A Behavioural Cost-of-Living Index

Hubert Wu
University of Oxford

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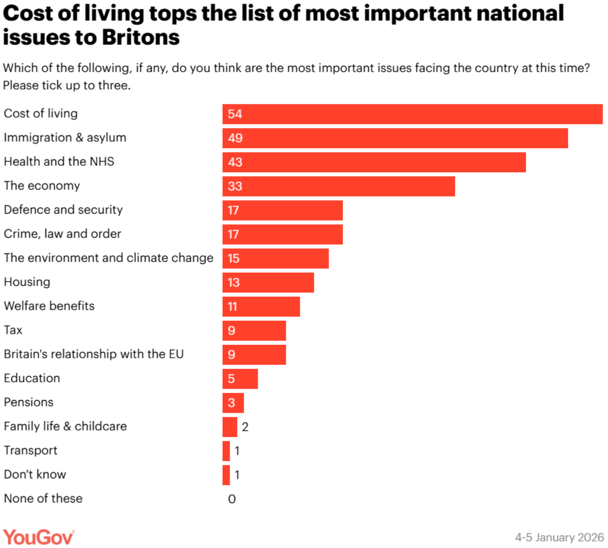
Motivation

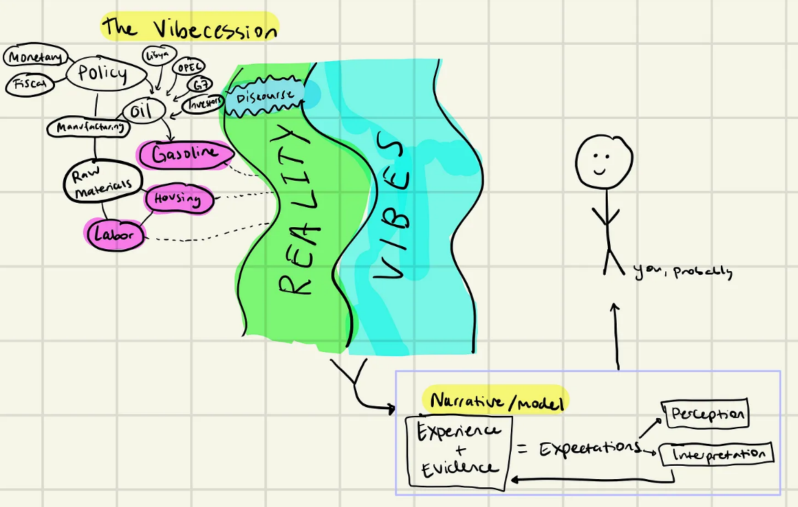


Source: YouGov, “Britons and the cost of living, January 2026”, 2026.

Motivation

- The cost of living (COLI) is a fundamental object in economics
  - → Formalises measurement of consumer welfare across time and space
  - → Provides theoretical justification for modern price and quantity indices
  - → A core object in contemporary public policy around the world

Motivation



Source: Kyla Scanlon, "The Vibecession: The Self-Fulfilling Prophecy", 2022.

Motivation

Image source: The Poke

Motivation

- The COLI's validity (and that of indices based on it) requires rational consumers to correctly solve their utility maximisation problems
- Although a hallmark of the economic approach, this assumption is nonetheless challenged by a large volume of evidence from behavioural economics
- **But remarkably little remains known about how to measure the COLI in the presence of behavioural distortions**

Focus of this paper

- I study the cost of living when it is subject to bounded rationality and imperfect optimisation on the part of consumers
  - → **Measurement concept**: a behavioural analogue to the classical Konüs (1939) cost-of-living index and associated index number formulae
  - → **Implementation**: welfare-corrected versions of superlative inflation statistics and the Distributional CPI of Jaravel (2026)

Selected related literature

- **Index number theory and the cost of living:** Konüs (1939); Pollak (1971); Diewert (1976); Hill (2006)
- **Salience, limited attention, and rational inattention:** Gabaix (2014); Sims (2003); Bracha and Tang (2024); Dietrich (2024)
- **Revealed preference theory:** Afriat (1967); Varian (1982); Manser and McDonald (1988)

