Challenges in Consumer Price Index Construction

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Introduction

The purpose of this paper is twofold:

- **Give readers a brief introduction to the various approaches to index number theory that are used to guide the construction of Consumer Price Indexes. This overview should be useful to researchers, academics and National Statistical Office employees who may not know much about the various approaches to index number theory that have been proposed and used over the last 150 years.**
- **The second main purpose is to discuss problems that compilers of a CPI face where the existing index number theories give conflicting advice or inadequate advice.**
- **The first third of the paper provides an overview of the 4 main approaches to bilateral index number theory : (i) Basket Approaches; (ii) Stochastic Approaches; (iii) Test Approaches and (iv) Economic Approaches.**

Introduction

- **A big problem with bilateral index number theory is that it does not address the problems that arise when constructing price indexes over more than two periods. This leads us into the second third of the paper which discusses various approaches to multilateral index number theory.**

- **The final third of the paper provides a brief discussion of 12 additional problems that arise when constructing a Consumer Price Index.**

2.2 Fixed Basket Approaches

- **Lowe (1823; 333-335) proposed that the government should form a representative annual basket $q \equiv (q_1,\ldots,q_N)$. If the prices of year t were $p^t \equiv (p_{t1},\ldots,p_{tN})$ and the prices of the base year 0 were $p^0 \equiv (p_{01},\ldots,p_{0N})$, then the Lowe price index which defines the aggregate level of prices in year t relative to the level of prices in year 0 as $P_{Lo} \equiv p^t \cdot q/p^0 \cdot q$. The Lowe index compares the cost of the same fixed basket of goods and services for the two years in question and it defines the price index as the ratio of the resulting costs.**
- **As time passed, economists and price statisticians demanded a bit more precision with respect to the specification of the basket vector q. There are two natural choices for the reference basket: the period 0 commodity vector q^0 or the period t commodity vector q^t.These two choices lead to the Laspeyres (1871) price index $P_L \equiv p^t \cdot q^0/p^0 \cdot q^0$ and the Paasche (1874) price index $P_P \equiv p^t \cdot q^t/p^0 \cdot q^t$.**

2.2 Fixed Basket Approaches

- **The problem with the Laspeyres and Paasche index number formulae is that they are equally plausible but in general, they will give different answers. This suggests that if we require a single estimate for the price change between the two periods, then we should take some sort of evenly weighted average of the two indexes as our final estimate of price change between periods 0 and 1.**
- **An examples of such symmetric averages are the arithmetic mean and the geometric mean, which leads to the Fisher (1922) ideal index, P_F, defined as the square root of the product of the Laspeyres and Paasche indexes, $[P_L P_P]^{1/2}$.**
- **The problem with the arithmetic mean of the Paasche and Laspeyres indexes is that the resulting index does not satisfy the Time Reversal Test. Thus if $P(p^0,p^t,q^0,q^t)$ denotes a generic index number formula between periods 0 and t going forward from period 0, then we would like the formula to satisfy $P(p^t,p^0,q^t,q^0) = 1/P(p^0,p^t,q^0,q^t)$. The Fisher index does satisfy the time reversal test. Moreover, it is the *only* index that is a homogeneous symmetric average of the Laspeyres and Paasche price indexes, P_L and P_P, that also satisfies the Time Reversal Test.**

2.3 Stochastic Approaches

- **The basic idea behind the unweighted stochastic approach is that each price ratio or price relative, p_{tn}/p_{0n} for $n = 1,2,\ldots,N$, can be regarded as an estimate of a common inflation rate α between periods 0 and t; i.e., it is assumed that**

(7) $p_{tn}/p_{0n} = \alpha + \varepsilon_n \;\; ; \;\; n = 1,2,\ldots,N$

- **The least squares estimator for α is the Carli (1764) price index P_C defined as follows:**

(8) $P_C(p^0,p^t) \equiv \sum_{n=1}^{N} (1/N)(p_{tn}/p_{0n})$.

- **Unfortunately, P_C does not satisfy the Time Reversal Test, i.e., $P_C(p^t,p^0) \neq 1/P_C(p^0,p^t)$.**
- **Now assume that the logarithm of each price relative, $\ln(p_{tn}/p_{0n})$, is an unbiased estimate of the logarithm of the inflation rate between periods 0 and t, β say. Thus we have:**

2.3 Stochastic Approaches

(9) $\ln(p_{tn}/p_{0n}) = \beta + \varepsilon_n$; $n = 1,2,\ldots,N$

- where $\beta \equiv \ln\alpha$ and the ε_n are independently distributed random variables with mean 0 and variance σ^2.
- The least squares or maximum likelihood estimator for β is the logarithm of the geometric mean of the price relatives. Hence the corresponding estimate for the common inflation rate α is the Jevons (1865) price index P_J defined as:

(10) $P_J(p^0,p^t) \equiv \prod_{n=1}^{N} (p_{tn}/p_{0n})^{1/N}$.

The Jevons price index P_J does satisfy the Time Reversal Test and hence is much more satisfactory than the Carli index P_C. However, both the Jevons and Carli price indexes suffer from an important flaw: each price relative p_{tn}/p_{0n} is regarded as being equally important and is given an equal weight in the index number formulae (2) and (4).

2.3 Stochastic Approaches

- **Theil (1967; 136-137) proposed a solution to the lack of weighting in (8) and (10). Theil's final measure of overall logarithmic price change was**

(11) $\ln P_T(p^0,p^t,q^0,q^t) \equiv \sum_{n=1}^{N} (1/2)(s_{0n} + s_{tn}) \ln(p_{tn}/p_{0n})$.

- **Taking the exponential of both sides of (11) leads to the *Törnqvist-Theil price index*, $P_T(p^0,p^t,q^0,q^t)$.**
- **There are many more stochastic approaches to index number theory that are available to price statisticians but for our purposes, we will regard $P_T(p^0,p^t,q^0,q^t)$ as a "best" index number formula from the viewpoint of the stochastic approach to index number theory.**

2.4 The Test Approach to Bilateral Index Number Theory

- **In this section, our goal is to assume that the bilateral price index formula, $P(p^0,p^1,q^0,q^1)$, satisfies a sufficient number of "reasonable" tests or properties so that the functional form for P is determined.**
- **The word "bilateral" refers to the assumption that the function P depends only on the data pertaining to the two situations or periods being compared; i.e., P is regarded as a function of the two sets of price and quantity vectors, p^0,p^1,q^0,q^1, that are to be aggregated into a single number that summarizes the overall change in the N price ratios, $p_1^1/p_1^0,\ldots, p_N^1/p_N^0$. Note that the price (and quantity) vectors are assumed to be strictly positive so that price (and quantity) ratios are well defined.**
- **Time does not permit me to list all 22 Tests that are listed in the paper but here are some of the 22 tests discussed in the paper:**

2.4 The Test Approach to Bilateral Index Number Theory

T3: Strong Identity or Constant Prices Test: $P(p,p,q^0,q^1) = 1$.

- That is, if the price of every good is identical during the two periods, then the price index should equal unity, no matter what the quantity vectors are. The controversial part of this test is that the two quantity vectors are allowed to be different in the above test.

T4: Fixed Basket or Constant Quantities Test:

$$P(p^0,p^1,q,q) = \sum_{i=1}^{N} p_i^1 q_i / \sum_{i=1}^{N} p_i^0 q_i = p^1 \cdot q / p^0 \cdot q.$$

- That is, if quantities are constant during the two periods so that $q^0 = q^1 \equiv q$, then the price index should equal the expenditure on the constant basket in period 1, $\sum_{i=1}^{N} p_i^1 q_i$, divided by the expenditure on the basket in period 0, $\sum_{i=1}^{N} p_i^0 q_i$.
- Recall the Fixed Basket Approach to bilateral index number theory: if $q^0 = q^1$, then the Laspeyres, Paasche and Lowe price indexes are all equal.

