Volume measures for non-market education output

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Insee
Mesurer pour comprendre

Context

$\rightarrow$ In accordance with the ESA (2010), the National Accounts Department (INSEE) currently uses an **output method** to estimate the volume of non-market education:

- Uses a **non-monetary variable**, related to the service, as a volume indicator (number of students)
- Volume $\approx$ number of students, with each education level aggregated by weighting them according to their expenditure per student

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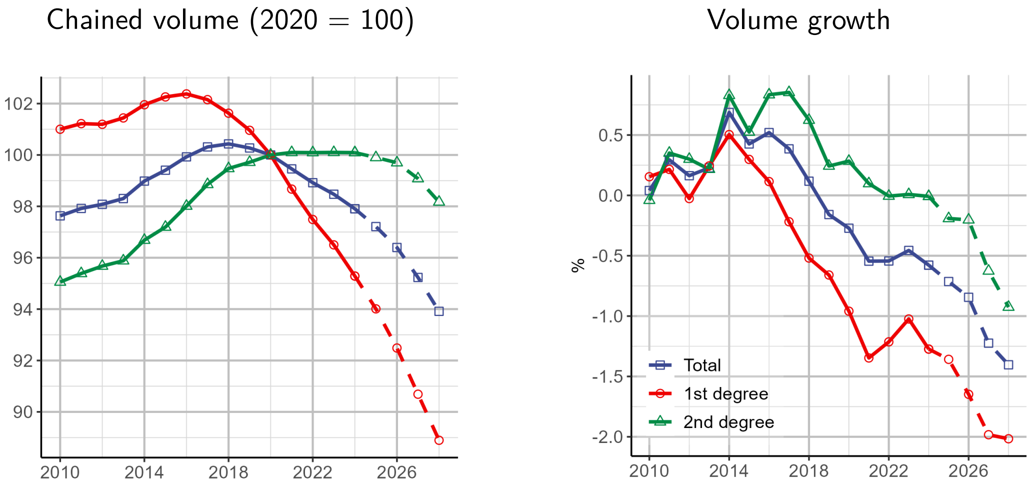
- Volume ≈ number of students, with each education level aggregated by weighting them according to their expenditure per student

Two main challenges:

- The volume indicator mainly reflects **demographic changes**
- The volume indicator does not account for changes in the **quality** of education provided

Motivation

Figure: Forecasts of education output volume for first and second degree education



Objectives

- Reports from national statistical institutes and academic studies explore various approaches to quality adjustments for non-market services (Fraumeni, 2008; Scheyer,2010; Statcan,2012; ONS, 2025)

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  - **Test different quality adjustments within an economically interpretable theoretical framework based on the concept of human capital**
- Quality is fundamentally a multidimensionnal and abstract concept, which can be proxied by :
  - **Input characteristics**: level of teacher experience
  - **Service characteristics**: class size
  - **Outcomes** of the service: PISA test scores, school climate.
- DCN and DEPP data, higher education **not included**
- **Quality adjustment prohibited by EU conventions** (ESA) for comparability reasons: **exploratory work**

Outline

From Value to Volume: The Non-Market Challenge

Non-market services (education, health...) are valued at cost in national accounts:

$$\text{Value}^t = C^t(y^t, w^t) = \sum_i X_i(y_i^t, w^t) \cdot w^t$$

Three approaches to measure *volume*:

Input measure

$$Q^t_{\text{input}} = \frac{C^t}{P^t_{\text{input}}}$$

Simple, but productivity is constant by construction

Eurostat: category C

Output measure

$$Q^t_{\text{output}} = \frac{\sum_i y_i^t \, c_i^{t-1}}{\sum_i y_i^{t-1} \, c_i^{t-1}}$$

Quantity y_i weighted by lagged unit costs

Eurostat: recommended

Outcome measure

$H = H(y, \Omega)$

Captures consumer welfare, but affected by external factors $\Omega \Rightarrow$ prohibited

Quality Adjustment: Conceptual Foundations

Problem: quantity y_i (e.g. pupil-hours) may miss quality attributes

Implicit quality adjustment

Stratify products into homogeneous groups i.

Quality $\approx$ *cost difference across strata*

Limit: cannot capture cost-neutral quality changes (e.g. teaching practices).

Explicit quality adjustment *(this paper)*

Introduce indicator q_i directly in the index:

$$v_i(y_i, q_i) = y_i\, f_i(q_i)$$

Challenge: how to calibrate f_i?

Requires a theoretical foundation.

