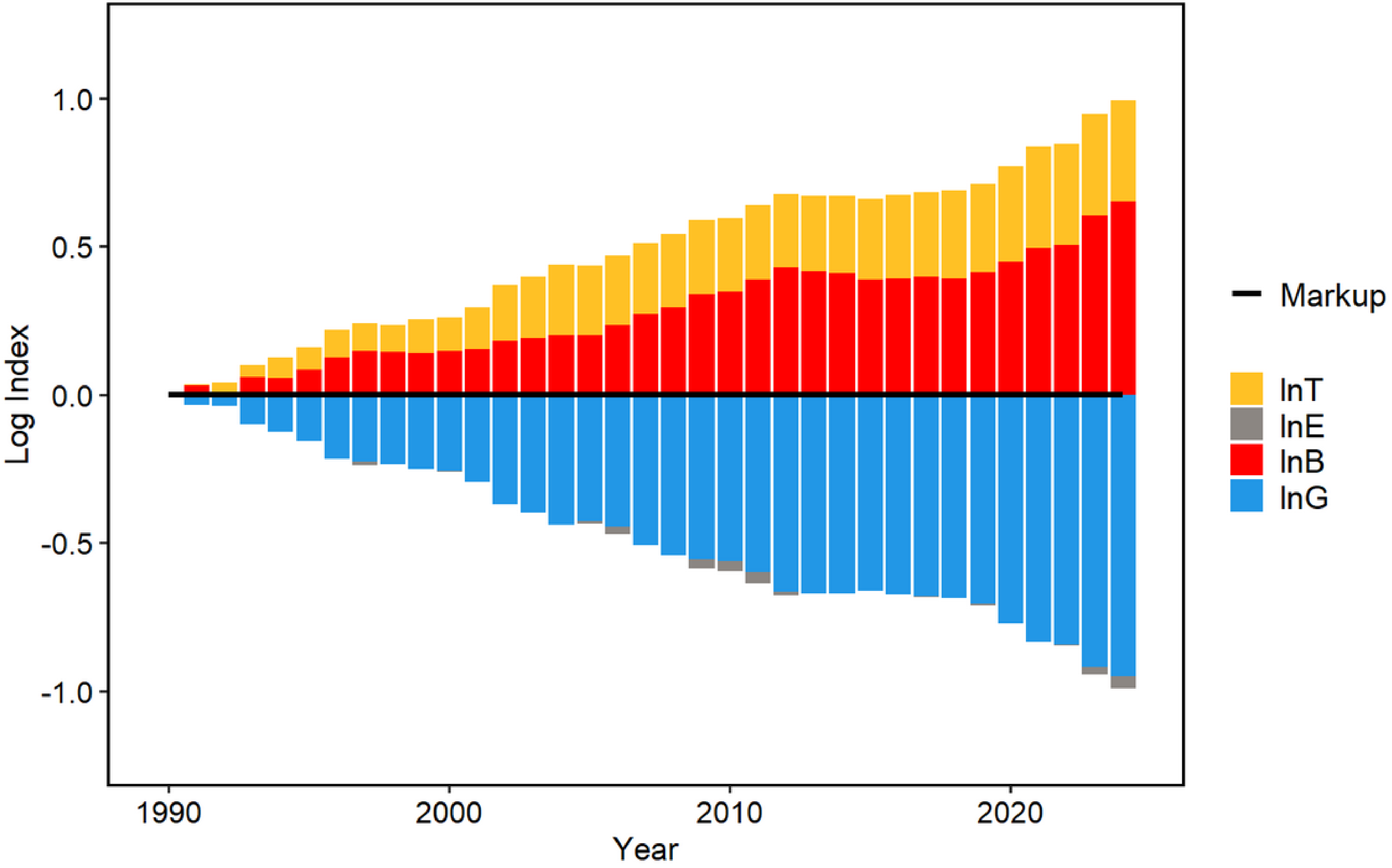


lnMFP DF is (the log of) multifactor productivity estimated from our markup function method

lnMFP ABS is (the log of) Australian Bureau of Statistics index number based estimate.

Quite different!