Notation

- **Time**: $t \in \{1, \ldots, T\}$
- **Goods**: $k \in \{1, \ldots, K\}$
- **Prices and quantities**: $\mathbf{p}_t \equiv (p_{t,1}, \ldots, p_{t,K}) \in \mathbb{R}^K_+$; $\mathbf{q}_t \equiv (q_{t,1}, \ldots, q_{t,K}) \in \mathbb{R}^K_+$

Classical index number bounds

The main object I work with is the Konüs (1939) cost-of-living index

$$C(\mathbf{p}_i, \mathbf{p}_j; u) \;\equiv\; \frac{e(\mathbf{p}_j, u)}{e(\mathbf{p}_i, u)}. \tag{1}$$

where $e(\mathbf{p}, u) \equiv \min_{\mathbf{q}}\{\mathbf{p} \cdot \mathbf{q} : u(\mathbf{q}) \geq u\}$ is the expenditure function.

Classical index number bounds

Recall that with $u = u_i$, (1) is bounded from above by the Laspeyres price index and from below by the minimum price relative.

Proposition 1 (Pollak, 1971)

$$\min_k \left\{ \frac{p_{j,k}}{p_{i,k}} \right\} \leq C(\mathbf{p}_i, \mathbf{p}_j; u_i) \leq \frac{\mathbf{p}_j \cdot \mathbf{q}_i}{\mathbf{p}_i \cdot \mathbf{q}_i} \tag{2}$$

These bounds result from substitution (e.g. consumers can shift their spending toward relatively cheaper goods with the minimum price relative being the logical extreme of this process).

Bounded rationality

- I relax the rationality assumption by introducing optimisation frictions into the consumer's problem in the form of limited attention.
- The basic idea is that consumers may not fully internalise price variation between periods or levels within them
- So behaviour need not solve

$$\max_{\mathbf{q}} u(\mathbf{q}) \text{ s.t. } \mathbf{p}.\mathbf{q} \leq m$$

Bounded rationality

- Many established ways of modelling this idea (e.g. see the rational inattention literature pioneered by Sims (2003));
- I work in the first instance with the sparsity framework introduced in the behavioural literature from Gabaix (2014)
- Sparsity implies that many prices receive zero or attenuated attention

Bounded rationality

Classical consumer's problem:

$$\mathbf{q} \in \arg\max_{\mathbf{q}} \; u(\mathbf{q}) \quad \text{s.t. } \mathbf{p} \cdot \mathbf{q} \leq m$$

The sparse consumer's problem:

$$\mathrm{smax} \;\equiv\; \arg\max \circ \mathcal{A} \tag{3}$$

where the attention operator $\mathcal{A}$ maps $\mathbf{p}$ into perceived prices $\mathbf{p}^s$.

Optimisation procedure under smax**:**

1. Choose the attention weights $\mathbf{m} \in [0,1]^K$.
2. Optimise given perceived prices (with $p^d_{i,k}$ being a 'default' price)

$$p^s_{i,k} = m_{i,k} p_{i,k} + (1 - m_{i,k}) p^d_{i,k} \tag{4}$$

under the true budget constraint.

The attention operator $\mathcal{A}$

- There are many choices of $\mathcal{A}$
- E.g. a popular microfoundation in the rational attention literature is the LASSO-like procedure

$$\mathbf{m}^* = \arg\min_{\mathbf{m}} \frac{1}{2}(1-\mathbf{m})'\Lambda_{kk}(1-\mathbf{m}) + \kappa \sum_k |m_k|,$$

$$\Lambda_{kk} \propto \sigma^2_{p_k} |\epsilon_k| \lambda^d p^d_k q^d_k$$

  which yields m^*_k that increases in budget shares, price volatility, and price elasticity
- **My focus is on studying (and measuring) the COLI itself given almost any choice of $\mathcal{A}$**