T5: Proportionality in Current Prices: $P(p^0,\lambda p^1,q^0,q^1) = \lambda P(p^0,p^1,q^0,q^1)$ for $\lambda > 0$.

2.4 The Test Approach to Bilateral Index Number Theory

T7: Invariance to Proportional Changes in Current Quantities:

$P(p^0,p^1,q^0,\lambda q^1) = P(p^0,p^1,q^0,q^1)$ for all $\lambda > 0$.

T9: Commodity Reversal Test (or Invariance to Changes in the Ordering of Commodities):

$$P(p^{0*},p^{1*},q^{0*},q^{1*}) = P(p^0,p^1,q^0,q^1)$$

T10: Invariance to Changes in the Units of Measurement (Commensurability Test):

$P(\alpha_1 p_1^0,\ldots,\alpha_N p_N^0;\ \alpha_1 p_1^1,\ldots,\alpha_N p_N^1;\ \alpha_1^{-1} q_1^0,\ldots,\alpha_N^{-1} q_N^0;\ \alpha_1^{-1} q_1^1,\ldots,\alpha_N^{-1} q_N^1) =$

$P(p_1^0,\ldots,p_N^0;\ p_1^1,\ldots,p_N^1;\ q_1^0,\ldots,q_N^0;\ q_1^1,\ldots,q_N^1)$ for all $\alpha_1 > 0, \ldots, \alpha_N > 0$.

T11: Time Reversal Test: $P(p^0,p^1,q^0,q^1) = 1/P(p^1,p^0,q^1,q^0)$.

T21: Factor Reversal Test (Functional Form Symmetry Test):

$$P(p^0,p^1,q^0,q^1)P(q^0,q^1,p^0,p^1) = p^1 \cdot q^1 / p^0 \cdot q^0 .$$

2.4 The Test Approach to Bilateral Index Number Theory

- **It turns out that the Fisher formula emerges as a "best" index from the viewpoint of the test approach to bilateral index number theory: it satisfies all 21 Tests listed in the paper.**
- **There is one additional test that has emerged in recent years as being very important:**

T22: *Circularity Test* (Transitivity over Time Test or Path Independence Test) :

$$P(p^0,p^2,q^0,q^2) = P(p^0,p^1,q^0,q^1)P(p^1,p^2,q^1,q^2).$$

- **The index number on the left hand side of T22, compares prices in period 2 directly with prices in period 0 and $P(p^0,p^2,q^0,q^2)$ is called the *fixed base price index* for period 2.**
- **The *chained price index* for period 2, $P(p^0,p^1,q^0,q^1)P(p^1,p^2,q^1,q^2)$, on the right hand side of T22 compares prices in period 2 with those in period 0 by first comparing prices in period 1 with those in period 0 and multiplies that index by the chain link index that compares prices in period 2 to those of period 1, $P(p^1,p^2,q^1,q^2)$.**

2.4 The Test Approach to Bilateral Index Number Theory

- **If the index number formula P satisfies the Circularity Test T22, then it does not matter whether we use the *chained index* (the right hand side of T22) to compare prices in period 2 with those of the base period 0 or if we use the *fixed base index* (the left hand side of T22): *we get the same answer either way*.**
- **Obviously, it would be very useful if we could find an index number formula that satisfied the Circularity Test and had satisfactory axiomatic properties with respect to the other tests that we have considered.**
- **It is straightforward to show that the Laspeyres, Paasche, Fisher and Törnqvist indexes do not pass the Circularity Test. The Jevons and Lowe indexes do pass this test.**

2.4 The Test Approach to Bilateral Index Number Theory

. In the paper, we have some new material on the interaction of the Identity, Commensurability and Circularity tests. We prove a result (basically due to Wolfgang Eichhorn) that proves to be important:

- Suppose that the bilateral index number formula, $P(p^1,p^2,q^1,q^2)$ satisfy the following tests: T1 (Positivity), T3 (Strong Identity) and T22 (Circularity). Then $P(p^1,p^2,q^1,q^2)$ must be equal to the following ratio where the function of prices p, $c(p) > 0$ for $p >> 0_N$:

(19) $P(p^1,p^2,q^1,q^2) = c(p^2)/c(p^1)$.

In summary, our candidate for "best" bilateral price index from the viewpoint of the test approach is the Fisher price index. However, a limitation on the above Test Approach to bilateral index number theory is that the above axioms assumed that all prices p_{tn} and q_{tn} were positive. The axiomatic approach to index number theory has in general not dealt adequately with the case where some prices are missing in one of the two periods being compared.

2.5 The Economic Approach to Index Number Theory

- **The economic approach to index number theory works as follows: suppose all purchasers of the N products in scope have the same preference or utility function, $f(q)$, where $q \equiv [q_1,\ldots,q_N]$ is a generic vector of aggregate purchases over a time period and $Q^t \equiv f(q^t)$ is the period t aggregate quantity that corresponds to the period t vector of purchases, q^t.**
- **Assume that $f(q)$ is linearly homogeneous, nondecreasing and concave function in q. It is further assumed that all purchasers of the N products face the same period t price vector p^t and they collectively choose q^t as a solution to the following period t utility maximization problem:**

(27) $\max_q \{f(q)\,;\, p^t\cdot q \le e^t\,;\, q \ge 0_N\} \equiv Q^t$

where e^t is observed period t aggregate expenditures on the N products.

- **The unit cost function that corresponds to the utility function $f(q)$, $c(p)$, is defined as the minimum cost of achieving the utility level 1. If purchasers face the period t vector of prices p^t, we have:**

(28) $c(p^t) \equiv \min_q \{p^t\cdot q : f(q) = 1;\, q \ge 0_N\} \equiv P^t.$

2.5 The Economic Approach to Index Number Theory

- Thus the **period t aggregate price level**, P^t, is equal to the **unit cost $c(p^t)$** and we have the following **decomposition of period t expenditure e^t into the product of P^t and Q^t**:

(29) $e^t = p^t \cdot q^t = c(p^t)f(q^t) = P^tQ^t$.

- Using this approach, we find that the bilateral price index that compares the prices of period t to the prices of period 0 is equal to $c(p^t)/c(p^0) = P^t/P^0$ (this is the **Konüs (1924) true cost of living index** between periods 0 and t when we have linearly homogeneous utility functions) and the **corresponding quantity index** is equal to $f(q^t)/f(q^0) = Q^t/Q^0$.
- Note that this Economic Approach to bilateral index number theory has led to the same type of price index that was defined by (19) and (24) in the previous section on the Test Approach to bilateral index number theory.

