

Model-Based Multilateral Methods for Estimating Quality-Adjusted Price Indexes

Jan de Haan

UNSW-ESCoE Conference on Economic Measurement
8-9 December 2025, Sydney

Aim of the paper

“Modelling can be a powerful tool for analyzing data and for compiling statistics. This is also true for constructing quality-adjusted price indexes from scanner data. In this paper we review model-based multilateral methods that estimate price indexes simultaneously for multiple periods.”

More quotes

“The combination of model and estimation method should produce a measure of price change with a mathematical structure that can be assessed from the perspective of index number theory.”

“We believe that methods where aggregation cannot be made explicit are not particularly helpful.”

Purely data-driven approaches can lead to misinterpretation if not grounded in a theoretical understanding

Quality and quality differences

“... product quality from the consumer’s perspective in absolute terms is a meaningless notion. It is only meaningful to speak of differences in quality between products; it is relative quality-adjustment parameters that matter and that we can try to estimate.”

Adding up quality-adjusted quantities

Assumption

“... adding up quality-adjusted quantities is justifiable for broadly comparable products (Dalén, 2001; De Haan, 2004; Von Auer, 2014; Chessa, 2016; De Haan and Krsinich, 2018). The change in the sum of quality-adjusted quantities then measures aggregate quantity change.”

(Linear utility function)

What is a quality-adjusted unit value index?

Homogeneous product

Unit value: expenditure divided by quantities sold

Unit value index: ratio of unit values in periods compared

Heterogenous product

Quality-adjusted unit value: expenditure divided by quality-adjusted quantities

Quality-adjusted unit value index: ratio of quality-adjusted unit values

Quality-adjusted unit value index: formula

$$P_{QAUV}^{0t} = \frac{\sum_{i \in S^t} p_i^t q_i^t / \sum_{i \in S^t} \theta_i q_i^t}{\sum_{i \in S^0} p_i^0 q_i^0 / \sum_{i \in S^0} \theta_i q_i^0} = \frac{\left[\sum_{i \in S^t} s_i^t (p_i^t / \theta_i)^{-1} \right]^{-1}}{\left[\sum_{i \in S^0} s_i^0 (p_i^0 / \theta_i)^{-1} \right]^{-1}} = \frac{\left[\sum_{i \in S^t} s_i^t (\tilde{p}_i^t)^{-1} \right]^{-1}}{\left[\sum_{i \in S^0} s_i^0 (\tilde{p}_i^0)^{-1} \right]^{-1}}$$

Ratio of expenditure-share weighted harmonic average of **quality-adjusted prices** (prices divided by quality-adjustment parameters)

Transitive index – not subject to chain drift

A geometric approximation

$$P_{GQAUV}^{0t} = \frac{\prod_{i \in S^t} (p_i^t / \theta_i)^{s_i^t}}{\prod_{i \in S^0} (p_i^0 / \theta_i)^{s_i^0}} = \frac{\prod_{i \in S^t} (\tilde{p}_i^t)^{s_i^t}}{\prod_{i \in S^0} (\tilde{p}_i^0)^{s_i^0}}$$

Ratio of expenditure-share weighted geometric average of quality-adjusted prices – also transitive

$$\frac{P_{GQAUV}^{0t}}{P_{QAUV}^{0t}} \approx \exp \left[\frac{1}{2} \left((CV^t)^2 - (CV^0)^2 \right) \right] \quad (\text{Not in paper})$$

How to estimate quality-adjustment parameters?

“Price is a signal for quality; under optimal conditions, we can expect the prices of broadly comparable products to reveal the consumers’ valuation of the difference in quality (or marginal utility).

.... relative prices are natural measures of the quality-adjustment parameters.”

Average of deflated prices

Relative prices are not constant over time, and so the choice of period matters.

Solution: use all the prices in the periods a product is sold. To make those prices comparable, express them in values for a common period by deflating them by an aggregate price index. Take an average of the deflated prices to obtain the 'best possible' estimate of the quality-adjustment parameter.