Key choice

What reduction in class size exactly offsets a 1% fall in enrolment in terms of volume?
$\Rightarrow$ Pinned down by the **elasticity** of f_i.

Calibrating Quality Weights via Outcome Elasticity

Framework: utility $U = U(H)$, outcome $H = H(y, q, \Omega)$.
Volume measure $v_i = y_i f_i(q_i)$ should replicate the substitutability between y and q in terms of outcome:

$$\underbrace{\frac{f_i'(q_i)\, q_i}{f_i(q_i)}}_{\text{elasticity of } f_i} = \alpha = \underbrace{\frac{\partial H/\partial q_i}{\partial H/\partial y_i} \frac{q_i}{y_i}}_{\text{outcome substitution elasticity}}$$

Application to education — outcome = human capital $H^t = y^t W^t(q^t, \Omega)$:

$$\alpha = \frac{\partial W}{\partial q_i} \frac{q_i}{W} \quad \Longrightarrow \quad \text{elasticity of } \textit{future wages} \text{ to quality indicator } q_i$$

Calibrating Quality Weights via Outcome Elasticity

First-order quality contribution:

$$\text{contrib}^t_{\text{quality},i} \approx \frac{C_i^{t-1}}{C^{t-1}} \times \alpha \times \frac{\Delta q_i^t}{q_i^{t-1}}$$

cost share $\times$ wage elasticity $\times$ quality growth

Common functional forms for f

Form	$f(q)$
Constant elasticity	q^α
Linear	$1 + \gamma q$
Semi-elasticity	$e^{\gamma q}$

Quality Indicators: Completeness vs. Robustness

Quality-adjusted output index:

$$Q^t_{\text{adj}} = \sum_i \frac{C_i^{t-1}}{C^{t-1}} \, \frac{y_i^t \, f_i(q_i^t)}{y_i^{t-1} \, f_i(q_i^{t-1})}$$

Two desirable properties of a quality indicator $q = g(X, \Omega)$:

Completeness

Share of outcome variation explained by q:

$$\frac{\partial H}{\partial q} \frac{\Delta q}{\Delta H} \text{ large}$$

Example: standardized test scores have large effects on future wages.

Robustness

q should not be driven by external factors Ω:

$$\frac{\partial q}{\partial \Omega} d\Omega \ll dq$$

Example: class size is fully controlled by the school system.

If indicators are independent, adjustment functions can be **combined multiplicatively**.

Bonus : Productivity Decomposition (1)

Output price index decomposes into input prices and technology *(Schreyer, 2012)*:

$$\underbrace{\frac{C^t(y^t, w^t)}{C^{t-1}(y^t, w^{t-1})}}_{P^t_{\text{output}}} = \underbrace{\frac{C^t(y^t, w^t)}{C^t(y^t, w^{t-1})}}_{\approx P^t_{\text{input}} \times sub^t} \times \underbrace{\frac{C^t(y^t, w^{t-1})}{C^{t-1}(y^t, w^{t-1})}}_{= A^{t-1}/A^t}$$

The term $C^t(y^t, w^{t-1})$ is **unobservable** $\Rightarrow$ we recover the residual apparent factor productivity:

$$\frac{\text{AFP}^t}{\text{AFP}^{t-1}} = \frac{Q^t_{\text{output}}}{Q^t_{\text{input}}} = \frac{P^t_{\text{input}}}{P^t_{\text{output}}} = \underbrace{\frac{A^t}{A^{t-1}}}_{\text{TFP growth}} \times \underbrace{\frac{1}{sub^t}}_{\leq 1}$$

Bonus : Productivity Decomposition (2)

Value growth decomposition:

$$\frac{\text{Value}^t}{\text{Value}^{t-1}} = \underbrace{Q^t_{\text{output}}}_{\text{volume}} \times \underbrace{P^t_{\text{input}}}_{\text{input prices}} \times \underbrace{\left(\frac{\text{AFP}^t}{\text{AFP}^{t-1}}\right)^{-1}}_{\text{inverse productivity}}$$

Interpretation

Higher productivity **reduces** value growth (same output costs less).

AFP $\leq$ TFP because $sub^t \geq 1$: substituting inputs lowers cost, so part of the productivity gain is absorbed by substitution and not reflected in AFP.