2.5 The Economic Approach to Index Number Theory

- **One way to regard index number theory is that index numbers can be used to *summarize information* in an efficient way.**
- **In the economic approach to index number theory, the 2N prices and quantities for period t, p^t and q^t, are reduced down to the aggregate period t price, $c(p^t)$ and the aggregate period t quantity, $f(q^t)$.**
- **Thus index number theory can be considered a part of *descriptive statistics*, which also tries to summarize information in an efficient manner.**
- **The practical question arises at this point: *how exactly are we to determine the functions* c(p) or f(q) in order to come up with an index number formula that National Statistical Offices could use? There are at least two methods that could be used:**
- ***Econometric methods* could be used to estimate c(p) or f(q). This approach is discussed below.**
- **The *theory of exact index number formulae* could be used.**

2.5 The Economic Approach to Index Number Theory

- The exact index number approach works as follows: we make certain specific assumptions about the functional form for either f(q) or c(p) and then we show under the assumption of collective utility maximizing behavior that $c(p^t)/c(p^0)$ is equal to a known bilateral index number formula.
- This theory was developed by Konüs and Byushgens (1926), Afriat (1972), Pollak (1971) (1989) and Diewert (1976). We do not have time to discuss this approach in the allotted time so refer to the paper for the details of this approach.
- The bottom line is that this approach to index number theory leads to the Fisher and Tornqvist-Theil indexes as being "best" indexes using this approach.
- An important limitation of the above 3 approaches to bilateral index number theory is that all three approaches implicitly or explicitly assume that all prices in the two periods being compared are positive. This assumption is far from being satisfied, particularly at the first stages of aggregation.
- Econometric methods can overcome this limitation as is discussed later in the paper.

2.7 Summary of Bilateral Index Number Theory and the Chain Drift Problem

- In sections 2.2-2.5, we outlined four main approaches to bilateral index number theory: the fixed basket approach, the stochastic approach, the test approach and the economic approach.
- These four approaches led to the Fisher index in section 2.2, the Törnqvist Theil index in section 2.3, the Fisher index in section 2.4 and the Fisher and Törnqvist Theil indexes in section 2.5.
- In section 2.5, it was noted that in situations where the fluctuations in prices and quantities were not large and there were no missing prices in the two periods being compared, then the Fisher and Törnqvist Theil indexes would give much the same answer.
- This is an encouraging result in that all approaches seem to lead to the same two bilateral indexes which "normally" give much the same answer.

2.7 Summary of Bilateral Index Number Theory and the Chain Drift Problem

- But the above result does not tell us how we should proceed over more than two periods. The problem of defining aggregate prices and quantities over multiple periods showed up in the test approach to index number theory.
- Test 22 (the Circularity Test) asked that the bilateral index number formula give the same answer in going from period 0 to period 2 directly or by going from 0 to 1 and then 1 to 2; i.e., we would like $P(p^0,p^2,q^0,q^2)$ to equal $P(p^0,p^1,q^0,q^1)P(p^1,p^2,q^1,q^2)$. However, both the Fisher and Törnqvist index fail this test. The failure of $P(p^0,p^2,q^0,q^2)$ to equal $P(p^0,p^1,q^0,q^1)P(p^1,p^2,q^1,q^2)$ is called the chain drift problem.
- The problem becomes severe when prices and quantities fluctuate substantially either due to discounted prices (sales) or seasonal factors and chained indexes are used. For a recent example of how big the chain drift problem can be using chained Fisher or Törnqvist indexes that utilize scanner data, see Fox, Levell and O'Connell (2025).
- A possible solution to solving the chain drift problem is to use Multilateral Indexes that satisfy the Circularity Test for all T periods in scope.
- In the following section, we will present some of the main multilateral methods that have been applied in recent times.

3. Multilateral Approaches

- In this section, we no longer assume that every product is present in each of the T periods under consideration. We assume that there is a total of N products that are present in at least one of the T periods.
- Denote the set of products that are present in period t as S(t). Thus if $n \in S(t)$, then $p_{tn} > 0$ and $q_{tn} > 0$; i.e., the observed price and quantity for product n in period t is positive. If $n \notin S(t)$, then we define $p_{tn} = 0$ and $q_{tn} = 0$. As usual, $p^t \equiv [p_{t1},\ldots,p_{tN}]$, $q^t \equiv [q_{t1},\ldots,q_{tN}]$ and $e^t \equiv p^t \cdot q^t \equiv \Sigma_{n=1}^{N} p_{tn}q_{tn}$ for $t = 1,\ldots,T$.

3.1 The GEKS Multilateral Price Indexes

- The multilateral GEKS method is due to Gini (1924) (1931) and was further developed by Eltetö and Köves (1964) and Szulc (1964). This method is based on constructing a set of Fisher index "star" indexes, where each observation acts as a base period in a complete set of bilateral comparisons. We then take the geometric mean of the N "star" indexes as our final price index.

3.1 The GEKS Multilateral Price Indexes

There are two problems with the GEKS multilateral method:

- **The underlying bilateral Laspeyres and Paasche indexes can only be calculated over matched products.**
- **Thus unmatched products in each bilateral comparison are simply ignored.**
- **The lack of matching problem can lead to very unreliable Laspeyres and Paasche indexes if the two periods being compared are widely separated but these unreliable comparisons get the same weight in the overall index as a more reliable bilateral comparison that has fewer unmatched prices.**

3.2 Geary Khamis Multilateral Indexes

- The **GK multilateral method** was introduced by Geary (1958) in the context of making international comparisons of prices. Khamis (1970) showed that the equations that define the method have a positive solution under certain conditions. A modification of this method has been adapted to the time series context and is being used to construct some components of the Dutch CPI; see Chessa (2016). The GK index was the multilateral index chosen by the Dutch to avoid the chain drift problem for the segments of their CPI that use scanner data.
- Recall that $S(t)$ was the set of products n that were purchased in period t. Define $S^*(n)$ as the set of periods t where product n was sold. Define the total sum vector q as the sum of the q^t:

(72) $q \equiv \Sigma_{t=1}{}^{T} q^t$.

- The equations which determine the *GK price levels* $P^1,\ldots, P^T$ and *quality adjustment factors* $\alpha_1,\ldots,\alpha_N$ (up to a scalar multiple) are the following ones:

3.2 Geary Khamis Multilateral Indexes

(73) $\alpha_n = \sum_{t\in S^*(n)} [q_{tn}/q_n][p_{tn}/P^t] = \sum_{n=1}^{N} [1/q_n][p_{tn}q_{tn}][1/P^t]$;

$n = 1,\ldots,N$;

(74) $P^t = p^t \cdot q^t/\alpha \cdot q^t = e^t/\alpha \cdot q^t$; $t = 1,\ldots,T$.

- where $\alpha \equiv [\alpha_1,\ldots,\alpha_N]$ is the vector of GK quality adjustment factors, $q_n \equiv \sum_{t=1}^{T} q_{tn} = \sum_{t\in S^*(n)} q_{tn}$ for $n = 1,\ldots,N$ and $e^t \equiv p^t \cdot q^t$ is period t expenditure on the N products.
- Once a solution α and $P^1{}_{,\ldots,}$ P^T to equations (73) and (74) has been found, the period t quantity levels Q^t are defined as follows:

(75) $Q^t \equiv \alpha \cdot q^t$; $t = 1,\ldots,T$.

Equations (75) tell us that the GK price and quantity levels are consistent with purchasers having linear preferences over the N products in scope.

3.2 Geary Khamis Multilateral Indexes

- The GK price and quantity levels are also consistent with purchasers having Leontief (no substitution preferences) but this case is empirically implausible (quantities purchased would vary in strict proportion over time and there would be no product churn).
- The GK multilateral method can be viewed as a (non-econometric) method for estimating linear preferences but it is not exactly the same as estimating linear preferences since the method simultaneously estimates two sets of parameters: the price levels P^t and the quality adjustment parameters α_n.
- The GK method generates two separate estimates for the period t price levels P^t: the direct ones using equations (74) and the indirect onex that uses equations (75) to generate the quantity levels Q^t and the indirect price levels are defined by $P^{t*} \equiv e^t/Q^t$.
- The following multilateral methods rely explicitly on econometric estimation.

