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ESCoE Discussion Paper 2026-03

February 2026

ISSN 2515-4664

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Keywords: Pollution, green accounting, externalities, welfare, productivity

JEL classification: Q51, D24, D62

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Published by:
Economic Statistics Centre of Excellence
King's College London
Strand
London
WC2R 2LS
United Kingdom
www.escoe.ac.uk

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I. The Consumer Approach to Environmental Bads

Weitzman (1976) provided a welfare interpretation for net national product. This has since been used extensively in the growing literature on environmental economic accounting to measure the negative welfare effects of pollution, environmental degradation, and other environmental bads. In addition to the restrictiveness of assumptions used by Weitzman (closed economy, no government, no technical progress, no tax distortions), the problematic lack of general discrete time counterparts to his results derived from continuous time optimization has been shown (Diewert and Schreyer 2006).[1] There exists, however, an additional problem – the Weitzman approach cannot fully capture the welfare effects of externalities. This makes it unsuitable for drawing connections between welfare and net national product in any context where externalities exist.

Weitzman's justification of net national product from a welfare perspective is that it is a proxy for the present discounted value of future consumption, and that this can be obtained by capitalizing current consumption and net investment (using a constant real interest rate). Subsequent interpretations of this result in the literature on environmental economic accounting relate to net investment including environmental depletion and degradation. Weitzman's results depend on the levels of consumption and net investment being choices made by consumers. If the net investments include externalities (e.g. environmental degradation) that are not chosen by the consumers yet affect the consumption choices that consumers make, then Weitzman's results will not hold.

We demonstrate this in the case of environmental externalities as follows. Suppose consumer utility u is a function of market goods and services, the N dimensional vector x and a K dimensional vector of environmental "bads" b (such as pollution) so that $u = f(x,b)$. Let p be the positive vector of prices for market goods and services and let e be the "income" or expenditure on market products in a given period. The consumer's utility maximization problem is:

$$(1)\ \max_x \{f(x,b) : p\cdot x = e\}.$$

[1] Others who have raised problems with the Weitzman approach include Dasgupta and Mäler (2000).

Note that the maximization is by the choice of the vector x, and b is taken as given, as consumers do not get to choose the vector of bads, b. The first order necessary conditions for an interior solution to the utility maximization problem are given by (2):

(2) $\nabla_x f(x,b) = \lambda p$;

(3) $p \cdot x = e$.

Premultiply both sides of (2) by x^T and solve the resulting equation for $\lambda = x^T \nabla_x f(x,b)/e$ where we have used (3) to derive this expression for λ. Substitute this expression into (2) and we obtain the following system of estimating equations for the consumer's system of (inverse) demand functions:

(4) $p = e\, \nabla_x f(x,b)/x^T \nabla_x f(x,b)$.

It is not possible to use equations (4) to estimate a fully flexible functional form for f(x,b).[2] To see this, suppose f(x,b) is a linear function of x and b. Then the marginal utility parameters of the linear function for the components of b do not enter the estimating equations (4) so it is not possible to recover the underlying preferences over combinations of goods and bads using a consumer framework.

Essentially, as the maximization of utility through choice of market goods and services is conditional on the level of environmental bads, we do not know how consumers will change their consumption choices with changes in the environmental externalities they face. A consequence of this is that Weitzman's connection between welfare and net national product is lost in the presence of externalities.[3]

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[2] A functional form is "flexible" if it can provide a second order approximation to an arbitrary twice differentiable linearly homogeneous function at a point (Diewert, 1974; p. 113).

[3] This result is analogous to the problem of estimating unconditional equivalence scales identified by Pollak and Wales (1979): "Even if all families have identical unconditional preferences, conditional equivalence scales estimated from observed differences in the consumption patterns of families with different demographic profiles cannot be used to make welfare comparisons; for example, we cannot use such data to determine the amount needed to make families with three children as well off as those with two children and $12,000. Unconditional equivalence scales are required to make welfare comparisons."

II. Switching to the Producer Perspective

The situation is more promising if we use a producer approach to measure the effects of bads on production. From a welfare perspective, we want good outputs to be maximized while minimizing bad outputs. However, the framing of the producer's cost minimization problem when bads are produced as a by-product of production is not completely straightforward. We consider three possible approaches to the treatment of bads in the production accounts of a firm or production unit.