Leontief special case

If inputs are non-substitutable (reasonable for education where labor dominates):

$$sub^t = 1 \Rightarrow \text{AFP} = \text{TFP}$$

Outline

Implicit quality adjustment (1)

Context: rapid growth in disability inclusion

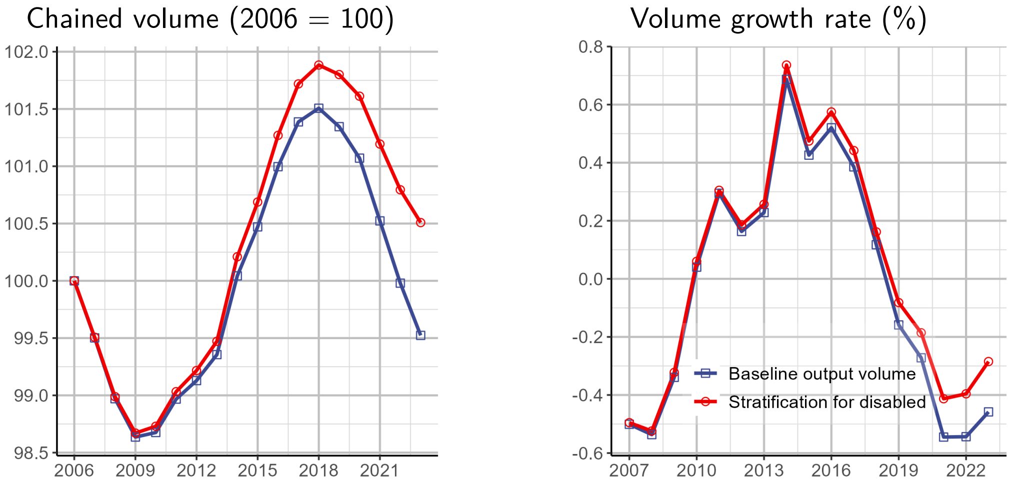
- 2005 law tripled enrolment: 151k $\rightarrow$ 496k pupils (2024)
- Cost per disabled pupil: **+22%** above average in 2006, **+74%** in 2026
- Most new pupils integrated into **regular classes** — not a separate stratum

Solution: finer stratification

- Split each education level into **disabled** / **non-disabled** sub-strata
- Reallocate headcounts & costs using budget data (Programme 230, action 3)
- Compliant with Eurostat conventions

Implicit quality adjustment (2)

Figure: Volume of education (first and second degree)



Explicit quality adjustment with PISA scores (1)

- We model the quality-adjusted volume of education as $v(y_i^t, q_i^t) = y_i^t e^{\gamma q_i^t}$ where y_i^t denotes the number of students, q_i^t is a measure of students' performance, and $\gamma \geq 0$ is a parameter;

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$$\begin{aligned}
&\text{weight of adjusted strata } (\approx 28\% \text{ of total cost}) \\
&\times \text{ change in } q (\approx \pm 2 \text{ points/year} \approx \pm 0.02 \text{ standard deviations}) \\
&\times \text{ semi-elasticity } (9.5\% \text{ wage gain per standard deviation}) \\
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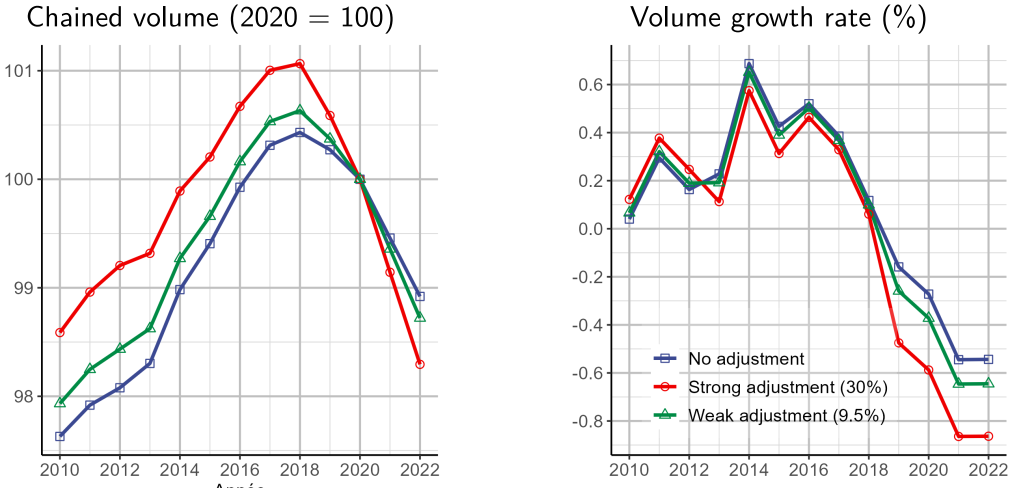
- Earlier levels are also adjusted with a **cohort allocation model**;

Explicit quality adjustment with PISA scores (2)