4.1 Time Product Dummy (TPD) Hedonic Regressions

- **The unweighted Time Product Dummy (TDP) model dates back to Court (1939) and Summers (1973).**
- **These models are based on the price data satisfying (to some degree of approximation) the following equations:**

$$(78)\quad p_{tn} \approx P^t\alpha_n\ ; \qquad\qquad t = 1,\ldots,T;\ n \in S(t)$$

- **where P^t is interpreted as the period t price level and α_n is a parameter which reflects the quality (or marginal utility) of product n.**
- **Taking logarithms of both sides of the approximate equalities defined by (78) leads to the following approximate equalities:**

4.1 Time Product Dummy (TPD) Hedonic Regressions

(79) $\ln p_{tn} \approx \ln P^t + \ln \alpha_n$; $t = 1,\ldots,T$; $n \in S(t)$

$$= \rho_t + \beta_n$$

- where $\rho_t \equiv \ln P^t$ for $t = 1,\ldots,T$ and $\beta_n \equiv \ln \alpha_n$ for $n = 1,\ldots,N$. The second set of approximate equations in (79) is a linear regression model.
- However, the ρ_t and β_n parameters in (79) are not uniquely determined; we require a normalization on one of these parameters. Choose the following normalization:

(80) $\rho_1 = 0$ (which corresponds to $P^1 = 1$).

- The linear regression that corresponds to (79) and (80) is the following least squares minimization problem:

(81) $\min_{\rho\text{'s}\ \beta\text{'s}} \{\Sigma_{t=1}^{T} \Sigma_{n\in S(t)} [\ln p_{tn} - \rho_t - \beta_n]^2\}$

- where ρ_1 is set equal to 0 in the above minimization problem.

4.1 Time Product Dummy (TPD) Hedonic Regressions

- **Assume for the moment that the approximate equations (78) hold as exact equations.**
- **Multiply both sides of equation p_{tn} by q_{tn} for $n \in S(t)$ and sum the resulting equations over n. For each t, we obtain the following equations for $t = 1,\ldots,T$:**

(82) $\Sigma_{n\in S(t)}\, p_{tn}q_{tn} = p^t \cdot q^t \equiv e^t = P^t\, \Sigma_{n\in S(t)}\, \alpha_n q_{tn} = P^t \alpha \cdot q^t = P^t Q^t$

- **where $e^t \equiv \Sigma_{n\in S(t)}\, p_{tn}q_{tn} = p^t \cdot q^t$ is period t expenditure on the N products and the period t quantity aggregate is**

$$Q^t \equiv \alpha \cdot q^t \equiv \Sigma_{n\in S(t)}\, \alpha_n q_{tn} \qquad \text{for } t = 1,\ldots,T.$$

- **Thus if equations (78) or (79) hold as equalities rather than as approximate equalities, the Time Product Dummy Hedonic Regression model implicitly assumes that purchasers of the N products in scope have the same linear preferences $f(q) \equiv \alpha \cdot q$ over the N products.**
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4.1 Time Product Dummy (TPD) Hedonic Regressions

- Define the T price levels P^{t*} by definitions (83) and the N quality adjustment parameters $\alpha_n{}^*$ by definitions (84):

(83) $P^{t*} \equiv \exp[\rho_t{}^*]$; $t = 1,\ldots,T$;

(84) $\alpha_n{}^* \equiv \exp[\beta_n{}^*]$; $n = 1,\ldots,N$.

- There are **two ways to finalize estimates for the period t price and quantity levels**.

Method 1 chooses the P^{t*} defined by (83) and defines the corresponding quantity levels Q^{t*} as deflated expenditures:

(85) $Q^{t*} \equiv e^t/P^{t*}$; $t = 1,\ldots,T$.

Method 2 uses the $\alpha_n{}^*$ defined by (84) to define the period t quantity levels Q^{t**} as follows:

(86) $Q^{t**} \equiv \Sigma_{n=1}{}^N \alpha_n{}^* q_{tn}$; $t = 1,\ldots,T$.

- The corresponding period t price levels P^{t**} are defined residually as follows:

(87) $P^{t**} \equiv e^t/Q^{t**}$; $t = 1,\ldots,T$.

4.1 Time Product Dummy (TPD) Hedonic Regressions

- **If the fit in the regression defined by (81) is perfect, then the two sets of aggregate price and quantity levels will coincide.**
- **Using the properties of the least squares minimization problem defined by (81), it can be shown that P^{t*} and Q^{t*} are invariant to changes in the units of measurement as are P^{t**} and Q^{t**}.**
- **An advantage of the unweighted (or more properly, the equally weighted) TPD price indexes defined by (83) and (85) is that they can be constructed using just price information for the N products in scope.**
- **But there is a disadvantage associated with the use of the TPD hedonic regression model: products with low volumes of purchases should not get the same weight in the regression as highly popular products.**
- **The paper considers the corresponding Weighted Time Product Dummy Multilateral Method with missing observations but time does not permit us to develop the associated algebra here (see the paper for the algebra).**
- **It turns out that this model is consistent with both linear preferences and Cobb-Douglas preferences on the part of purchasers.**

4.1 Time Product Dummy (TPD) Hedonic Regressions

- The bottom line is that the GK and Weighted Time Product Dummy Multilateral Methods are consistent with purchasers maximizing a linear utility function and also consistent with maximizing a Leontief utility function for the GK model and consistent with maximizing a Cobb-Douglas utility function for the WTPD model.
- Both methods give us two ways of calculating the set of price and quantity levels generated by the respective model. This is similar to our results using the GK model.
- In the following section, we will use a different econometric specification in order to estimate linear preferences.

4.2 The Econometric Estimation of Linear Preferences

- **Sections 3.2 and 4.1 described two multilateral methods (the GK and WTPD indexes) that were consistent with consumers maximizing a linear preference function subject to a budget constraint.**
- **In this section, we again assumed that consumers of the N products in scope maximize a linear utility function but we will take an econometric approach that is based on the estimation of a system of direct demand functions of the type defined by equations (35) in section 2.5, $q^t = e^t\nabla_p c(p^t)/c(p^t)$, or a *system of inverse demand functions* defined by equations (39), $p^t = e^t\nabla_q f(q^t)/f(q^t)$.**
- **However, linear preferences do not lead to a differentiable unit cost function so we cannot use the system of direct demand functions (35) to estimate linear preferences.**
- **We can use the following system of indirect demand functions to estimate linear preferences.**

4.2 The Econometric Estimation of Linear Preferences

(97) $p_{tn} = e^t\alpha_n/\Sigma_{j\in S(t)}\, \alpha_j q_{tj} = e^t\alpha_n/\alpha\cdot q^t$; $t = 1,..,,,,T$; $n\in S(t)$

- where α is the vector $[\alpha_1,\ldots, \alpha_N]$ of quality adjustment parameters.
- If we added an error term to the right hand side of each equation in (97), we could attempt to estimate the α_n by solving the following nonlinear least squares minimization problem:

(98) $\min_{\alpha}\ \Sigma_{t=1}^{T}\, \Sigma_{n\in S(t)}\, \{p_{tn} - [\alpha_n/\alpha\cdot q^t]\}^2$.

- If $\alpha^* \equiv [\alpha_1^*,\ldots, \alpha_N^*]$ is a solution to (98), then it can be seen that $\lambda\alpha^*$ is also a solution to (98) where λ is any positive number.
- This non-uniqueness always occurs when we attempt to estimate utility functions or their dual unit cost functions.
- The scale of utility is arbitrary so we need to impose at least one normalization on the estimated parameters in order to obtain a cardinal measure of utility.