A. Baseline Approach: No taxes on Bads

Let y be a vector of outputs that can be produced by a vector of inputs x. Denote the set of feasible inputs that can produce the vector of outputs y as S(y). If $x \in S(y)$, then the vector of outputs y can be produced by the vector of inputs x. As a by-product of using the vector of inputs x in producing outputs, a vector of bads,[4] b, is also produced. Hence we have:

$$
\begin{aligned}
(5)\quad & b_1 = g_1(x); \\
& b_2 = g_2(x); \\
& \quad \ldots. \\
& b_K = g_K(x)
\end{aligned}
$$

where the bads production functions $g_k(x)$ are in principle observable.[5] We assume that when x is zero each b_k is zero.[6] We summarize equations (5) with the vector equation, $b = G(x) \equiv [g_1(x), g_2(x), \ldots, g_K(x)]^T$ where z^T denotes the transpose of a vector z. If bads are not taxed or regulated, the producer's cost minimization problem is defined as follows:

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[4] Färe and Grosskopf (1983; 173) introduced the term "bads" to describe environmental pollutants.

[5] For example, let $x = x_1$ where x_1 is the amount of oil or other fossil fuel that is used in production. In this case, the vector b could represent the amount of various pollutants that are produced by the production unit under consideration that can be associated with the utilization of x_1 units of oil used by the unit during the time period under consideration. In general, it is not an easy task to determine these empirical bads production functions.

[6] Shephard and Fare (1974) proposed the concept of null jointness, where bads must be produced when goods are produced. Our approach encompasses this possibility without restricting our set up to have only this type of production.

(6) $\min_x \{w \cdot x : x \in S(y)\} \equiv C(y,w)$

where w is a vector of positive input prices. If the joint cost function C(y,w) is differentiable with respect to the components of w, then we can apply Shephard's (1953; 11) Lemma and get the vector of cost minimizing inputs x(y,w) as the following vector of first order derivatives with respect to the components of w:

(7) $x(y,w) = \nabla_w C(y,w)$.

The "optimal" vector of bads produced by the production unit, b(y,w) is obtained by substituting (7) into (5):

(8) $b(y,w) \equiv G[x(y,w)]$.

If the production unit is behaving in a competitive manner, then output prices, p, should be approximately equal to marginal costs. Then if the joint cost function is differentiable with respect to outputs y, we obtain the following equations:

(9) $p(y,w) = \nabla_y C(y,w)$.

Equations (7) and (9) become a system of estimating equations for the competitive case after a functional form for the joint cost function C(y,w) has been specified. The bads production functions $b = G(x)$ are estimated separately.

B. Bads are Taxed

We now consider the case where bads are taxed. Let τ be a vector of tax rates on the production of bads; i.e., $\tau \equiv [\tau_1,\ldots,\tau_K]$ where τ_k is the tax on one unit of production of the k^{th} bad, b_k, for $k = 1,\ldots,K$. The taxation of bads dramatically changes the producer's cost minimization problem, even if the tax rates are modest. With positive taxes on pollutants, the producer now has a positive incentive to reduce production of pollutants. This may lead to the use of new inputs that curtail production of certain pollutants or to the dropping from production certain pollution intensive outputs or to the production of new outputs that are less pollution intensive. In order to minimize notation, we denote the new vector of outputs by y, the new vector of inputs by x and the new vector of bads by b. The new production possibilities set of inputs that are associated with the production of the vector of outputs y is again denoted by S(y). The new set of functions G(x) that relate the production of bads b to the set of inputs x that are used by the production unit is given by equations (10):

(10) $b = G(x)$.

This treatment of bad outputs is broadly consistent with the framework pioneered by Färe and Grosskopf who maintained that "bad" outputs could not simply be treated as ordinary "good" outputs or as ordinary inputs; they argued for an asymmetric treatment of good and bad outputs.[7]

In this section, we assume that the functions $G(x) = [g_1(x), \ldots, g_K(x)]^T$ are known; i.e., if the vector of "regular" inputs is known, then the resulting vector of outputs y and the vector of pollutants $b \equiv G(x)$ can be calculated. The production unit's new cost minimization problem is:

(11) $\min_{x,b} \{w \cdot x + \tau \cdot b : x \in S(y) ; b = G(x)\} = \min_{x} \{w \cdot x + \tau \cdot G(x) : x \in S(y)\}$.