The behavioural COLI

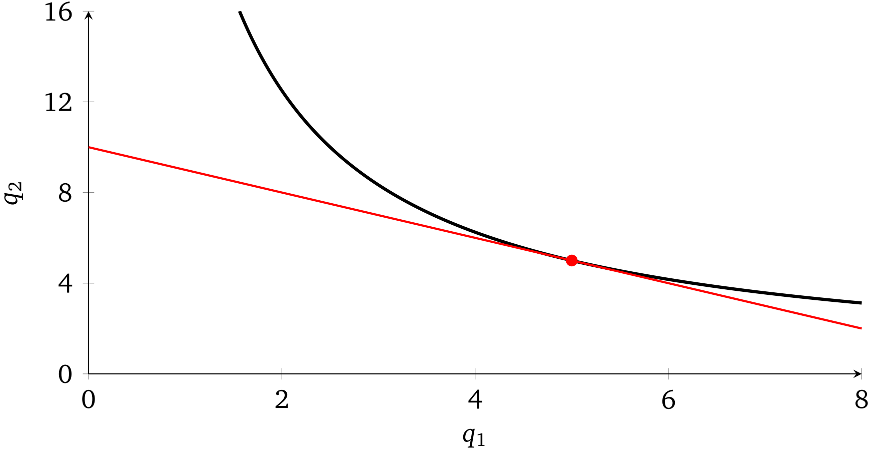
Define the sparse (or 'behavioural') COLI from (1) with sparse expenditure functions $e(\mathbf{p}, \mathbf{m}, u)$ resulting from the consumer's problem solved under smax:

$$C^s(\mathbf{p}_i, \mathbf{p}_j, \mathbf{m}_i, \mathbf{m}_j, u) \equiv \frac{e^s(\mathbf{p}_j, \mathbf{m}_j, u)}{e^s(\mathbf{p}_i, \mathbf{m}_i, u)} \tag{5}$$

Note:

- for ease of exposition default prices $\mathbf{p}^d$ are fixed
- $\mathbf{m}_i = \mathbf{m}_j = \mathbf{1}$ nests the classical COLI
- two choice variables $(\mathbf{q}, \mathbf{m})$ and a distorted state variable $\mathbf{p}$
- rational behaviour occurs only conditional on attention

Intuition



$u(q_1, q_2) = q_1 q_2 \quad u_0 = 25 \quad \mathbf{p}_0 = (1,1),\ y_0 = 10,\ \mathbf{q}_0 = (5,5)$

$\mathbf{p}_1 = (4,1),\ \mathbf{q}_1 = (2.5, 10),\ e(\mathbf{p}_1, u_0) = 20$

$\mathbf{p}_s = (2,1),\ \mathbf{m} = (\frac{1}{2}, 1),\ \mathbf{q}_2^s = (3.536, 7.071)$

$e^s(\mathbf{p}_1, \mathbf{m}, u^0) = \mathbf{p}_1 \cdot \mathbf{q}^s = 21.215$

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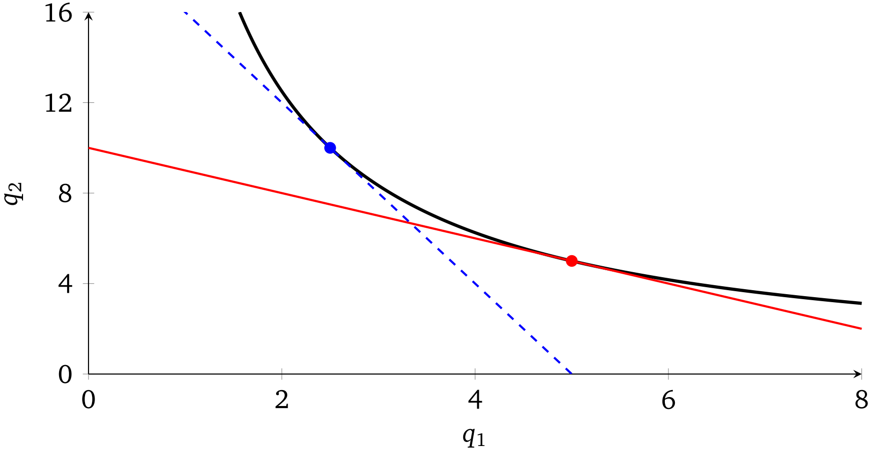