4.2 The Econometric Estimation of Linear Preferences

- **The simplest method of normalization is to set $\alpha_{n^*} = 1$ for some product n^* that is purchased in every period.**
- **There is another possible problem with the minimization problem defined by (98): it can be the case that there is no solution to (98). For example, suppose that there are only 2 periods and 2 products in scope. Suppose further that product 1 is only available in period 1 and product 2 is only available in period 2. In this case, there is no solution to (98). Thus we need a certain amount of product overlap in order to obtain a solution to (98).**
- **There is another more serious problem with (98): even if a unique solution to (98) and a normalization like $\alpha_{n^*} = 1$ is imposed, the resulting quantity levels, $Q^{t*} \equiv \alpha^* \cdot q^t$ for $t = 1,\ldots,T$, may not be invariant to changes in the units of measurement in the sense that the ratios Q^{t*}/Q^{1*} may change if the units of measurement are changed.**

4.2 The Econometric Estimation of Linear Preferences

- **Invariance to changes in the units of measurement is a fundamental requirement for national statistics.**
- **This problem can be cured if we convert the inverse demand system of estimating equations (97) into share equations. Thus multiply both sides of equation t,n for $t = 1,\ldots,T$ and $n \in S(t)$ in (97) by q_{tn} and divide both sides of the resulting equation by e^t. We obtain the following system of inverse demand share equations:**

(99) $s_{tn} = \alpha_n q_{tn} / \Sigma_{j\in S(t)}\, \alpha_j q_{tj} = \alpha_n q_{tn}/\alpha\cdot q^t\ ; \quad t = 1,.,,,,T\ ;\ n\in S(t).$

- **The corresponding nonlinear least squares minimization problem is:**

(100) $\min_{\alpha}\ \Sigma_{t=1}^{T}\, \Sigma_{n\in S(t)}\, \{s_{tn} - [\alpha_n q_{tn}/\alpha\cdot q^t]\}^2.$

-

4.2 The Econometric Estimation of Linear Preferences

- We require a normalization like $\alpha_{n^*} = 1$ as well as a certain amount of product overlap in order to get a unique solution to (100).
- Once the solution vector α^* has been calculated, as usual, define the quantity levels as $Q^{t*} \equiv \alpha^* \cdot q^t$ and the corresponding price levels as $P^{t*} \equiv e^t/\alpha^* \cdot q^t$ for $t = 1,\ldots,T$.
- If we want P^{1*} to equal 1, then multiply the vector α^* by $\lambda^1 \equiv e^1/\alpha^* \cdot q^1$ and define a new $\alpha^{**} \equiv \lambda^1 \alpha^*$.
- Define the new $Q^{t**} \equiv \alpha^{**} \cdot q^t$ and $P^{t**} \equiv e^t/\alpha^{**} \cdot q^t$ for $t = 1,\ldots,T$.
- The new price and quantity levels will be proportional to the corresponding initial price and quantity levels and the new price levels will have $P^{1**} = 1$.

- We have many different methods for estimating linear preferences. Which method is best?

4.3 The Estimation of CES Preferences

- **Our second example of the methodology explained in section 2.5 that uses inverse demand functions to estimate preferences is the case where the utility function is a CES (Constant Elasticity of Substitution) function, where f(q) is defined as follows in the case of N products:**

(101) $f(q) \equiv [\Sigma_{n=1}^{N} \alpha_n (q_n)^r]^{1/r}$

- **where the α_n are positive parameters and the parameter r satisfies the following inequalities:**

(102) $0 < r \leq 1.$

- **Note that if the parameter r equals 1, then the CES utility function defined by (101) becomes the linear utility function that was discussed in previous sections.**
- **The extension of the system of inverse demand equations (35) to the case where there are missing observations is the following system of estimating equations:**

(103) $p_{tn} = e^t[\partial f(q^t)/\partial q_n]/f(q^t)$; $t = 1,\ldots,T;\ n \in S(t).$

4.3 The Estimation of CES Preferences

- When the above equations do not hold exactly, the estimates which result from using the above equations to estimate the quantity levels $Q^{t*} = f(q^t,\alpha^*)$ for t = 1,…,T will not be invariant to changes in the units of measurement; i.e., the resulting bilateral indexes, Q^{t*}/Q^{1*} will in general change when the units of measurement are changed.
- As noted earlier, this is not an acceptable situation for producers of consumer price and quantity indexes.
- As in the previous section, the lack of invariance problem can be solved by moving to the system of inverse demand *share* equations.
- Thus multiply both sides of equation t,n in equations (103) by q_{tn}/e^t and we obtain the following system of estimating equations, with shares as the dependent variables instead of prices:

104) $s_{tn} = q_{tn}[\partial f(q^t)/\partial q_n]/f(q^t)$; $t = 1,\ldots,T;\ n \in S(t).$

- Add error terms (with means 0 and constant variances) to the right hand side of equations (104) and we obtain the following nonlinear least squares estimation problem:

4.3 The Estimation of CES Preferences

(105) $\min_{\alpha} \Sigma_{t=1}^{T} \Sigma_{n\in S(t)} \{s_{tn} - q_{tn}[\partial f(q^t)/\partial q_n]/f(q^t)\}^2$.

- Assume that $f(q)$ is the CES utility function defined by (101) and (102). Equations (104) become the following system of estimating equations:
- (106) $s_{tn} \equiv p_{tn}q_{tn}/e^t = q_{tn}[\partial f(q^t)/\partial q_n]/f(q^t) = \alpha_n (q_{tn})^r/\Sigma_{i\in S(t)} \alpha_i (q_{ti})^r$; $t = 1,\ldots,T$; $n\in S(t)$.
- The nonlinear least squares minimization problem for CES preferences using inverse demand share equations is defined by (108) where we also set $\alpha_{n^*} = 1$:

(108) $\min_{\alpha, r} \Sigma_{t=1}^{T} \Sigma_{n\in S(t)} \{s_{tn} - [\alpha_n(q_{tn})^r/\Sigma_{i=1}^{N} \alpha_i(q_{ti})^r]\}^2$.

- We require a normalization on the α_n like $\alpha_1 = 1$.

4.3 The Estimation of CES Preferences

- **It is also possible to estimate CES preferences by estimating the unit cost function c(p) that corresponds to the CES utility function defined by (101).**
- **The unit cost function c(p) that corresponds to the CES utility function f(q) defined (101) with the $\alpha_n > 0$ and $r < 1$ but $r \neq 0$ turns out to be the following function assuming that $p >> 0_N$:**

(110) $c(p) = [\Sigma_{n=1}^{N} \beta_n (p_n)^{\rho}]^{1/\rho}$

where the parameters β_n and ρ are defined in terms of the α_n and r as follows:

(111) $\beta_n \equiv (\alpha_n)^{1/(1-r)}$ **for** $n = 1,\ldots,N$ **and** $\rho \equiv - r/(1-r)$.

- **When $r = 1$, $\rho = -\infty$; when $r = 0$, $\rho = 0$; when $r = -\infty$, $\rho = 1$.**
- **Thus the CES unit cost function regularity conditions for ρ are consistent with the regularity conditions on r when we estimated the CES utility function.**

4.3 The Estimation of CES Preferences

- When there are missing prices, the estimating equations (35) become the following equations when $c(p^t)$ is defined by (115)

(116) $q_{tn} \equiv e^t\beta_n p_{tn}^{\rho-1}/\Sigma_{i\in S(t)} \beta_i(p_{ti})^{\rho}$; $t = 1,\ldots,T;\ n\in S(t)$.

- As usual, not all of the parameters β_n can be identified using the estimating equations (116) so in order to eliminate exact multicollinearity, we require a normalization such as $\beta_{n*} = 1$ for a product n^* that is present in all T periods.
- As was the case when estimating the CES utility function, using equations (116) to estimate the unknown β_n and ρ will lead to price and quantity levels that are not invariant to changes in the units of measurement. As in the previous section, the lack of invariance problem can be solved by moving to the system of *share* equations.