Weighted average of deflated prices

Take, for example, a weighted geometric average of deflated prices

$$\theta_i = \prod_{t=0}^T \left(\frac{P_i^t}{P^{0t}} \right)^{w_i^t}$$

But the deflator is unknown

And what weights should be used?

Endogenous approach: Time Product Dummy (TPD)

Weighted TPD: expenditure shares as weights

$$\theta_i = \prod_{t=0}^T \left(\frac{p_i^t}{P_{GQAUV}^{0t}} \right)^{s_i^t / \sum_{t=0}^T s_i^t} \quad \text{where (as before)} \quad P_{GQAUV}^{0t} = \frac{\prod_{i \in S^t} (p_i^t / \theta_i)^{s_i^t}}{\prod_{i \in S^0} (p_i^0 / \theta_i)^{s_i^0}}$$

System of equations for θ_i and $P_{GQAUV}^{0t} = P_{TPD}^{0t}$

Can be solved in iterative fashion

TPD is a **purely endogenous** approach. Where does the name come from?

A simple multiplicative model

Price is the product of the general price level, a product term and an error term

$$p_i^t = P^t \theta_i \mu_i^t$$

P^t / P^0 is the price index (ratio of price levels) to be estimated

Taking logarithms (and setting $\ln \mu_i^t = \varepsilon_i^t$)

$$\ln p_i^t = \ln P^t + \ln \theta_i + \varepsilon_i^t$$

Weighted TPD regression

Dummy variables (fixed effects) for **time** and **product** (plus intercept)

$$p_i^t = \alpha + \sum_{t=1}^T \delta^t D^t + \sum_{i=1}^{N-1} \gamma_i D_i + \varepsilon_i^t$$

$$\alpha = \ln(P^0 \theta_N) \quad \delta^t = \ln(P^t / P^0) \quad \gamma_i = \ln(\theta_i / \theta_N)$$

Confirms we are estimating **relative** quality-adjustment parameters

Weighted Least Squares regression with expenditure shares as weights:
same result as iterative approach

When are conditions non-optimal?

This is when, for some reason, the quality-adjustment parameter estimates, i.e., relative prices, fail to accurately reflect the 'true' quality differences between products

The following two examples illustrate when a fully endogenous approach such as TPD may lead to problems: product relaunches and (some forms of) life cycle pricing

TPD is sensitive to choice of product identifier

Product relaunch: product identifier (such as GTIN/barcode) changes but product remains 'the same' from the consumer's perspective. The 'old' and 'new' products are erroneously treated as different products and their quality-adjustment parameter estimates will be different.

If price is raised during relaunch:

Quality-adjustment parameter estimate for the 'new' product tends to be higher than that for the 'old' product – most likely has a downward effect on the TPD index

Life cycle pricing could adversely affect TPD index

Example of life cycle pricing: price skimming during introduction of a model and inventory cleaning (Silver and Heravi, 2005).

“ The price may initially decline rather slowly and start to decline faster when more buyers are attracted. The issue with TPD then is that the quality-adjustment parameter estimates for recently introduced models would be ‘too high’, and hence the quality-adjusted prices ‘too low’ – the opposite would hold for obsolete models.”

More potential weaknesses and some important advantages

Other potential weaknesses

Products with a single observation across sample period are zeroed out

High product churn: potentially unreliable parameter estimates due to sparse data

Strong points

No omitted variables bias

No need to specify a functional form for quality-adjustment parameters

Sound econometrics?

Possible objections

Choice of regression weights should relate to heteroskedasticity

Expenditure shares are endogenous (they depend on price)

Least Squares may not be the best estimation option

Assumptions on the error terms are required (e.g., to test statistical significance of parameter estimates)

Fixed effects is a panel method, but here the panel is 'unbalanced'

Maybe econometrics is not the issue

Reply

Econometrics assume the model is 'true'. The errors and hence prices are stochastic due to sampling or measurement error

TPD is not really a stochastic approach (?)