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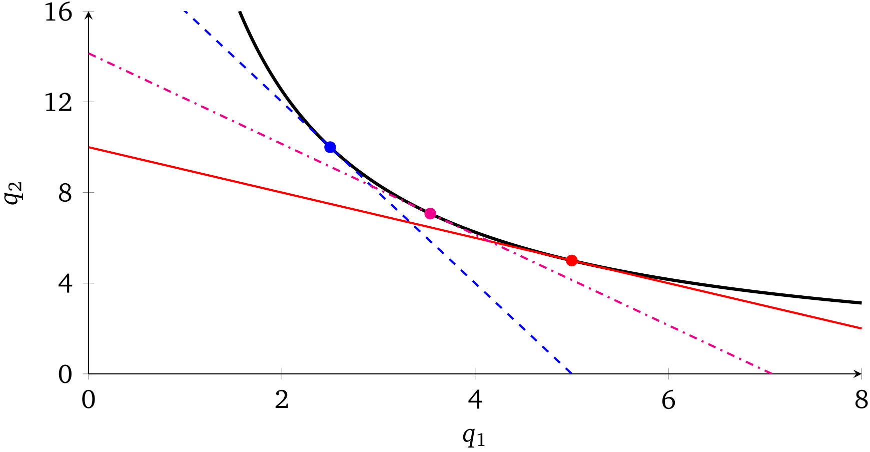

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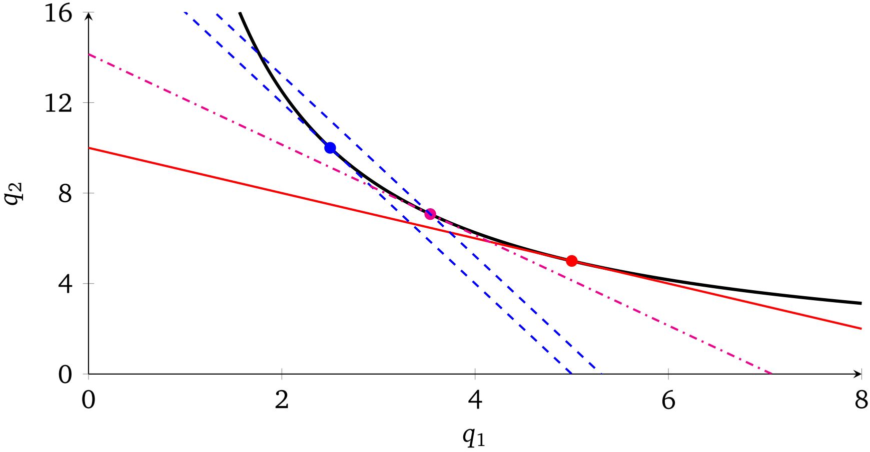

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Some easy properties of the behavioural COLI

Fact 1 (sparsity premium)

$$C(\mathbf{p}, \mathbf{p}^s, u) \geq 1 \tag{6}$$

For any common expenditure reference, the cost of living is always higher under limited attention vs. without.

Fact 2 (Laspeyres inattention limit)

Suppose $\mathbf{p}_0^s = \mathbf{p}_0$, $\mathbf{m}_1$ is a common scalar $\mu \in [0, 1]$ for all goods, and that $\mathbf{p}_2^d = \mathbf{p}_1$. Then

$$C(\mathbf{p}_i, \mathbf{p}_j, u_i) \leq C^s(\mathbf{p}_i, \mathbf{p}_j, \mathbf{m}_i, \mathbf{m}_j, u_i) \leq \frac{\mathbf{p}_j . \mathbf{q}_i}{\mathbf{p}_i . \mathbf{q}_i} \tag{7}$$

The Laspeyres price index is more prominent under uniform inattention increases.

Remark: prevailing substitution bias estimates

- A long-standing finding in the index number literature is that substitution bias from Laspeyres-type price indices is modest, e.g.
  - $\rightarrow$ **Hill (2006)**: 0.15–0.25 ppts annually
  - $\rightarrow$ **BLS (1996)**: 0.20 ppts
  - $\rightarrow$ **Manser and McDonald (1988)**: 0.18 ppts
- Fact 2 provides a new rationale for this through a behavioural channel: **bounded rationality**
- Nonetheless requires strong assumptions: for this to hold, must rule out factors including heterogeneous attention, other default rules, basket composition changes, etc.