The new joint cost function minimization problem defined by (11) has a potentially nonlinear objective function so the usual duality theory results do not go through for the above cost minimization problem.

Suppose the production of bads functions, $g_1(x), g_2(x), \ldots, g_K(x)$ are linear functions of x; i.e., we have:

(12) $g_k(x) = \alpha_k \cdot x$; $k = 1,\ldots,K$

[7] See Färe and Grosskopf (1984) (2004) and Färe, Grosskopf, Lovell and Pasurka (1989).

where α_k is a nonnegative, nonzero vector of constants for each k. Under these conditions, the cost minimization problem defined by (11) becomes the following problem:

$$(13)\ \min_x \{w\cdot x + \tau\cdot G(x): x\in S(y)\} = \min_x \{w\cdot x + \textstyle\sum_{k=1}^{K} \tau_k\alpha_k\cdot x: x\in S(y)\}$$
$$= \min_x \{w^*\cdot x : x\in S(y)\} \qquad \text{using (12) and (14)}$$
$$\equiv C(y,w^*)$$

where w^* is defined as

$$(14)\ w^* \equiv w + \textstyle\sum_{k=1}^{K} \tau_k\alpha_k\cdot .$$

In the competitive case, the usual estimating equations (7) and (9) can now be applied where w is replaced by the tax adjusted input prices w^* defined by (14). The application to Diewert and Fox (2018) nonparametric measurement of TFP or to the Diewert and Morrison (1986) translog methodology is reasonably straightforward using this framework.[8]

Typically, pollution bads can be associated with inputs of various types of energy that are burned such as coal, natural gas or oil. If there are no other pollutants associated with the production unit under consideration, then only a few τ_k will be positive and the associated α_k vectors will have only one positive component which should be determined by engineering studies. In practice, governments will simply impose a direct pollution tax on the quantity of the energy input that is purchased by the production unit.[9]

It is important for governments to impose even very small pollution taxes on inputs that are associated with the production of pollutants because this will force firms to introduce pollutants accounting into their accounting information framework. This in turn will lead to more accurate information on the period by period production of bads by firms. And of course, even small taxes on bads will lead firms to produce less bads. Improved information on the production of bads will

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[8] Färe and Grosskopf (1983; 176) observed that "bads" do not satisfy the usual free disposal assumption that are usually made in production theory. This valid observation means that we need to assume that the bads production functions $g_k(x)$ are linear and remain constant over time in order to apply these frameworks for measuring TFP.

[9] Governments could also impose physical emission limits rather than taxes, or use limits on some bads and taxes on others. Physical limits would constrain the set of functions G(x) that relate the production of bads b to the set of inputs x, but would not affect the general framework presented. Similarly for technical change in G(x), either autonomous or as part of government regulation.

be helpful to scientists, medical researchers and economists who are interested in measuring the influence of bads on aggregate welfare.

C. The Comprehensive Cost Function

There is another possible treatment of bads in a production function context and that is to combine the constraints $x \in S(y)$ and $b = G(x)$ into a comprehensive set of x and b combinations that can produce the output vector y. Call this comprehensive set of (x,b) combinations the set $S^*(y)$.[10] Using this comprehensive set, the cost minimization problem (11) becomes:

$$(15)\ \min_{x,b}\ (w \cdot x + \tau \cdot b: (x,b) \in S^*(y)\} \equiv C^*(y,w,\tau).$$

Shephard's (1953, 11) Lemma can be applied to the new cost function defined by (15) so if $C^*(y,w,\tau)$ is differentiable with respect to the components of w and τ, we obtain the following equations that define the cost minimizing demands for inputs x and "supplies" of bads b:

$$(16)\ x(y,w,\tau) = \nabla_w C^*(y,w,\tau)\ ;$$
$$(17)\ b(y,w,\tau) = \nabla_\tau C^*(y,w,\tau)\ .$$

Equations (16) and (17) show that bads can be treated in the same way as inputs are treated in an econometric study that estimates input demand equations.[11] As in Section A above, if the production unit is behaving in a competitive manner, then output prices, p, should be approximately equal to marginal costs. If the comprehensive cost function is differentiable with respect to outputs y, we obtain the following equations which could be used in an econometric study:

$$(18)\ p(y,w,\tau) = \nabla_y C^*(y,w,\tau).$$

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[10] This framework was used by Brandt, Schreyer and Zipperer (2014; 26).
[11] When treating bads in the production accounts of the international System of National Accounts, we recommend treating bads as an intermediate input.