- Index number theory is leading
- Sampling is irrelevant – given the data set, the estimation method should produce an index that 'makes sense'
- The data have no measurement error (by assumption)

Auxiliary information: Time Dummy Hedonic

Addresses TPD weaknesses by using product characteristics rather than product dummy variables in log-linear model

Expenditure-share Weighted Least Squares regression

- Leads to ratio of weighted geometric quality-adjusted prices, like weighted TPD, but
- **Prices are explicitly quality-adjusted** using exogenous/auxiliary information

Auxiliary information: Time Dummy Hedonic

Drawbacks

Omitted variables bias – TDH index will be upward biased if quality is improving

This is an interesting point. In statistics, auxiliary information is often used to improve estimates. However, TPD (which does not make use of any auxiliary information) has no omitted variables bias.

Linear combination of explanatory variables – a requirement for least squares regression – may not be optimal

Example 1 from previous research

De Haan, Hendriks and Scholz (2019): packaged men's t-shirts

Scanner data Dutch department store

49 months

Product price: monthly unit value across stores

Product identifier: GTIN

weighted TPD; weighted TDH; chained Törnqvist

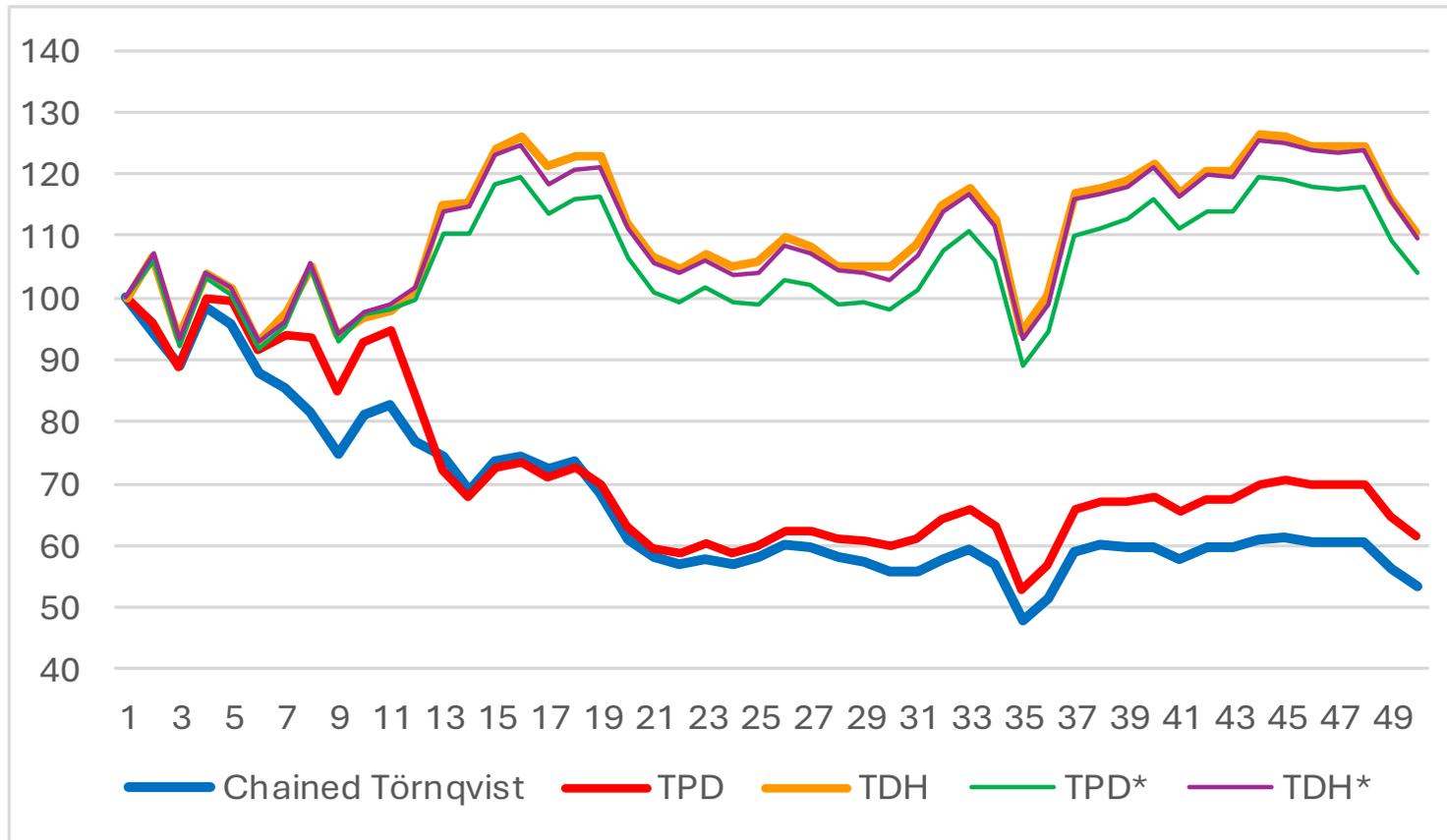
Product identifier: cluster via cross-classification of categorical variables