Behavioural index number bounds

One of the most important discoveries of microeconomic theory (and a fact underlying countless national accounts datasets) is that computable, non-parametric index formulae bound the COLI.

For example, under non-homothetic preferences we have seen that

$$\min_k \left\{ \frac{p_{j,k}}{p_{i,k}} \right\} \leq C(\mathbf{p}_i, \mathbf{p}_j; u_i) \leq \frac{\mathbf{p}_j.\mathbf{q}_i}{\mathbf{p}_i.\mathbf{q}_i}$$

Behavioural index number bounds

Let $\mathcal{P}_i^s$ be the observations revealed preferred to i under perceived prices in the sense of Varian (1982), and $\mathcal{W}_i^s$ those revealed worse than i:

$$\mathcal{P}_i^s = \{t : t \succeq_s i\}, \qquad \mathcal{W}_i^s = \{t : i \succeq_s t\}.$$

Then define

$$\underline{e}_i^s(\mathbf{p}) := \inf_{\mathbf{q} \geq 0} \{\mathbf{p} \cdot \mathbf{q} : \mathbf{p}_t^s \cdot \mathbf{q} \geq \mathbf{p}_t^s \cdot \mathbf{q}_t \; \forall t \in \mathcal{W}_i^s\},$$

and

$$\overline{e}_i^s(\mathbf{p}) := \min_{t \in \mathcal{P}_i^s} \mathbf{p} \cdot \mathbf{q}_t.$$

For the behavioural COLI, the relevant upper bounding envelope is

$$\overline{e}_{i,\mu}^{s,\mathrm{beh}}(\mathbf{p}_j) := \min_{t \in \mathcal{P}_i^s} \left[\mathbf{p}_j \cdot \mathbf{q}_t + \frac{1-\mu}{\mu}(\mathbf{p}_i \cdot \mathbf{q}_t - \mathbf{p}_i \cdot \mathbf{q}_i)\right].$$

Behavioural index number bounds

Proposition 2

Under the sparse analogue of the Generalised Axiom of Revealed Preference, $\mathbf{p}_i^s = \mathbf{p}_i$, and $\mathbf{p}_j^s = (1-\mu)\mathbf{p}_i + \mu\mathbf{p}_j$ with $\mu \in (0,1]$,

$$\min_k \left\{ \frac{p_{j,k}}{p_{i,k}} \right\} \leq \frac{e_i^s(\mathbf{p}_j)}{\mathbf{p}_i \cdot \mathbf{q}_i} \leq C_{ij}(u_i) \leq C_{ij}^s(u_i) \leq \frac{\overline{e}_{i,\mu}^{s,\mathrm{beh}}(\mathbf{p}_j)}{\mathbf{p}_i \cdot \mathbf{q}_i} \leq \frac{\mathbf{p}_j \cdot \mathbf{q}_i}{\mathbf{p}_i \cdot \mathbf{q}_i}. \quad (8)$$

Behavioural index number bounds

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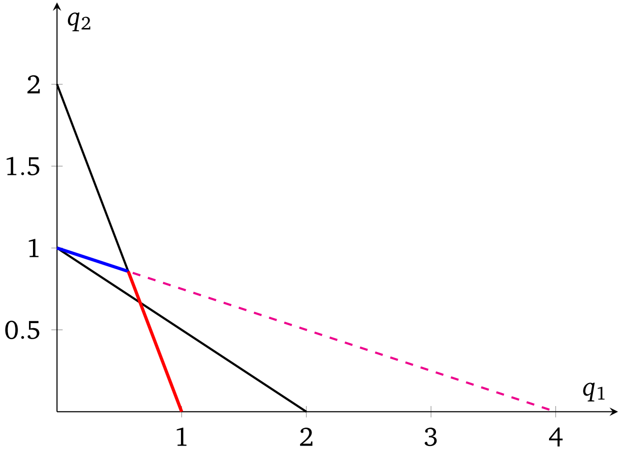
Empirical content of the behavioural COLI