Pittman (1981) (1983) used a variant of this model along with the growth accounting framework developed by Caves, Christensen and Diewert (1982).

There are some possible problems associated with the use of the model defined by (15)-(17). The cost minimization problem (15) does not capture the fact that there is a special structure to the set S*(y) which is exhibited in (11) but not in (15). This special structure creates problems for the usual nonparametric approaches to productivity and efficiency analysis that do not make use of price information.[12] However, in our framework which assumes competitive cost minimizing behavior on the part of producers, there are no problems in applying equations (16) and (17), provided the minimum in (15) exists and the resulting joint cost function is differentiable with respect to input prices. Convexity or free disposability assumptions on S*(y) are not required in order to show that $C^*(y,w,\tau)$ is concave and linearly homogeneous in w,τ.[13] However, there is a problem with applying our cost function approach to firm level data.[14] When a firm adopts a new technology which reduces the production of bads, typically there will be new inputs that are utilized by the firm. This creates a problem when indexes of input and output are constructed to measure the productivity of the firm moving from period $t-1$ to period t: there will be a missing price problem (in period $t-1$) for the new inputs that are introduced into period t and for the old inputs used in period $t-1$ but no longer used in period t. These missing prices need to be imputed in order to apply the economic approach to index number theory.[15] This problem will not show up using aggregated data.

The measurement framework proposed in the present section just requires information on the firm's production of bads, b, and the corresponding tax rates, τ. The approach explained in the previous section required additional information on the bads production functions, the $g_k(x)$ functions, or in the case of linear functions $g_k(x)$, information on the α_k parameters.

[12] See Färe and Grosskopf (1983) on this point.
[13] See Hotelling (1935), McFadden (1966) and Diewert (1974) on this point.
[14] Our approach is similar to the approach of Brandt, Schreyer and Zipperer (2014) when bads are taxed.
[15] See Freeman, Inklaar and Diewert (2021) on this missing price problem.

III. Conclusion

We have shown that the Weitzman (1976) consumer approach to modelling bads and justifying the use of net national product as a welfare measure, fails in the presence of externalities. We laid out three alternative ways of modeling bads from the producer perspective.

Reviewing these approaches, our conclusion is that treating environmental bads as inputs into production is probably not the best way to organize the data. It may be better to treat the bads as intermediate inputs, i.e., make the output aggregate an aggregate of the prices (p, τ) and quantities (y, $-b$). This recognizes that the vector of bads is actually produced by the production sector. This approach creates an aggregate output variable from which environmental bads have been netted out, consistent with the idea of net product, but without relying on Weitzman's consumer-based approach as a justification.

This net product approach to the treatment of bads is broadly consistent with the approaches of Pittman (1981) (1983) and Brandt, Schreyer and Zipperer (2014). It avoids most of the problems raised by the work of Färe and Grosskopf and their coauthors because we assume input and output price information is available and we assume cost minimizing behavior on the part of producers.

The net product approach captures some aspects of the overall welfare measurement problem in that the production of bad outputs is accounted for on the supply side, but the tradeoffs on the consumer side are not addressed. Accounting for these tradeoffs is a difficult problem. More generally, there is another problem which needs to be addressed for a full economic accounting of the welfare effects of environmental bads: the tax revenue which is raised by taxing bads can be used by the government to either reduce other taxes or to provide public goods. Thus, in addition to the problem of trying to quantify the effects on the welfare of the population of a country in reducing the production of bads, we have the problem of trying to estimate the marginal excess burdens of alternative taxes in order to determine the "best" use of tax revenues raised by taxing bads; see McKitrick (1997) and Jorgenson, Goettle, Ho and Wilcoxen (2013). A first step in addressing these problems is for governments to impose taxes on bads.

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