4.3 The Estimation of CES Preferences

- Multiply both sides of equation t,n in equations (116) for $n \in S(t)$ by p_{tn}/e^t and we obtain the following system of estimating equations, with shares as the dependent variables instead of quantities:

(117) $s_{tn} = \beta_n(p_{tn})^\rho / \Sigma_{i \in S(t)} \beta_i(p_{ti})^\rho$; $\quad t = 1,\ldots,T;\ n \in S(t)$.

- Add error terms (with means 0 and constant variances) to the right hand side of equations (117) and we obtain the following nonlinear least squares estimation problem:

(118) $\min_{\beta,\rho} \{\Sigma_{t=1}^T \Sigma_{n \in S(t)} [s_{tn} - \beta_n(p_{tn})^\rho / \Sigma_{i \in S(t)} \beta_i(p_{ti})^\rho]^2\}$.

- Note that the quantities q_{tn} are regarded as independent variables in the nonlinear regression (108) and the prices p_{tn} are the independent variables in the nonlinear regression (118).
- If there were no errors in (108) so that the data fitted the CES utility function exactly, then the data would fit the nonlinear regression defined by (118) exactly as well.
- However, when there are errors, the two methods of estimating CES preferences can give very different results.

4.3 The Estimation of CES Preferences

- **Some evidence on differences in estimates for the elasticity of substitution by estimating the CES utility function f(q) defined by (101) versus estimating the CES unit cost function c(p) defined by (110) can be found in the study on rice demand in Japan by Diewert, Abe, Tonogi and Shimizu (2025).**
- **In this study, the authors found that estimating the CES utility function led to an elasticity of substitution estimate equal to 23.09 whereas estimating the CES unit cost function led to an elasticity of substitution estimate equal to 5.04. The dependent variables in both nonlinear regressions were expenditure shares.**
- **The regression R^2 for the f(q) regression was 0.9967 (with a sum of absolute errors between actual shares and predicted shares equal to 7.26) whereas the c(p) R^2 was only 0.7080 (with a sum of absolute errors equal to 71.3).**
- **Using the same data source and estimation methodology that was used for Rice in Japan was used by the present authors for Japanese data on Cereals, Spaghetti and Cheese and led to the results listed in Table 3 which is listed on the next slide.**

Table 3: Table of Alternative Estimates for the Elasticity of Substitution for Some Japanese Products

Product	f(q) or c(p)	**sigma**	R^2	Sum of Absolute Errors
Rice	**f(q)**	**23.09**	**0.9967**	**7.26**
Rice	**c(p)**	**5.04**	**0.708**	71.3
Cereal	**f(q)**	**21.59**	**0.9982**	**5.97**
Cereal	c(p)	**2.18**	**0.7811**	**69.15**
Spaghetti	**f(q)**	**9.97**	**0.9969**	**7.68**
Spaghetti	**c(p)**	**5.11**	**0.7523**	**69.7**
Cheese	**f(q)**	**18.57**	**0.997**	**6.12**
Cheese	**c(p**)	2.82	0.7106	67.44

4.3 The Estimation of CES Preferences

- The above Table shows that the Rice results are not an isolated result. Estimating a CES unit cost function leads to a lower estimate of σ and a very poor fit to the data compared to estimating a CES utility function using the same data.
- Since most applications of the CES Feenstra (1994) methodology for measuring the gains from new products are based on estimates for σ that estimate a CES unit cost function, it is likely that these estimated gains are overstated since a higher σ estimate means a lower gain from additional product variety. The above results suggest that researchers should look at estimating a CES utility function rather than estimating the dual unit cost function when it is likely that the products in scope are strong substitutes.
- As noted above, a problem with the CES functional form is that there is only one parameter, the elasticity of substitution σ, that is used to describe substitution possibilities between every pair of products. In the following section, we describe a functional form for the purchaser's utility function that has a separate parameter to describe substitution possibilities for each product. Moreover, the reservation prices that are generated by the KBF functional form are finite.

4.4 The Estimation of KBF Preferences

- Konüs and Byushgens (1926) introduced the following functional form for a linearly homogeneous utility function:

(119) $f(q) \equiv (q \cdot Aq)^{1/2} = (\Sigma_{i=1}^{N} \Sigma_{j=1}^{N} a_{ij}q_iq_j)^{1/2}$; $a_{ij} = a_{ji}$; $1 \leq i \leq j \leq N$.

- Thus A is an N by N symmetric matrix that contains (N+1)N/2 unknown a_{ij} parameters. The matrix A satisfies certain restrictions which are spelled out in Diewert (1976) and Diewert and Hill (2010). Konüs and Byushgens and Diewert showed that this utility function is exact for the Fisher (1922) Ideal quantity and price indexes so we call preferences defined by (119) KBF preferences. Note that there is no problem in defining f(q) if some q_n are equal to 0.

- In most empirical applications where there are a large number of products in scope, it is not possible to estimate all (N+1)N/2 unknown parameters a_{ij} in the KBF utility function defined by (119). In order to reduce the number of parameters in the A matrix, we define A as the following matrix which has rank 2:

(120) $A \equiv \alpha\alpha^T - \beta\beta^T$

- where the transposes of the column vectors α and β are defined as $\alpha^T \equiv [\alpha_1,\ldots,\alpha_N]$ and $\beta^T \equiv [\beta_1,\ldots,\beta_N]$.

4.4 The Estimation of KBF Preferences

- **With A defined by (120), the system of inverse demand *share* equations (104) becomes the following system of estimating equations:**

(121) $s_{tn} = q_{tn}[\alpha_n \alpha \cdot q^t - \beta_n \beta \cdot q^t]/[(\alpha \cdot q^t)^2 - (\beta \cdot q^t)^2] + e_{tn}$;

$$t = 1,\ldots,T \; ; \;\; n = 1,\ldots N.$$

- **Equations (121) are valid even when there are missing products because when product n is missing in period t, $s_{tn} = q_{tn} = 0$.**
- **The utility function as a function of the unknown parameters α, β and the consumption vector q is defined as follows:**

(122) $f(q,\alpha,\beta) \equiv [q^T(\alpha\alpha^T - \beta\beta^T)q]^{1/2} = [(\alpha \cdot q^t)^2 - (\beta \cdot q^t)^2]^{1/2}$.

- **Note that if $\beta = 0_N$, then $f(q,\alpha,0_N) = [(\alpha \cdot q^t)^2]^{1/2} = \alpha \cdot q^t = \Sigma_{n=1}^N \alpha_n q_n$; i.e., the utility function collapses down to the linear utility function that was studied in earlier sections.**

4.4 The Estimation of KBF Preferences

- **This is an important point because it implies that starting coefficients α_n and β_n for the nonlinear least squares minimization problem that is defined below can be set equal to the estimates of the α_n^* that result in the estimation of linear preferences with the starting coefficients for the β_n^* set equal to 0.**
- **There are some tricky aspects to the new utility function as compared to the case of a linear utility function: we need to impose some restrictions on the vectors α and β in order to prevent exact collinearity. See the paper for details.**
- **We have had some success in estimating this functional form; see Diewert, Abe, Tonogi and Shimizu (2025). However, we need additional studies using this functional form in order for us to recommend its use to National Statistics Offices as a multilateral method.**

4.5 Summary on the Use of Multilateral Indexes

- **In the introduction to this paper, we mentioned two Challenges in Measuring Consumer Price Change:**

- **The chain drift problem (or lack of transitivity problem) and**
- **The lack of matching problem (caused by product churn).**