Identification challenges

- To compute a behavioural COLI one needs knowledge of (or assumptions about) $\mathbf{m}$ and $\mathbf{p}^d$
- Not necessarily straightforward
  1. Attention is not typically observed like prices and demands
  2. Similarly, there are many forms a ‘default’ price could take
- Without these objects any observed data can be rationalised as ‘behavioural’ and the theory cannot be falsified

Adjusting the PCE-PI

- Attention weights for 11 Personal Consumption Expenditures (PCE) categories estimated by Dietrich (2024) from Federal Reserve Bank of Cleveland data
  - $\rightarrow$ Current best estimates in the literature
  - $\rightarrow$ Attention assumed to be time-invariant
- Quarterly prices and expenditures from the US Bureau of Economic Analysis's data archive (2020 Q4-2021 Q3)
- Default prices: trailing 12-month averages
- Construct a **chained Törnqvist index satisfying Proposition 2** inputting perceived prices i.e.

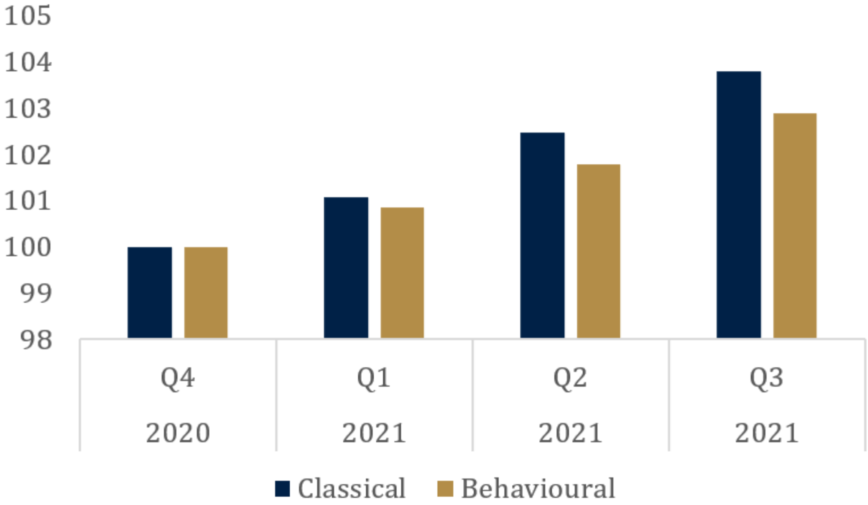
$$T_{ij}(\mathbf{p}_i^s, \mathbf{p}_j^s) = \prod_{k=1}^{K} \left(\frac{p_{j,k}^s}{p_{i,k}^s} \right)^{\frac{1}{2}\left(s_{i,k}+s_{j,k}\right)} . \tag{9}$$

Attention weights

Category	m_k
Motor vehicles	0.428
Recreational goods	0.571
Other durable goods	0.35
Food and beverages	0.757
Gasoline and other energy	0.88
Other nondurable goods	0.878
Housing and utilities	0.481
Health care	0.152
Transportation services	0
Food services	0
Other services	0.487

Notes: Estimates are from model (6) in Table 1 of Dietrich (2024), which includes demographic fixed effects and expenditure-weight controls. Attention weights for transportation services and food services are set to zero as the corresponding estimates are statistically insignificant.

Behavioural vs the classical COLI (2020–2021)



Experienced inflation vs. true welfare loss

- consumers *experience* lower inflation due to inattention-based attenuation
- but welfare loss is higher than perceived because
  1. prices rise more than believed
  2. failure to substitute reduces real income
- inattention creates a gap between experienced inflation and true welfare through two channels: **distorted price perceptions** and **suboptimal substitution**

Inequality with bounded rationality

- the **Distributional CPI (D-CPI)** of Jaravel (2026) decomposes the CPI by socio-economic groups
- introducing sparsity implies a correction term,

$$\pi^B_{g,t} - \pi^D_{g,t} \;=\; (1-m)\sum_k s_{g,k,t-1}(\Delta \ln p^d_{k,t} - \Delta \ln p_{k,t})$$