Figure: Volume of education production with a quality adjustment for PISA scores



Explicit Quality Adjustment with Class Size (1)

Trends in France (2016–2024)

- Pre-primary: -10% average class size
- Primary: -12% — driven by declining birth rates and priority education policies (REP/REP+)
- Secondary: **stable** across all grade levels

What the literature says

Level	Finding
Primary	Consistent positive effect: $+1.5$ to $+3\%$ s.d. per pupil fewer (Krueger, Piketty, Fredriksson)
Primary (heterog.)	Effect **twice larger** for disadvantaged pupils; saturation for large classes
Lower secondary	Small positive effect, roughly **half** that of primary
Long run	Mixed; **no robust effect on wages**

Explicit Quality Adjustment with Class Size (2)

Framework: Fully-Taught Pupils (FTP)

$$FTP(q) = q \cdot f(q)$$

- $f(q)$: share of fully-taught pupils at class size q
- No adjustment: $f(q) = 1 \Rightarrow FTP = q$
- FTP assumed **concave** (saturation effect)

Baseline: $f^{HC}(q) = q^{-0.04}$, calibrated on Fredriksson et al.).

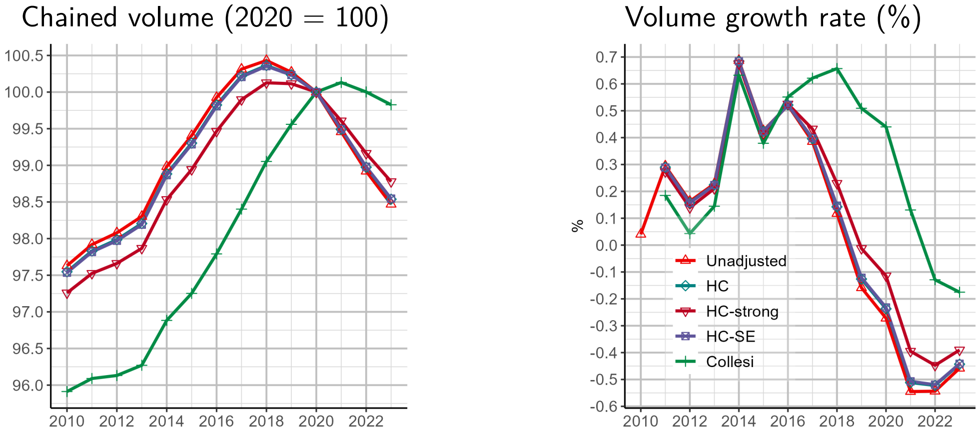
The effect on the annual volume growth rate is approximately:

$$\underbrace{\text{weight of adj. levels}}_{\approx 40\%} \times \underbrace{\Delta q}_{\approx \pm 1\%/\text{yr}} \times \underbrace{\text{wage elasticity}}_{0.04}$$

$$= \pm 0.02 \text{ ppts}$$

Explicit Quality Adjustment with Class Size (3)

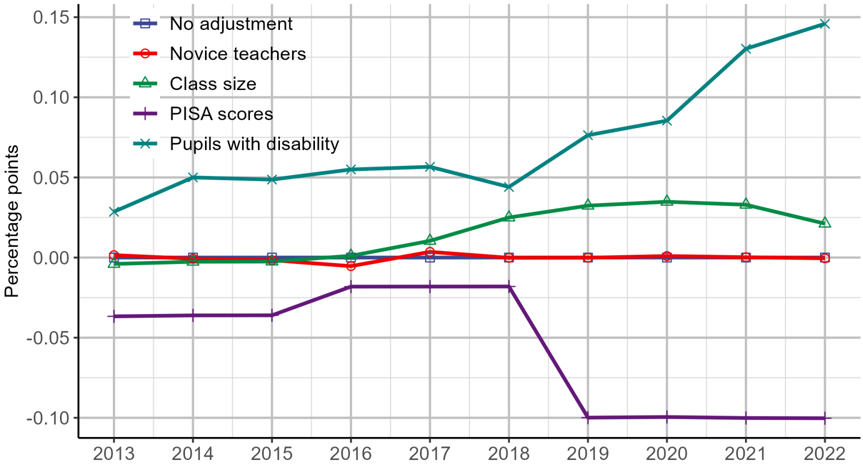
Figure: Volume of education production quality-adjusted for class size



Coverage: First and second degree education.

Summary of tested quality adjustments

Figure: Difference between the growth rates of adjusted and unadjusted volume



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  - Identifiable decomposition + link with output index $\Longrightarrow$ Fouliard (2026)