- **Following others, we suggested that using multilateral index numbers could be used to solve the chain drift problem.**
- **We looked at seven different multilateral index number models in the previous sections: GEKS, GK, WTPD, LP (the Linear Preferences Model using inverse demand functions), CESq (CES preferences estimated using the CES utility function f(q)), CESp (CES preferences estimated using the CES unit cost function c(p)) and the KBF utility function model with a rank one substitution matrix.**
- **All of these multilateral indexes produce transitive price and quantity levels and these methods generate price and quantity indexes that are invariant to changes in the units of measurement. Thus there is no chain drift problem using these multilateral indexes over periods 1 to T.**

4.5 Summary on the Use of Multilateral Indexes

- **The GK, WTPD and LP indexes are consistent with purchasers maximizing a linear utility function. The elasticity of substitution between every pair of products for these models is equal to $+\infty$, which is problematic. However, many National Statistical Offices have used GK or WTPD in constructing indexes at the first stage of aggregation where it is known that the products in scope are highly substitutable. An advantage of WTPD and LP is that one can see how well these methods fit the observed data. These methods can work well.**

- **GEKS, CESq, CESp and KBF are consistent with more flexible preferences but the elasticity of substitution for the two CES models is *constant* for all pairs of products (which is also problematic).**

4.5 Summary on the Use of Multilateral Indexes

- **GEKS is consistent with the most flexible preferences and so it is a preferred method if product turnover is low.**
- **However, in situations where there is a lot of product churn, GEKS can be unsatisfactory because it is constructed using bilateral Fisher indexes between all possible pairs of time periods using only the set of products that are present in both periods being compared.**
- **The "true" bilateral Fisher index between two periods where some products are unmatched should be based on Laspeyres and Paasche bilateral indexes that use reservation prices for the unmatched products in order to be consistent with the economic approach to index number theory.**
- **However, the GEKS indexes are constructed using only matched prices; i.e., reservation prices are not estimated.**
- **The KBF model gets around the lack of product matching and reservation prices for missing products are generated as a byproduct of the estimation method. However, the estimated preferences are not fully flexible. But the preferences that are estimated are more flexible than all other models with the exception of GEKS.**

4.5 Summary on the Use of Multilateral Indexes

- **Tentative conclusions about the merits of the seven methods for transitive index construction at the first stage of aggregation are as follows:**

(i) Use GEKS (or CCDI) if product turnover is low so that price matching is high.

(ii) Use WTPD or LP if product turnover is high and it is thought that the products in scope are highly substitutable.

(iii) If product turnover is high and it is not known if the products in scope are highly substitutable, use CESq (direct CES utility function estimation) or KBF. KBF is more flexible but is fairly complicated and difficult to explain to the public.

(iv) The use of CESp is not recommended since (limited) experience has shown that estimating a CES unit cost function leads to a model that does not fit the data as well as the comparable CESq model.

- **But the above conclusions are only tentative. In particular, the use of econometric methods for the KBF indexes is somewhat risky. Many econometricians would not agree that our stochastic specification is satisfactory. Other econometricians may object to the estimation of inverse demand functions. There can be difficulties with choosing alternative normalizations on the α and β vectors. Moreover, it would be difficult for national statistical offices to explain the KBF indexes to the public. More experience with estimating KBF preferences is required.**

5. Twelve Other Challenges in Measuring Consumer Price Change.

5.1. The Extension Problem

- How exactly are the data for period T+1 (p^{T+1} and q^{T+1}) to be linked to the indexes already constructed for periods 1 to T? This is the Extension Problem. The paper summarizes the methods that have been suggested in the literature to solve this problem.

5.2. Integration of Different Methods

- How should statistical agencies integrate portions of their CPI that *use very different methods* to construct different parts of their Consumer Price Indexes?
- At the first stage of aggregation, some statistical offices use multilateral methods as described above to construct parts of their CPI; e.g., see the Australian Bureau of Statistics (2016).

5.2. Integration of Different Methods

- **Other parts of the CPI are constructed using traditional price collection methods where price collectors collect a sample of prices from retail outlets or they collect prices from the internet.**
- **To make matters more complicated, most national CPIs use a Lowe index to aggregate the lower level price indexes at higher levels of aggregation. These Lowe indexes typically use a national annual basket from a prior year to form the Lowe quantity basket q and this basket is combined with the monthly subaggregate price indexes that have been constructed using entirely different methodologies.**
- **The annual basket weights come from Household Consumer Expenditure Surveys. The end result is an aggregate Consumer Price Index that has no clear methodological justification.**
- **There is a particular methodological problem with the treatment of strongly seasonal products in a Lowe index: for some months of the year, there will be no price quotes for such products when they are out of season but somehow, monthly prices for these products have to be imputed in order to calculate the Lowe index. For a more detailed discussion of this particular problem, see Chapter 9 in IMF (2025).**

5.3 How should Price Indexes for Owner Occupied Housing be Constructed?

- **The problem is this: how exactly should we construct a monthly (imputed) price for the services of a housing unit that is owned by the occupant of the property? There are at least six different approaches to this measurement problem:**
- ***The Acquisitions Approach*. In this approach, we impute a zero price for a housing property that was not purchased during the current period. If a new housing structure is purchased during the reference month, we charge the entire purchase price of the property (or just the value of the structure) as the price for the new structure. This approach of course does not measure the services of other housing properties that are owned but not newly constructed in the reference month. This approach seemingly avoids imputations but it does not measure the flow of services that the owner of a "used" structure gets from the property.**
- ***The Rental Equivalence Approach*. Using this approach, the monthly price for the services of an owned housing unit is the fair market rent for the unit. This approach runs into troubles when there are no "similar" housing units that are rented so that "objective" measures of the market rent for high end housing units in particular may be difficult to obtain.**

5.3 How should Price Indexes for Owner Occupied Housing be Constructed?

- ***The User Cost Approach*. This approach is a hypothetical financial opportunity cost approach. The price is what it would cost to purchase the property at the beginning of the month and then sell it at the end of the month. Suppose that the beginning of the period value of the property is V^0 and the end of the period value is V^1. The property owner forgoes the beginning of the period value of the property so if the owner's opportunity cost of capital is the monthly interest rate is r, then the total initial cost (including interest foregone) is $V^0(1+r)$. The benefit at the end of the month is V^1 so the net opportunity cost of living in the dwelling instead of investing the funds is the (end of period) *user cost value* equal to $V^0(1+r) – V^1$.**
- **Decomposing this value into constant quality price and quantity components involves some additional assumptions about the form of depreciation of the structure; see Jorgenson and Griliches (1967), Christensen and Jorgenson (1969) and Diewert (1974b) for more details on this approach.**
- **There are additional complications due to the fact that the housing unit consists of separate land and structure components and the structure component depreciates while the land component does not. For a more complete discussion of these complications and references to the literature, see Chapter 10 in IMF (2025). We mention that Jorgenson and his coauthors have argued in favour of the user cost approach in the CPI to the valuation of *all* durable goods that are purchased by households.**

5.3 How should Price Indexes for Owner Occupied Housing be Constructed?

- ***The Opportunity Cost Approach*. This approach suggests that the rent foregone and the financial opportunity cost of tying up capital in an owner occupied housing unit are *both* opportunity costs of living in the house for the reference month and so the "true" opportunity cost of living in ones home is the maximum of the rental foregone and the user cost.**
- ***The Payments Approach*. This approach to pricing the services of an owner occupied dwelling unit is a kind of cash flow approach: it simply adds up the costs that are associated with living in the owned unit. These cash costs include mortgage interest on any loans that were made to purchase the unit and property taxes on the unit. This approach ignores imputed costs of ownership like depreciation on the structure and possible imputed benefits such as possible capital gains on the property that might have occurred over the reference period. However, cash flow is an important accounting concept and a measure of it can serve some purposes.**