weighted TPD*; weighted TDH*

Example 1: results

- Chained Törnqvist downward biased – chain drift
- Weighted TPD downward biased due to lack of matching when the fabric of many t-shirts changed to ‘organic’
- Weighted TPD* resolves this issue
- Weighted TDH* and TDH very similar but perhaps somewhat upwards biased – omitted variables

Men's t-shirts



Example 1: rebased index

Shows last 35 months, after introduction organic t-shirts (when a lack of matching occurred)

Product identifier: GTIN

Includes unit value index

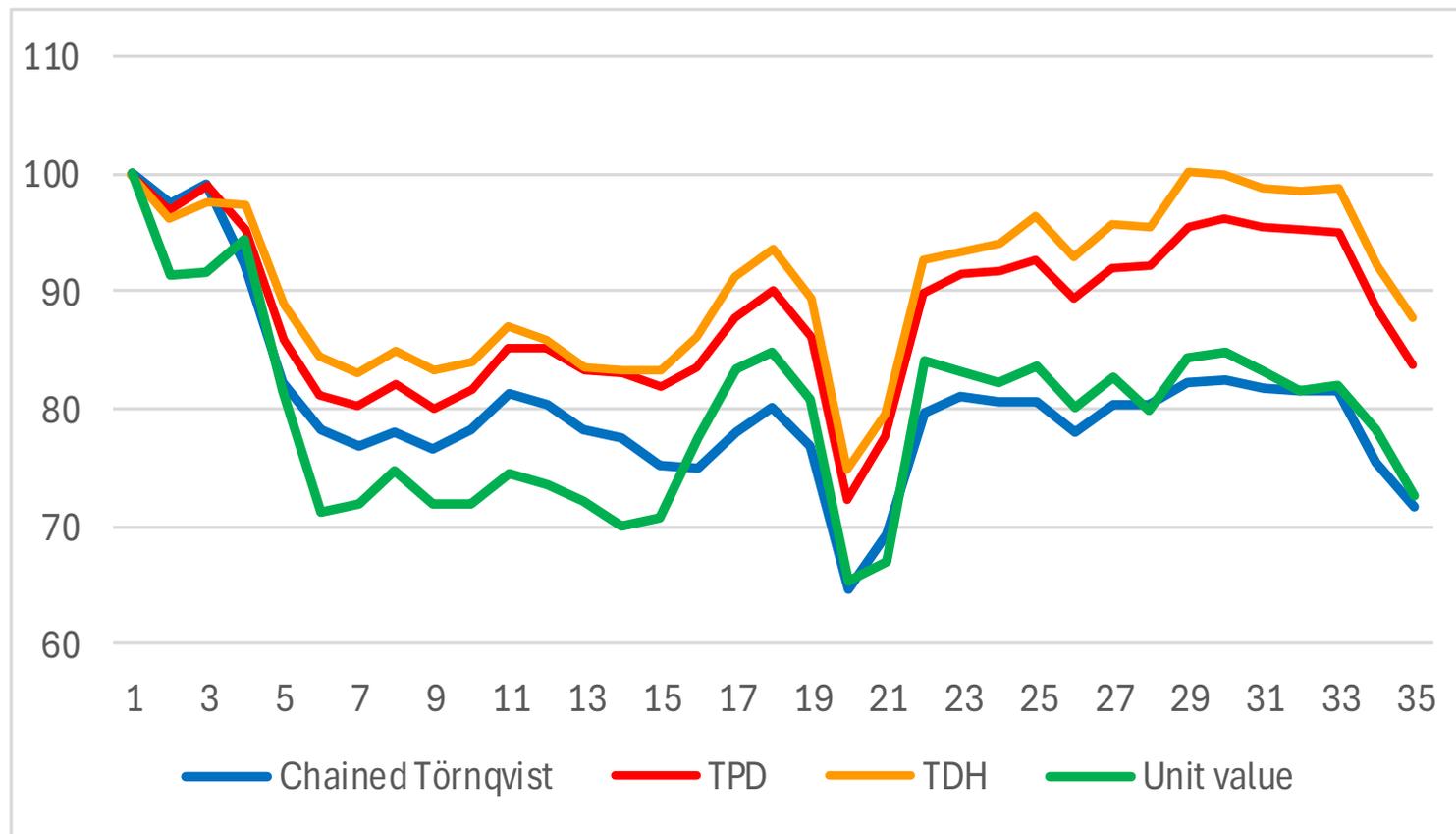
Product churn in GTIN terms not very high in this sub-period