Application to Australian Retail Sector: $m^t = 0$



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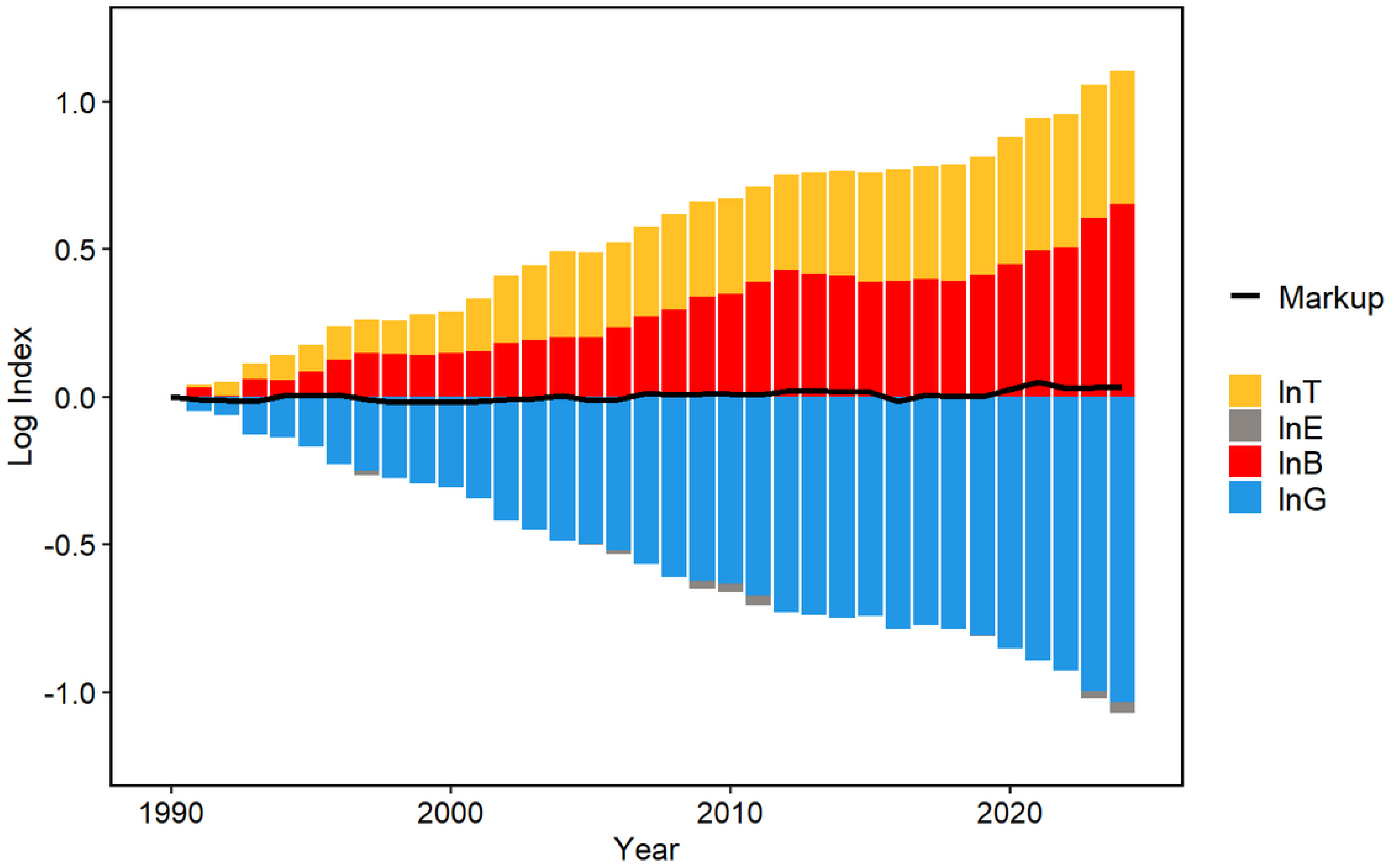
Application to Australian Retail Sector

t



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Summary

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- Past vectors of inputs and outputs for the production unit are used to build up an approximation to the “true” technology
- Our markup growth estimate can be decomposed into contributions from output and input price changes, efficiency change, and technical progress and returns to scale