5.3 How should Price Indexes for Owner Occupied Housing be Constructed?

- ***The Deprival Value Approach*. The above five approaches to valuing the services of an owner occupied dwelling unit are well known but this sixth approach is perhaps a new approach. This approach is based on the amount of money it would take to deprive the owner(s) of the services of the owned dwelling unit for the accounting period.**
- **The formal methodology that justifies this approach is exactly the same as the methodology that was developed to measure the money it would take to compensate a user of the services of a free product (like the use of the internet) for the loss (or deprival) of these services for a reference period.**
- **Conceptually this approach is appealing. A problem with the rental equivalence approach is that an owner of a highly valuable property may find that the market rent for the property is much lower than the owner's valuation of the benefit of living in the property. It is the latter value that measures the utility derived from the use of the housing unit for the reference period.**
- **The practical problem with this approach is the difficulty in obtaining objective measures of the deprival value.**

5.4 What is the Right Price of Household Time?

- **The economic approach to the construction of a CPI is based on the assumption of households maximizing a utility function subject to a budget constraint. But consumers typically do not derive utility from *purchases* of goods and services; they need to combine their purchases with inputs of *time* using the services of the purchased products to derive overall utility.**
- **Overall utility is not just a function of the vector of purchases q; it is a function of q and a vector of times spent on various household leisure and work activities.**
- **Households maximize utility subject to both a budget constraint and a time constraint.**
- **The pioneering work on integrating the time constraint with the budget constraint is Becker (1965). For additional work on Becker's theory of the allocation of time and household production, see Pollak and Wachter (1975), Hill (2009) and Schreyer and Diewert (2014).**

5.4 What is the Right Price of Household Time?

- **Becker (1965) used household market wage rates to value household time. However, if all members of the household are retired or unemployed; then there is no market opportunity wage rate to value household time.**
- **Furthermore, even if there are market wage rates of household members to value time, it is not clear whether this opportunity cost wage rate should be used to value household work.**
- **Instead one could use market wage rates for hiring workers to do the various types of household work.**
- **These issues are discussed more fully in Schreyer and Diewert (2014) and in Diewert, Fox and Schreyer (2017).**
- **A related issue is whether household production should be included in a Consumer Price Index. This problem has been with us for a long time but it has assumed new importance with households installing solar panels and supplying electricity not only to themselves, but also to the transmission company that supplies electricity at peak times. How exactly should the purchase of solar panels be treated?**

5.5 How can we Measure the Preferences of Individual Households?

- **The problem with individual household data is that many purchases are infrequent. This leads to a big lack of matching problem.**
- **However, for some recent progress on this problem of sparse data for individual households, see Crawford (2025).**
- **A related problem is: how should we aggregate over individual household CPIs to produce an overall Consumer Price Index?**
- **This leads us into the problems associated with the construction of indexes of Social Welfare. For an introduction to this literature, see Chapter 5 in IMF (2025).**

5.6 Unique Products

- **In different periods, different products are produced and purchased. This prevents the routine matching of prices. Most of the present paper attempts to deal with this problem but the methods that were suggested do not deal adequately with a service or product that changes in its quality every period so there is absolutely no matching of prices.**
- **An example of this extreme lack of matching occurs when we attempt to construct residential property price indexes. Each property has a unique location but also the property has a unique structure which does not have the same quality over time; i.e., the structure depreciates. In this case, the matched product approaches that we discussed above cannot be applied; we need to apply hedonic regressions with *characteristics of the product* as explanatory variables rather than the product itself as an explanatory variable as in the Time Product Dummy regression approach described above.**
- **For explanations of this Time Product Characteristics approach and references to the literature, see Chapters 7 and 8 in IMF (2025).**

5.7 Complex Products

- Many service products are very complicated; e.g., telephone and internet access service plans. It is probably necessary to use Time Product Characteristics hedonic regressions in this context.

5.8 Bundled Products and Discounts for Volume Purchases

- At times, two or more products are bundled together and offered at a discounted price as compared to the total price of the products if they were sold separately. See the paper for some thoughts on how to deal with this problem.

5.9 How to Value the Household Production of Environmental "Bads"

- **Households produce undesirable outputs (such as air pollution) as a byproduct of their household product of their consumption of various goods. If governments put a price on the production of household "bads", then this price and the associated quantity of the undesirable output could be treated as just another price and quantity that enter a Consumer Price Index.**
- **But this treatment of undesirable outputs seems counterintuitive: an increase in the "consumption" of a bad should not increase utility; it should decrease utility.**
- **A practical way of dealing with "bads" in a CPI has not been worked out to our knowledge.**
- **For an introduction to the treatment of this issue in the production accounts, see the controversy between Pittman (1983) and Färe, Grosskopf, Lovell and Pasurka (1989).**
- **For a more general discussion of how to integrate the environment into the economic accounts, see Fleurbaey and Blanchet (2013).**

5.10 Heavily Subsidized Products

- **In the limit, subsidized products can be supplied to consumers free of (explicit) charges.**
- **Is zero the "right" price for this type of product? The answer is no. We mentioned above the pioneering deprival value methodology that could be used to obtain prices for free products.**
- **For free educational and medical services provided to households by governments, market prices for comparable privately supplied services could be used to value these free services.**

5.11 Financial Products

- **What is the "correct" price of a household's monetary deposits?**
- **This question has not yet been resolved in a definitive manner. For various approaches to the treatment of financial services, see Barnett (1978) (1980), Feenstra (1986), Fixler and Zieschang (1991), Hill (1996b), Diewert (2014) and Diewert, Fixler and Zieschang (2016) and the references in these papers.**

5.12 Products Involving Risk and Uncertainty

- **What is the correct pricing concept for gambling and insurance expenditures?**
- **This question has assumed new importance with climate change leading to larger and larger insurance claims on property damage including residential housing.**
- **The treatment of insurance in a CPI is not at all settled. The gross insurance approach regards property insurance payments as the correct value for insurance in a CPI while the net insurance approach subtracts claims from gross insurance and regards the resulting value as the correct one.**
- **There is also controversy on how to treat changes in risk: does a change affect the price of insurance or the real quantity of insurance?**
- **The academic literature on the utility of insurance and gambling payments is not easy to explain to the public and so National Statistical Offices have tended to use very simple approaches to the treatment of these expenditures in a CPI. There is a need for better approaches in this area.**

Final Thoughts

- **Although the measurement challenges discussed in this paper arise in many national CPIs, their relative importance differs considerably across countries.**
- **For example, Statistics Canada continues to face long-standing difficulties with the treatment of owner-occupied housing and the integration of scanner data with traditional survey-based collection.**
- **Japan experiences exceptionally high rates of product churn in digital devices and household electronics, which makes price matching and quality adjustment particularly problematic.**
- **In the United States, major challenges include the extensive use of hedonic methods for consumer durables and the reliance on rental equivalence for measuring housing services.**
- **Australia has pioneered the implementation of multilateral index methods such as GEKS–Tornqvist, yet still confronts practical issues in real-time updating and the linking of rolling-window indexes.**
- **Many European statistical offices are rapidly expanding their use of web-scraped and big-data sources, generating new types of missing-price and classification problems. These country-specific examples illustrate how CPI theory and practice are shaped by local data environments and institutional settings, reinforcing the need for context-sensitive research and methodological development.**

It can be seen that there are many challenges that need to be addressed.

For references to the literature, see the paper!

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