Weighted TPD seems to work well

Weighted TDH slightly upwards biased due to omitted variables?

Quality-adjustment raises index

Men's t-shirts, rebased



What about GEKS-Törnqvist?

Additivity is not supported (in general) by economic theory

Alternative to quality-adjusted unit value index: **quality-adjusted GEKS-Törnqvist index**

- Imputes 'missing prices' of unmatched products in bilateral Törnqvist price indexes using hedonics
- Estimation method does not affect index formula – model-assisted rather than model-based approach
- Transitive

Hedonic imputation GEKS-Törnqvist

De Haan and Krsinich (2014)

- Bilateral TDH method
- Regression weights

average of expenditure shares in the periods compared

which means half of expenditure shares for unmatched products
in the periods they are sold

Produces bilateral hedonic imputation Törnqvist price indexes

Down-weights unmatched items compared with standard bilateral
weighted TDH

Example 2 from previous research

De Haan and Daalmans (2022): TVs

Scanner data Dutch department store

17 months

Product price: monthly unit value across stores

Product identifier: GTIN

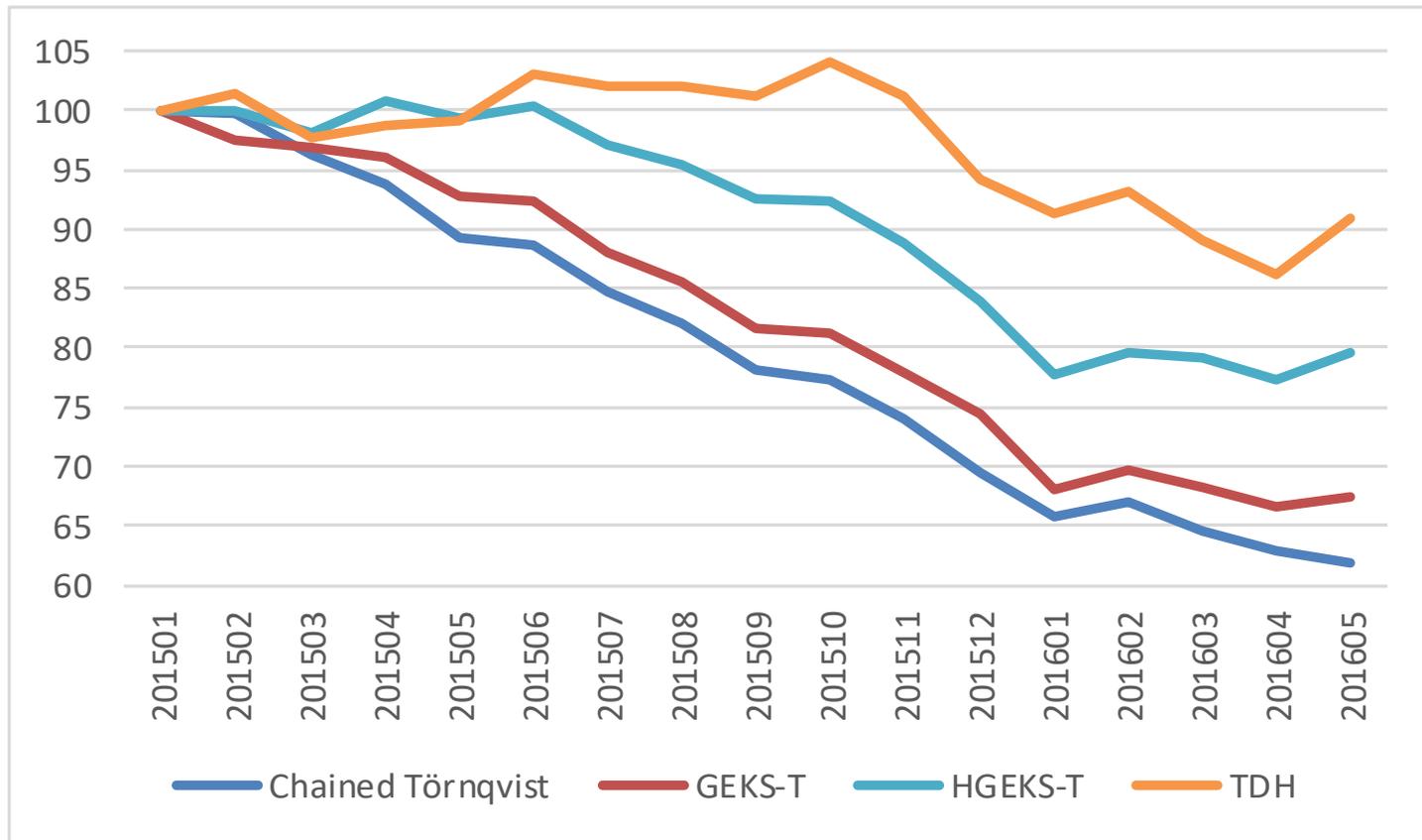
GEKS-Törnqvist, hedonic imputation GEKS-Törnqvist, chained Törnqvist,
weighted TPD

Example 2: results

- Chained Törnqvist index downward biased
- GEKS-Törnqvist index most likely downward biased
- Hedonic imputation GEKS-Törnqvist (De Haan-Krsinich type)
 - much lower than weighted TPD index – probably due to down-weighting unmatched products
 - Imputations raise index (as compared with GEKS-Törnqvist)
- Weighted TDH index possibly upward biased

We should have estimated weighted TPD, but data were unavailable to us

TVs



Is weighted TPD useful?

From a practical perspective, weighted TPD is a very useful model-based multilateral method

- Only requires prices, quantities or expenditures, and a product identifier
- No omitted variables bias – product fixed effects comprise the contributions of all price-determining characteristics, including unobservable ones

Very useful, but

there are **some issues**

- Approximately additive – products should be broadly comparable
- Dependent on choice of product identifier
- Endogenous approach – may not work well under non-optimal conditions (i.e., when relative prices do not accurately reflect quality differences)
- Needs enough matches in the data – difficult to say what the minimum number is

Want to know more?

Consumer Price Index: Theory (Diewert, 2025)

Thank you