- explore this correction w a grid over $m \in \{0, 0.25, 0.50, 0.75, 1.0\}$ and three default rules

Rule	Definition
lag1: previous month	$p^d_{k,t} = p_{k,t-1}$
lag12: year-on-year month	$p^d_{k,t} = p_{k,t-12}$
ma12: 12-mo trailing average	$p^d_{k,t} = \frac{1}{12}\sum_{j=1}^{12} p_{k,t-j}$

Welfare inequality by income quintile

Group	Classical (%)	Behavioural (%)	Bias (pp)
Q1 (Lowest)	18.11	17.42	−0.69
Q2	17.67	16.92	−0.75
Q3	17.51	17.11	−0.40
Q4	16.98	16.67	−0.32
Q5 (Highest)	16.14	16.05	−0.09
Q1−Q5 gap: π^D			1.97 pp
Q1−Q5 gap: π^B			1.37 pp
Compression due to attention			−0.60 pp (−30%)

Table: Cumulative COLIs by income quintile (2020-2023, `lag12`, $m = 0.5$)

Welfare inequality over time

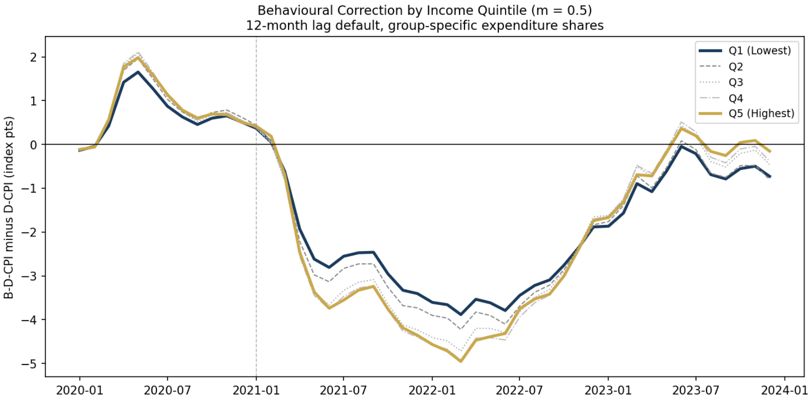


Figure: Bias ($\pi^B - \pi^D$, pp) by income quintile (2020-23, `lag12`, $m = 0.5$.)

Sensitivity to varying (in)attention

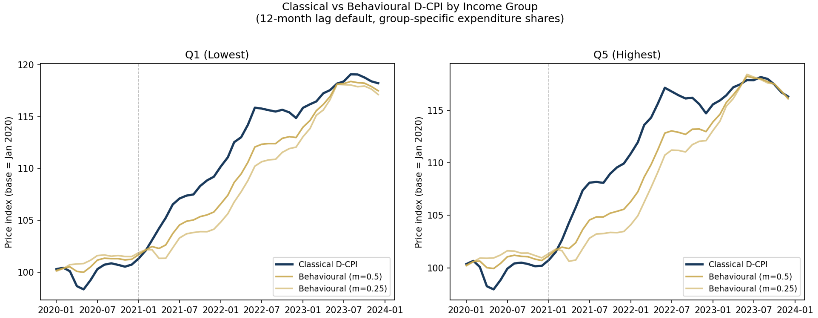


Figure: Welfare bias for $m = \{0.25, 0.5\}$; Q1 and Q5

Grid values

m	Classical (%)	Behavioural (%)			Bias (pp)		
		lag12	ma12	lag1	lag12	ma12	lag1
0.00	20.55	18.24	20.05	20.61	−2.32	−0.50	+0.05
0.25	20.55	18.83	20.19	20.60	−1.73	−0.37	+0.05
0.50	20.55	19.41	20.31	20.59	−1.15	−0.24	+0.04
0.75	20.55	19.99	20.44	20.58	−0.57	−0.12	+0.02
1.00	20.55	20.55	20.55	20.55	0.00	0.00	0.00

Aggregate COLIs in the D-CPI sample, 2020–2023.

Thank you!